

Robust H_∞ switching MIMO control for a plasma time-varying parameter model with a variable structure in a tokamak

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Abstract: The paper deals with design and simulation of hierarchical multi-loops feedback magnetic plasma control system on the base of experimental data for the operating Globus-M spherical tokamak in line with the special methodology developed. The components of the system are designed to achieve high-performance operation during short pulses of plasma discharges about 200 ms. The plasma equilibrium reconstruction algorithm with regularization in terms of moving filaments without iterations was developed. The LPV plant model was derived from reconstructed plasma equilibriums as a distributed parameter system with time-varying parameters. A switching plasma shape multivariable controller was designed by the H_∞ robust loop-shaping approach for magnetic field at X-point and poloidal fluxes on plasma separatrix (isoflux control). The novelty of the plasma shape control system is the application of robust switching control with a controller state vector matching. The feedback control was applied to the whole discharge with the transition from a limiter phase to a diverter phase when the plasma changed its magnetic topology as a controlled structure.

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1. INTRODUCTION

Modern operating tokamaks with vertically elongated plasma (Wesson, 1997) for instance DIII-D (US), JET (UK), ASDEX Upgrade (Germany), TCV (Switzerland), EAST (China), KSTAR (South Korea) have plasma position, current, and shape control systems (Ariola et al., 2008) of different types and there is no any general standard on magnetic plasma control systems so far. Moreover, the known systems are not so fast-acting in comparison with plasma position stabilization system of spherical Globus-M tokamak (Ioffe Institute, S-Petersburg, Russia) (Gusev et al., 2013) on the base of high-speed current inverters (Kuznetsov et al., 2005). Therefore, the paper treats a *special methodology* of a plasma magnetic cascade multiloop control system design and analysis on the tokamak Globus-M. The methodology covers plasma equilibrium reconstruction with the usage of the magnetic measurements outside plasma, linear plasma models arrays derivation relative to the varying plasma equilibrium reconstructed for creation of the Linear Parameter Varying (LPV) plasma models, design and usage of the plasma reconstruction code in the feedback on the base of the experimental scenario, design H_∞ robust switching MIMO controller for plasma shape and application of the controller to a whole plasma discharge with variable plasma structure when transition from the limiter to the diverter phase.

We directed our efforts to design multivariable plasma shape control system to use it in real time with maximum speed of response. This is because of short Globus-M

plasma pulses of about 200 ms. So plasma equilibrium reconstruction algorithm was designed without iterations using moving filaments that gave a chance to control plasma separatrix location by action on magnetic field at X-point and poloidal fluxes on the separatrix. The original approach suggested by the authors, which uses linear models and experimental data allows the synthesis of controllers without using bulky plasma-physical codes like TSC, CORSICA (US) or DINA, PET (RF). The reconstruction and control algorithms designed were embedded into the new Globus-M multi-loop hierarchy structure of the magnetic plasma control system and were simulated around experimental scenario.

Section 2 gives the plasma magnetic control problem statement for the Globus-M tokamak. In section 3 the plasma equilibrium reconstruction algorithm is described, verified, and applied to the plasma discharge data by the realized moving filaments numerical code. Section 4 shows obtaining mathematical and numerical LPV models. Section 5 describes plasma position, current, and shape control subsystems. In Section 6 we propose the robust switching plasma shape controller and give the results of the control system simulations with LPV models. Conclusions underline the reasons of the high performance of the plasma shape control system presented.

2. PLASMA CONTROL PROBLEM STATEMENT

2.1 Plasma Current and Shape Control in Globus-M Tokamak

There are two SISO fast control closed-loops for plasma vertical and horizon position stabilization on the Globus-M tokamak (Gusev et al., 2013). In these loops, current invertors in self-oscillations modes with frequency of about 3 kHz (Kuznetsov et al., 2005) are used as actuators together with analog PID-controllers. Six SISO feedback systems for currents control in the Poloidal Field (PF) coils with multiphase thyristor rectifiers as actuators and analog P-controllers form the inner MIMO cascade of the whole hierarchical (Kartsev et al., 2017) multivariable control system (Fig. 1). The model of multi-phase thyristor rectifier is designed and investigated in (Mitrishkin et al., 2016a) and the simplest model of the first order is used in the present research. Programs of the references of the PF currents control cascade specify the plasma configuration in real time scenarios (Gusev et al., 2013).

Based on the above, we planned in this work first to design a scalar plasma current control feedback system with the help of the Central Solenoid (CS) current, then to design high-performance robust multivariable plasma shape control system with the plasma equilibrium reconstruction algorithm without iterations in the feedback loop for poloidal flux control on plasma separatrix. This MIMO system of scalar and multivariable loops was supposed to be the outer cascade of the complete system hierarchy (Fig. 1). As this takes place, we kept the plasma position control systems and PF currents cascade at their original configurations.

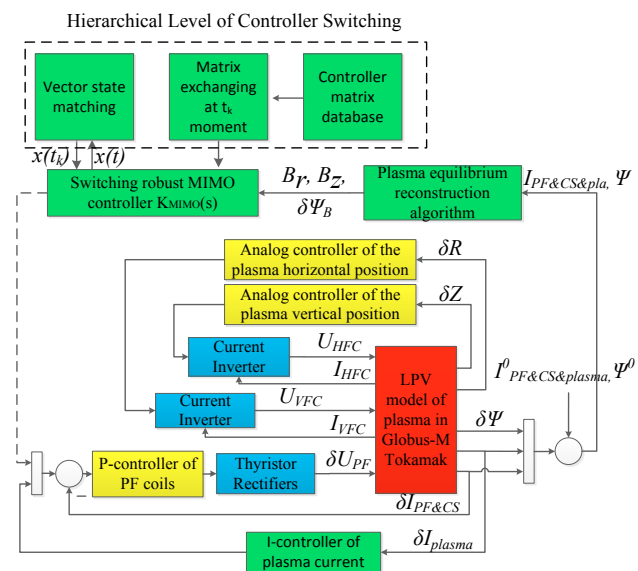


Fig. 1. Block diagram of multiloop hierarchical plasma magnetic control system of Globus-M tokamak.

On this way, we developed plasma reconstruction code which gave a chance to generate linear plasma models relatively plasma equilibriums reconstructed as plasma toroidal current and poloidal field distributions. The linear models were LPV type and were used in simulations in closed-loop system around experimental scenarios. Because of that the plasma shape MIMO robust controller was planned to be modified to get a new feature: switching on the array of robust controllers at a set of time moments to increase system performance accuracy and robust stability.

In doing so, the special methodology of hierarchical plasma control system design mentioned in Introduction was supposed to be developed.

2.2 Experimental Data for Plasma Equilibrium Reconstruction and Linearized Model Deriving

The vertical cross-section of Globus-M tokamak with locations of PF coils and vacuum vessel (VV) elements is shown in Fig. 2a. The basic parameters of Globus-M tokamak are following: maximum plasma current $I_{p \max} = 0.35$ MA, maximum toroidal magnetic field $B_t \max = 0.6$ T, major radius $R_0 = 0.36$ m, minor radius $a_p = 0.24$ m, aspect ratio $A = R_0/a_p = 1.5$, maximum vertical elongation $k_{\max} = 2.2$.

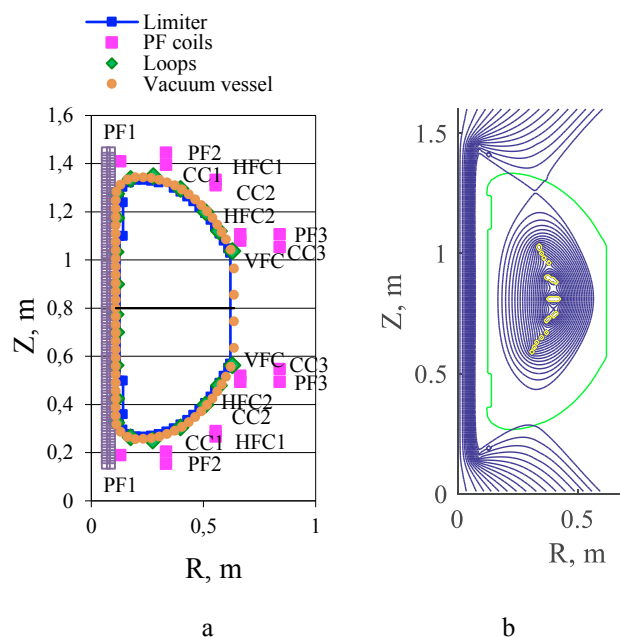


Fig. 2. a) Vertical cross section of Globus-M tokamak where PF, CC, HFC, and VFC are poloidal, correction, horizon and vertical field coils respectively. b) Reconstructed diverted configuration of the Globus-M plasma with the upper X-point, shot 31648, at 190 ms.

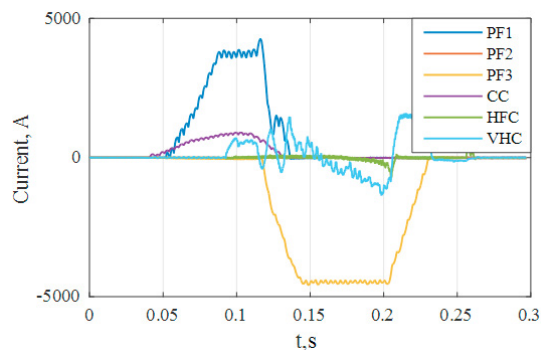


Fig. 3. Scenario currents in the control coils of the plasma discharge.

To demonstrate design and simulation of the advanced feedback control system shot No 31648 was chosen. The scenario currents in the control coils of the discharge are represented in Fig. 3. These currents, the intergrated signals

on 21 magnetic loops (Fig. 4) along with the plasma current and the summary current in the plasma and the VV (Fig. 5) are used for the plasma equilibrium reconstruction and the linear models deriving.

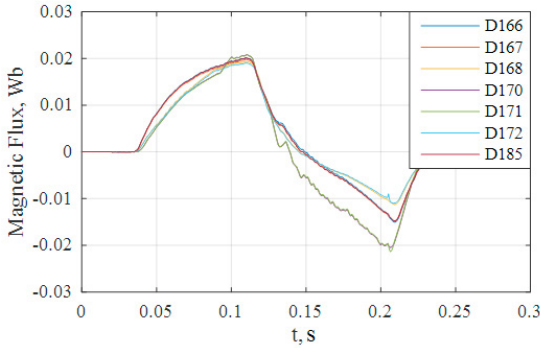


Fig. 4. Magnetic flux on loops (integrated signals on loops).

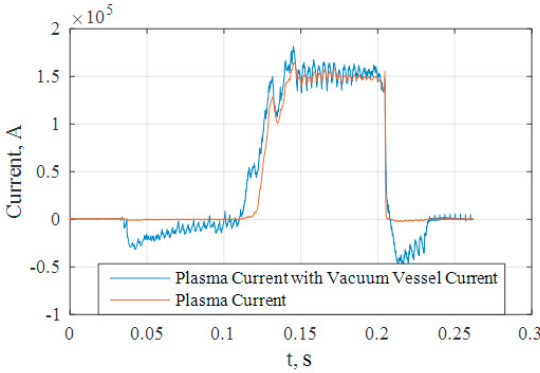


Fig. 5. Plasma current and total current of plasma and vacuum vessel.

3. PLASMA EQUILIBRIUM RECONSTRUCTION

3.1 Reconstruction by Moving Filaments

One of possible approaches to obtain the poloidal flux distribution is to approximate the toroidal plasma current distribution by a set of ring-filament currents (Swain et al., 1982). The sum of these currents is equal to the total plasma current. In this case, analytical solution for the poloidal flux ψ can be found with the aid of the Green's function method (Ariola et al., 2008). Locations and currents of the filaments are estimated by minimization of the following functional

$$\chi^2 = \sum_{k=1}^{N_{\text{meas}}} [(M_k - R_k) / \sigma_k]^2 \rightarrow \min \quad (1)$$

where M_k and R_k are measured and reconstructed signals respectively, and σ_k is an error associated with the k -th measurement. The minimization problem can be solved with the singular value decomposition methodology (Forsythe et al., 1977) which also provides regularization for the problem that is ill-posed.

Reconstructed signals R_k (poloidal flux ψ and toroidal plasma current) in (1) are functions of the currents in the filaments and their coordinates expressed in terms of the Green's functions (Ariola et al., 2008). For poloidal flux the

function has the form: $\psi(r) = \sum_{n=1}^N I_n G(r_n, r)$, where I_n is the current in the n -th conductor (plasma filament, poloidal coil, or vacuum vessel element (vacuum vessel is divided into 67 elements)), r_n is the coordinate of this conductor, G is the Green's function, r is the coordinate in the poloidal plane.

The minimization problem (1) can be solved by the linearization of the currents and the Green's functions namely

$$I_m = I_m^0 + \delta I_m, \quad G(r_m, r) = G(r_m^0, r) + \left. \frac{\partial G(r_m, r)}{\partial r_m} \right|_{r_m^0} \delta r_m$$

where I_m and r_m are the current and the coordinate of the m -th filament, I_m^0 and r_m^0 are their some initial values, $G(r_m, r)$ is the Green's function. Dropping the terms of the second order of smallness in the linearization of the functional (1) the problem is reduced to the following optimization statement: $\chi^2 = [Kx - P]^2 \rightarrow \min_y$

$x = (\delta \vec{r}^T, \delta \vec{I}^T)^T$, $\delta \vec{r}$ are coordinates of filaments, $\delta \vec{I}$ is the vector of currents in filaments and VV elements, K is the matrix, P is the vector. The solution of the underdetermined system of linear algebraic equations set $Kx = P$ is found by the singular value decomposition methodology (Forsythe et al., 1977). Fig. 2b demonstrates an example of the plasma configuration reconstructed by the developed numerical Moving Filaments code for the diverted phase at 190 ms of the discharge chosen.

Moving filaments code was verified on plasma-physics DINA code (Khayrutdinov et al., 1993). DINA code separatrix and reconstructed separatrix by Moving Filaments code are in agreement with an acceptable accuracy.

4. PLASMA LPV MODEL

4.1 LPV Model of Plasma in a Tokamak

The dynamics of the tokamak-plasma system is described by the Kirchhoff's voltage law. We assume that the plasma is massless, can move as a rigid body and can change its total current holding its current profile (Walker et al., 2006). In terms of the filament representation this means that all filaments have the same vertical and radial displacements δz_p and δr_p , and perturbations of the total plasma current δI_p do not change proportions between plasma filaments currents. Then the linearized model around the plasma equilibrium Kirchhoff's law for conductors and plasma currents is written as a set of linear differential equations with time varying parameters because the plasma equilibrium changes during a discharge:

$$M_{cc} \delta \dot{I} + M_{cp}(\Gamma(t), Y(t)) \delta \dot{I}_p + R \delta I + \frac{\partial \Psi_c(\Gamma(t), Y(t))}{\partial \vec{r}_p} \delta \dot{\vec{r}}_p = \delta U, \quad (2)$$

$$M_{pc}(\Gamma(t), Y(t)) \delta \dot{I} + M_{pp}(\Gamma(t), Y(t)) \delta \dot{I}_p + \frac{\partial \Psi_p(\Gamma(t), Y(t))}{\partial \vec{r}_p} \delta \dot{\vec{r}}_p = 0,$$

$$\delta \vec{r}_p = [\delta r_p \quad \delta z_p]^T, \quad \partial \Psi_c / \partial \vec{r}_p = [\partial \Psi_c(t) / \partial r_p \quad \partial \Psi_c(t) / \partial z_p]$$

$\partial\Psi_p / \partial\vec{r}_p = [\partial\Psi_p(t)/\partial r_p \ \partial\Psi_p(t)/\partial z_p]$. Here $\vec{r}_p = [r_p \ z_p]^T$, “c” denotes conductors and “p” denotes plasma filaments, M_{ab} is an inductance matrix between a and b , Ψ_a is a vector of fluxes at a due to the plasma, R is the diagonal matrix of conductors resistances. The plasma resistivity is neglected. The inductance matrices M_{ab} and fluxes Ψ_a are calculated from a number of positions $\Gamma(t)$ and a set of currents $Y(t)$ of plasma filaments evaluated by the plasma equilibrium reconstruction algorithm.

Because the plasma is assumed to be massless the total vertical and radial forces on the plasma equal zeroes. That gives a force balance equation

$$\frac{\partial\vec{F}}{\partial\vec{r}_p}(\Gamma(t), Y(t))\delta\vec{r}_p + \frac{\partial\vec{F}}{\partial I}(\Gamma(t), Y(t))\delta I = 0. \quad (3)$$

Combining (2) and (3) one can obtain:

$$\begin{aligned} \tilde{M}(\Gamma(t), Y(t))\delta\dot{I} + R\delta I &= \delta U, \\ \tilde{M}(t) &= M_{cc} - \frac{\partial\Psi}{\partial\vec{r}_p} \left(\frac{\partial F}{\partial\vec{r}_p} \right)^{-1} \frac{\partial F}{\partial I} - M_{cp} M_{pp}^{-1} \left(M_{pc} - \left(\frac{\partial F}{\partial\vec{r}_p} \right)^{-1} \frac{\partial F}{\partial I} \right). \end{aligned}$$

So linear model can be written in a standard state space form (Korenev et al., 2016):

$$\begin{aligned} \delta\dot{I}(t) &= A(\Gamma(t), Y(t))\delta I(t) + B(\Gamma(t), Y(t))\delta U(t), \\ \delta y(t) &= C(\Gamma(t), Y(t))\delta I(t), \end{aligned}$$

$$A(t) = -\tilde{M}(\Gamma(t), Y(t))^{-1} R, \quad B(t) = \tilde{M}(\Gamma(t), Y(t))^{-1} [E_{N \times N} \ 0_{N \times V}]^T.$$

The model parameters depend on time because the plasma equilibrium changes during discharges so matrices A , B , C depend on time parameter t as has been shown in (2). The vector output δy includes variations of the plasma vertical and horizontal coordinates, plasma current, conductors currents, and magnetic flux at the locations of magnetic loops.

4.2 LPV Numerical Models Generation

The models of the model array generated for Globus-M tokamak have 10 states corresponding to the currents in the poloidal field coils and 67 states from the filaments of the VV presentation. Therefore, the total number of states is equal to 77. All generated models have a single unstable pole corresponding to the plasma vertical instability caused by the vertical elongation. For the diverted plasma that eigenvalue ranges from 200 s^{-1} (180 ms) to 500 s^{-1} (200 ms). LPV models are obtained by linear interpolation of LTI models using LPV System block provided in Simulink of the MATLAB environment.

To verify the generated models plasma position controllers have been designed and applied on DINA code (Dokuka et al., 2016). The modeling has shown the efficiency of the controllers, that demonstrates the reliability of the models.

5. PLASMA POSITION, CURRENT, AND SHAPE CONTROL SUBSYSTEMS

5.1 Plasma Position Control Subsystem

Globus-M tokamak already has stabilization subsystems of the vertical and horizontal plasma positions (Mitrishkin et al., 2014). The controllers of these subsystems marked in yellow in Fig. 1 have been implemented into physics experiments and are working for more than 10 years. The unites marked in green are designed to control the plasma current and poloidal fluxes on the plasma separatrix. The actuators specifically current invertors and thyristor rectifiers are marked in blue.

5.2 Plasma Current Control Subsystem

To control the total plasma current the I-controller has been synthesized and discretized with sampling time $100 \mu\text{s}$. This controller receives the measured signal of the plasma current from the Rogovski loop and through the negative feedback sends its output to the cascade reference of the multiphase thyristor voltage rectifier of the CS (Central Solenoid) coil as this coil actually operates the plasma current by the current transformer principle.

5.3 Robust H_∞ Isoflux Plasma Shape Control Subsystem on a Diverter Phase

In order to control the plasma shape the following approach is suggested for simulation of the whole system on the base of the experimental data with the reconstruction algorithm in the feedback and possible application in practice.

The output signals of deviations with regard to the plasma reconstructed equilibrium come from the LPV model specifically poloidal flux values at the 21 points of the magnetic loop locations $\delta\Psi$, the plasma current δI_{plasma} , poloidal coil currents $\delta I_{PF\&CS}$. These signals are added to the magnetic experimental scenario signals $I_{PF\&CS\&plasma}^0$ and Ψ^0 (Fig. 1) respectively, reletavily which the linear models were calculated. Based on the summary signals the plasma equilibrium reconstruction algorithm calculates r -, z -components of a magnetic field B_r , B_z in a place of a desirable X-point location, as well as a difference in a poloidal flux between the X-point and points of a desirable location of the plasma boundary $\delta\Psi_B$ (Fig. 1). The objective of the multivariable controller $K_{MIMO}(s)$ is to reduce these values to zero. This objective is the consequence of the tokamak plasma physics: the magnetic field at the X-point for any discharge is zero and the plasma boundary is defined as the greatest isolevel line of the poloidal flux (isoflux control (Hoffman et al., 1990, Walker et al, 2000)).

A set of MIMO controllers $K_{MIMO}(s)$ for their switching is synthesized by the Loop Shaping methodology based on the Normalized Coprime Factorization approach (McFarlane et al., 1989).

Tokamak Globus-M has 7 poloidal field coils (PF1x2, PF2x2, PF3x2, Correction Coils CC) which are not involved in the control system of the plasma horizontal and vertical position, and plasma current. A nominal plant model in terms of the used approach has 7 inputs $\delta I_{PF\text{Ref}}$ corresponding to the current variation references in 7 PF coil currents control loops. Therefore the controller

$K_{MIMO}(s)$ has 7 outputs and 4 inputs (2 components of the magnetic field and 2 differences between poloidal fluxes in points 1 and 2 and X-point (Fig. 1)). The inputs and outputs of the nominal plant model G are scaled with scaling factors $D_{IN}=500 \times E^{7 \times 7}$ A and $D_{OUT}=\text{diag}([10^{-3} \text{ T}, 10^{-3} \text{ T}, 10^{-4} \text{ Wb}, 10^{-4} \text{ Wb}])$ respectively where E is an identity matrix. Matrix weighting functions for the open loop shaping are as follows: $W_1 = E^{7 \times 7}$, $W_2(s) = \frac{0.01s + 180}{s} E^{4 \times 4}$. The final robust

controller in the multivariable shape control loop takes the form: $K_{VS}(s) = D_{IN} W_2(s) K_{\infty}(s) W_1 D_{OUT}^{-1}$, where $K_{\infty}(s)$ is the synthesized H_{∞} optimal central controller which provides the robust stability margin $\varepsilon \sim 0.40 \div 0.41$ of the closed-loop control system with the same weighting functions for various models in the LPV unite for shot No 31648. The augmented nominal plant model G is factorized by means of the left coprime factorization $G_w = W_2 G W_1 = M_l^{-1} N_l$ where the factors M_l and N_l are to satisfy the equality $M_l M_l^* + N_l N_l^* = E$. The perturbed plant with uncertain transfer functions Δ_M and Δ_N may be presented as $G_p = (M_l + \Delta_M)^{-1} (N_l + \Delta_N)$. The maximum possible H_{∞} norm of the uncertainty $[\Delta_N \ \Delta_M]$ when the closed-loop system is stable is the robust stability margin ε (McFarlane et al., 1989). To simplify essentially the control system the MIMO plasma shape controller was reduced from 81 to 20 order.

6. SWITCHING MIMO PLASMA SHAPE CONTROLLER DESIGN AND FEEDBACK SYSTEM SIMULATION

6.1 Switching robust MIMO controller design

The algorithm of the Switching Control with pre-calculated controllers for each model in the LPV unite was used in the development of plasma shape control systems to improve the accuracy and stability margin of control since the plasma parameters change during the discharge (see Section 4). The scheduled algorithm is to change controllers at the set of the time moments $\{t_k\}_{k=1}^N$. The controller matrix database is created in advance as a set of controller matrices $\{A_c(k) \ B_c(k) \ C_c(k) \ D_c(k)\}_{k=1}^N$ in accordance with the plasma discharge scenario and plasma linear models obtained from the experimental data. The controller matrices are exchanged at each time moment t_k by the matrix exchanging procedure, which corresponds to the controller switching (Fig. 1).

To eliminate the transient response in the system when switching to the next controller which has new matrices along with the scenario the corresponding initial state of the controller vector is used. The multivariable switching controller equation has the form depended on two time scales namely continuous time t and discrete time t_k :

$$\dot{x}(t) = A_c(t_k)x(t) + B_c(t_k)u(t), \quad y(t) = C_c(t_k)x(t) + D_c(t_k)u(t)$$

where A_c, B_c, C_c, D_c are matrixes of the state space representation of the controller, $x(t), y(t), u(t)$ are the state vector, the output, and the input of the controller respectively. The controller operates in continuous time t

but the matrices are switched in discrete moments t_k . It means that the set of matrices from the database is mapped into the controller matrices set

$$\{A_c(k), B_c(k), C_c(k), D_c(k)\}_{k=1}^N \xrightarrow{\wp} \{A_c(t_k), B_c(t_k), C_c(t_k), D_c(t_k)\}_{k=1}^N$$

where the mapping algorithm \wp finds the proper controller state vector with the new matrices as well and solves the linear algebraic equations set:

$$\begin{bmatrix} C_c \\ F - A_c \end{bmatrix} x_{new}(t_k) = \begin{bmatrix} y(t_k) - D_c u(t_k) \\ B_c u(t_k) \end{bmatrix}$$

where $F_{i,i} = x_{inew}(t_k) / x_i(t_k)$, F is a diagonal matrix. The new state of controller vector $x_{inew}(t_k)$ is loaded into the next controller at the time t_k of the change of the plasma model and ensures a smooth transition process.

Because the linear plasma model changes its parameters and when the plasma shape controller is an LTI unit the robust stability margin depends on t . The switching control makes possible to increase ε on average during a finite time interval T with the following estimation

$$\int_0^T \varepsilon(t) dt / T \leq \sum_{k=1}^N \int_{t_{k-1}}^{t_k} \varepsilon_k(t) dt / T,$$

where ε_k is given by the proper controller after switching.

6.2 Transition from the limiter phase to the diverter phase

Since the plasma shape control problem takes place when the plasma reaches the diverter phase with X-point, the plasma equilibrium reconstruction algorithm determines the moment of the plasma separation from the wall and closes the plasma shape control loop (the dashed line in Fig. 1). For shot No 31648 the plasma shape control loop is closed at about 0.182 second (the green vertical line in Fig. 6 and 7) when the plasma changes its structure.

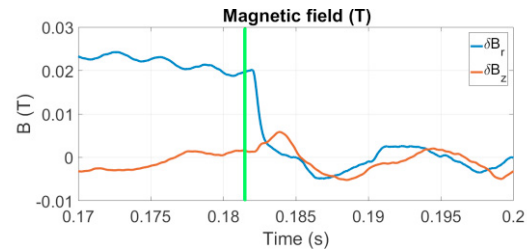


Fig. 6. Deviations of the magnetic field in the location of X-point for shot No 31648.

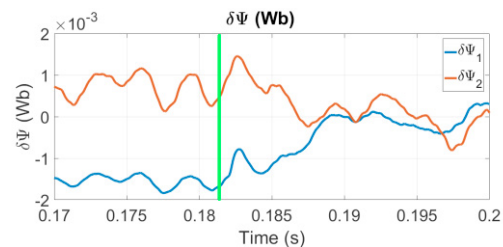


Fig. 7. Deviations of the flux in the desired location of the plasma separatrix (see Fig. 8) for shot No 31648.

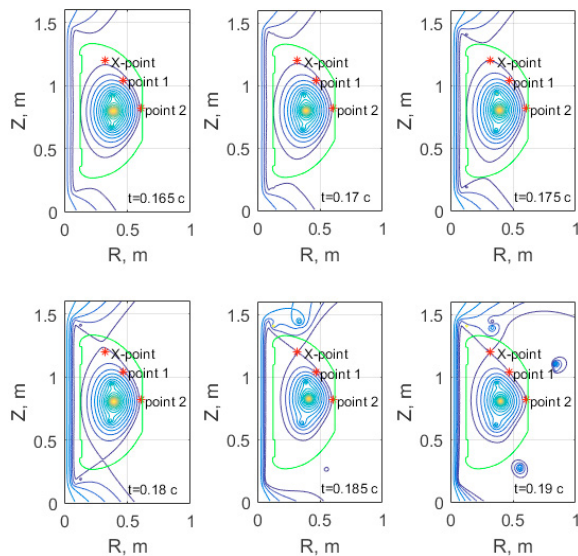


Fig. 8. Evolution of the plasma boundary for shot No 31648 with 5 filaments at 0.165, 0.17, 0.175, 0.18, 0.185, 0.19 seconds.

6.3 Numerical Simulations of the Feedback Control System for the Whole Plasma Discharge

Fig. 6 and 7 demonstrate the results of the flux control system operation. Reducing the number of plasma filaments to 5 did not lead to considerable loss of the plasma equilibrium reconstruction accuracy. The transient response is acceptable with the settling time of about 15 ms. The red dots indicate the desired location of the plasma separatrix in Fig. 8. Fig. 6-8 show that the developed plasma shape control system is able to transfer the plasma boundary to the desired location within 20 ms of the divertor phase of the discharge.

6. CONCLUSIONS

The high performance of the plasma shape control system is achieved by the usage of a set of advanced physics-control-engineering solutions: the plasma reconstruction algorithm without iterations in the feedback and with the reduced number of filaments, obtaining the array of linear plasma models from the experimental data, the switching plasma shape controller designed by H_∞ loop shaping methodology on the base of the model array and significantly reduced, separation of plasma current and shape loops.

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