FORMAL RECONSTRUCTION OF THE ASSERTORIC SYLLOGISTIC OF N. A. VASILIEV

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In Aristotelian and traditional syllogistics the propositions of types **a**, **i**, **e**, **o** are considered as basic. The famous Russian logician N. A. Vasiliev in his article "On Particular Statements, Triangle of Oppositions and the Law of Excluded Fourth" proposed to found the logic of syllogistic type on the ground of three kinds of propositions: **a**, **e** and the so-called *accidental* propositions "Only some (not all) *S* are *P*". The last kind of proposition will be denoted as **t**.

V. A. Smirnov [*1989*] made the first attempt to formalize Vasiliev's syllogistic. He set out the axiomatic system **C2V** in the language, where elementary formulas are of the types: $S\mathbf{a}P$ ("Every *S* is *P*"), $S\mathbf{e}P$ («Every *S* is not *P*») and $S\mathbf{t}P$ («Only some *S* are *P*»), and complex formulas are composed by means of propositional connectives. **C2V** axiom schemes are:

A0. Axiom schemes of classical propositional calculus,

A1. $(M\mathbf{a}P \mathbin{\&} S\mathbf{a}M) \supset S\mathbf{a}P$,

A2. $(M\mathbf{e}P \mathbin{\&} S\mathbf{a}M) \supset S\mathbf{e}P$,

A3. $S\mathbf{e}P \supset P\mathbf{e}S$,

A4. $\neg(S\mathbf{a}P \mathbin{\&} S\mathbf{e}P)$,

A5. $\neg(S\mathbf{a}P \,\&\, S\mathbf{t}P)$,

A6. $\neg(S\mathbf{e}P \,\&\, S\mathbf{t}P)$,

A7. $S\mathbf{a}P \vee S\mathbf{e}P \vee S\mathbf{t}P$,

A8. $S\mathbf{e}P \vee S\mathbf{a}S$.

There is only one rule in **C2V** — *modus ponens*.

A1 is a formal notation for *modus Barbara*, **A2** — for *modus Celarent*, **A3** — for e-conversion law, **A4–A7** — for Vasiliev's triangle of oppositions laws. The sense of **A8** is that the subject of any false general negative proposition is non-empty ($S\mathbf{a}S$ formula of **C2V** contains information that S is non-empty).

C2V calculus is definitionally equivalent to Smirnov's system **C2** formulated in standard syllogistic language with constants **a**, **e**, **i**, **o**.
C2 postulates are: **A0**, **A1**, **A2**, **A3**, and also $S\mathbf{a}P \supset S\mathbf{i}P$, $S\mathbf{i}P \supset S\mathbf{a}S$, $S\mathbf{e}P \equiv \neg S\mathbf{i}P$, $S\mathbf{o}P \equiv \neg S\mathbf{a}P$ and *modus ponens*.

In the system **C2** the definition of type **t** propositions is:

$$S\mathbf{t}P \Leftrightarrow S\mathbf{i}P \,\&\, S\mathbf{o}P.$$

In **C2V** system the definitions of **i** and **o** propositions are:

$$S\mathbf{i}P \Leftrightarrow \neg S\mathbf{e}P,$$

$$S\mathbf{o}P \Leftrightarrow \neg S\mathbf{a}P.$$

V. A. Smirnov demonstrated that the **C2V** system is embedded into the classical predicate calculus under the translation ψ_1:

$\psi_1(S\mathbf{a}P) = \forall x(Sx \supset Px) \,\&\, \exists x Sx$,

$\psi_1(S\mathbf{e}P) = \forall x(Sx \supset \neg Px)$,

$\psi_1(S\mathbf{t}P) = \exists x(Sx \,\&\, Px) \,\&\, \exists x(Sx \,\&\, \neg Px)$,

$\psi_1(\neg \mathbf{A}) = \neg\psi_1(\mathbf{A})$,

$\psi_1(\mathbf{A} \nabla \mathbf{B}) = \psi_1(\mathbf{A}) \nabla \psi_1(\mathbf{B})$, where ∇ is any binary connective.

C2 is based on Ockham's interpretation of categorical propositions. According to it, each affirmative proposition with empty subject is regarded as false, and each negative one as true. However, Vasiliev's paper contains no mention of such an interpretation.

That is why it is important to carry out Vasiliev's idea of logical systems with **a**, **e** and **t** basic types of propositions preserving the same interpretation of $S\mathbf{t}P$, but varying $S\mathbf{a}P$ and $S\mathbf{e}P$ interpretations.

In this paper we try to reconstruct in Vasiliev's style three well-known syllogistic systems: the fundamental Brentano-Leibnitz syllogistic, the positive syllogistic fragment of Bolzano's logic, and the traditional syllogistic formalized by Lukasiewicz.

In fundamental syllogistic each general proposition with empty subject is true, and each particular one is false. Its axiomatization, based on classical propositional calculus (**FC** system), was offered by V. Markin [*1991*]. **FC** postulates are: **A0**, **A1**, **A2**, **A3**, and also $S\mathbf{a}S$, $S\mathbf{i}P \supset S\mathbf{i}S$, $S\mathbf{o}P \supset S\mathbf{i}S$, $S\mathbf{e}P \equiv \neg S\mathbf{i}P$, $S\mathbf{o}P \equiv \neg S\mathbf{a}P$ and *modus ponens*.

To construct Vasiliev's type of **FCV** calculus, definitionally equivalent to **FC**, one has to eliminate **A4** and **A8** axiom schemes from **C2V** and to accept the new axiom schemes:

A9. $S\mathbf{a}S$,

A10. $S\mathbf{e}S \supset S\mathbf{e}P$,

A11. $S\mathbf{e}S \supset S\mathbf{a}P$.

A9 is the syllogistic identity law for the type **a** propositions. The sense of **A10** and **A11** is the following: if subject S is empty, then propositions $S\mathbf{e}P$ and $S\mathbf{a}P$ are true ($S\mathbf{e}S$ formula of **FCV** contains information that S is empty).

The definition of type **t** propositions in **FC** and the definitions of **i** and **o** propositions in **FCV** are the same as in **C2** and **C2V** systems.

We have proved the theorem: *Vasiliev's type* **FCV** *syllogistic is embedded into the predicate calculus under the following* ψ_2 *translation*:

$\psi_2(S\mathbf{a}P) = \forall x(Sx \supset Px)$,

$\psi_2(S\mathbf{e}P) = \forall x(Sx \supset \neg Px)$,

$\psi_2(S\mathbf{t}P) = \exists x(Sx \;\&\; Px) \;\&\; \exists x(Sx \;\&\; \neg Px)$,

$\psi_2(\neg\mathbf{A}) = \neg\psi_2(\mathbf{A})$,

$\psi_2(\mathbf{A} \nabla \mathbf{B}) = \psi_2(\mathbf{A}) \nabla \psi_2(\mathbf{B})$.

The system **FCV** is not an adequate formalization of Vasiliev's syllogistic. One of the triangle of oppositions laws — $\neg(S\mathbf{a}P \;\&\; S\mathbf{e}P)$ — is not provable in **FCV**. Only the weakening of this law — $\neg S\mathbf{e}S \supset \neg(S\mathbf{a}P \;\&\; S\mathbf{e}P)$ — is an **FCV** theorem. This formula means that $S\mathbf{a}P$ and $S\mathbf{e}P$ propositions are incompatible, if their subject S is non-empty.

In Bolzano's syllogistic the propositions of all types are false if their subjects are empty. The axiomatization of the positive syllogistic fragment of Bolzano's logic, based on classical propositional calculus (**BC** system), was offered by V. Markin [*1991*]. **BC** postulates are: **A0**, **A1**, **A2** and also $S\mathbf{i}P \supset P\mathbf{i}S$, $S\mathbf{a}P \supset S\mathbf{i}P$, $S\mathbf{i}P \supset S\mathbf{a}S$, $S\mathbf{e}P \equiv \neg S\mathbf{i}P \;\&\; S\mathbf{i}S$, $S\mathbf{o}P \equiv \neg S\mathbf{a}P \;\&\; S\mathbf{i}S$ and *modus ponens*.

To construct the **BCV** calculus with basic **a**, **e** and **t** constants, definitionally equivalent to **FC**, one has to eliminate **A3**, **A7** and **A8** axiom schemes from **C2V** and to accept the new axiom schemes:

A12. $(S\mathbf{a}P \vee S\mathbf{t}P) \supset (P\mathbf{a}S \vee P\mathbf{t}S)$,

A13. $(S\mathbf{a}P \vee S\mathbf{t}P) \supset S\mathbf{a}S$,

A14. $S\mathbf{a}S \supset (S\mathbf{a}P \vee S\mathbf{e}P \vee S\mathbf{t}P)$.

In **BC** system the definition of type **t** propositions is:

$$S\mathbf{t}P \Leftrightarrow S\mathbf{i}P \;\&\; S\mathbf{o}P.$$

In the **BCV** system the definitions of types **i** and **o** propositions are:

$$S\mathbf{i}P \Leftrightarrow S\mathbf{a}P \vee S\mathbf{t}P,$$

$$S\mathbf{o}P \Leftrightarrow S\mathbf{e}P \vee S\mathbf{t}P.$$

Now we are able to explicate the sense of the **A12** axiom scheme: it is the **i**-conversion law counterpart. **A13** asserts that the subject of each true $S\mathbf{a}P$ or $S\mathbf{t}P$ proposition is non-empty ($S\mathbf{a}S$ formula of **BCV** as well as in

C2V contains information that the term S is non-empty). **A14** means that if the term S is non-empty, then one of Vasiliev's triangle of oppositions laws — $S\mathbf{a}P \vee S\mathbf{e}P \vee S\mathbf{t}P$ — holds.

We have proved the theorem: *Vasiliev's type* **BCV** *syllogistic is embedded into the predicate calculus under the following* ψ_3 *translation*:

$$\psi_3(S\mathbf{a}P) = \forall x(Sx \supset Px) \,\&\, \exists xSx,$$

$$\psi_3(S\mathbf{e}P) = \forall x(Sx \supset \neg Px) \,\&\, \exists xSx,$$

$$\psi_3(S\mathbf{t}P) = \exists x(Sx \,\&\, Px) \,\&\, \exists x(Sx \,\&\neg Px),$$

$$\psi_3(\neg\mathbf{A}) = \neg\psi_3(\mathbf{A}),$$

$$\psi_3(\mathbf{A} \nabla \mathbf{B}) = \psi_3(\mathbf{A}) \nabla \psi_3(\mathbf{B}).$$

The **BCV** system as well as the **FCV** system are not adequate formalizations of Vasiliev's syllogistic because one of the triangle of oppositions laws — $S\mathbf{a}P \vee S\mathbf{e}P \vee S\mathbf{t}P$ — is not a **BCV** theorem. Only the weakening of this law — axiom scheme **A14** — is provable in **BCV**.

Traditional syllogistic has the initial presupposition that all syllogistic terms are non-empty. Traditional syllogistic could be formalized by Smirnov's axiomatic system **C4**. **C4** is the extension of **C2** obtained by adding the new axiom scheme: $S\mathbf{i}S$ — syllogistic identity law for the type **i** propositions. The **C4** theorem set is deductively equivalent to Lukasiewicz's well-known syllogistic.

Given the language with basic constants **a**, **e**, **t**, we construct a system which is definitionally equivalent to **C4** system. This is the **C4V** calculus obtained from **C2V** by adding the new axiom scheme:

A15. $\neg S\mathbf{e}S$.

The definition of type **t** propositions in **C4** and the definitions of types **i** and **o** propositions in **C4V** are the same as in the **C2** and **C2V** systems.

We have proved the theorem: **C4V** *calculus is embedded into the predicate calculus under the translation* ψ_4:

$$\psi_4(\mathbf{A}) = (\exists xS_1x \,\&\, \ldots \,\&\, \exists xS_nx) \supset \psi_2(\mathbf{A}),$$

where **A** *is any formula of* **C4V** *language,* $S_1, \ldots, S_n$ *is the list of all syllogistic terms in* **A**, ψ_2 *is the "fundamental" translation of Vasiliev's type* **FCV** *syllogistic formulas into the predicate calculus.*

C4V as well as **C2V** can be regarded as an adequate formalization of Vasiliev's assertoric syllogistic, because all syllogistic principles accepted by Vasiliev are provable in this system.

References

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