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On applicability of the Tikhonov–Cagniard magnetotelluric model for sounding of a non-uniform medium

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A laterally uniform Earth model excited by a plane wave has been proposed by Tikhonov and Cagniard (T–C) for magnetotelluric sounding. The T–C model applicability for an Earth model with a non-uniform surface layer is discussed. The analytical expression for the kernel of integral transformation of the horizontal electric field into a magnetic one at the surface of a stratified section is derived. It is shown that the T–C model is applicable if $L_f \gg \max(|\lambda_0|, |\lambda_L|)$, where L_f is the characteristic field scale, $\lambda_0 = Z_0^- / (-i\omega\mu_0)$, $\lambda_L = [\mathscr{J}/(S^{-1} + Z_0)]^{1/2}$, S is conductance of subsurface layer, and Z_0^- and $\mathscr{J}$ are the impedance and transverse resistance of a laterally uniform stratified basement. The influence of spatial filtration of the electromagnetic field on MT-impedance evaluation is discussed.

1. Introduction

The magnetotelluric (MT) method of the Earth sounding is based on the fundamental Tikhonov–Cagniard (T–C) model. For validity of the T–C model it is necessary that tangential components of electric $\mathbf{E}_\tau(\mathbf{r})$ and magnetic $\mathbf{H}_\tau(\mathbf{r})$ fields at point $\mathbf{r}$ on the Earth's surface were connected with the local impedance $Z(\mathbf{r})$ at the same point via Leontovich's condition (Landau and Lifshitz, 1957)

$$\mathbf{n} \times \mathbf{H}_\tau(\mathbf{r}) = \mathbf{E}_\tau(\mathbf{r})/Z(\mathbf{r}) \tag{1}$$

where $\mathbf{n}$ is the upward unit vector normal to the Earth's surface. It has been believed for years that the T–C model is in practice applicable if the electromagnetic field varies slowly along a horizontal direction.

Dmitriev and Berdichevsky (1979) have considered the laterally uniform model excited by an arbitrary external field. They showed that the electric field at the Earth's surface can be expressed as the result of convolution of the magnetic field with the induction impedance filter (IIF) and of vertical currents in the atmosphere with the galvanic impedance filter (GIF). These filters are axially symmetrical and decay with distance $r \to \infty$ faster than $1/r^m$, where m is any number. In the case of a uniform Earth, both filters decay with distance r as $\exp(-r/\delta)$, where δ is skin depth. On the basis of numerical calculations, this inference was extended to behaviour of the IIF for the stratified model. This numerical calculation showed also that GIF decays slower than IIF. As a result, Dmitriev and Berdichevsky have concluded that the T–C model leads to the correct evaluation of the local impedance, provided vertical currents are negligible and the horizontal magnetic field component at the Earth's surface varies linearly. Although these results have extended our knowledge on applicability of the T–C model, the question of the filter's width for a stratified

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medium and the effect of lateral inhomogeneities still remains open.

In the present paper, the applicability of the T–C model for a more general Earth section consisting of a non-uniform surface sheet with conductance $S(\mathbf{r})$ and a laterally uniform underlying section is investigated. Such a thin sheet model is widely used in deep geoelectrics.

The tangential component of the electric field $\mathbf{E}_\tau(\mathbf{r})$ is known to remain continuous at the sheet, whereas the tangential component of the magnetic field suffers a discontinuity given by Price's relation

$$\mathbf{n} \times [\mathbf{H}_\tau(\mathbf{r}) - \mathbf{H}_\tau^-(\mathbf{r})] = S(\mathbf{r}) \cdot \mathbf{E}_\tau(\mathbf{r}) \tag{2}$$

where $\mathbf{H}_\tau^-(\mathbf{r})$ is a magnetic field tangential component on the surface of a laterally uniform underlying section. If the T–C impedance for the underlying section is designated by Z_0^-, then the local impedance $Z(\mathbf{r})$ in (1) is equal to

$$1/Z(\mathbf{r}) = 1/Z_0^- + S(\mathbf{r}) \tag{3}$$

If eqn. (1) connects tangential components $\mathbf{E}_\tau$ and $\mathbf{H}_\tau$ at the Earth's surface by means of the local impedance (3) then (2) leads to the conclusion that $\mathbf{E}_\tau$, $\mathbf{H}_\tau^-$ at the surface of a laterally uniform section satisfy the equation

$$\mathbf{n} \times \mathbf{H}_\tau^-(\mathbf{r}) = \mathbf{E}_\tau(\mathbf{r})/Z_0^- \tag{4}$$

i.e., the impedance of the underlying section can also be determined on the basis of the T–C model, and vice versa, if eqn. (4) is valid just below the non-uniform sheet, then eqn. (1) is valid at the Earth's surface. Thus, the problem of T–C model applicability to the inhomogeneous Earth reduces to the problem of its applicability at the surface of a laterally uniform basement underlying the sheet. This does not mean, however, that availability of the non-uniform surface sheet in the model fails in practice to affect the model fields. Indeed, the existence of the non-uniform sheet at the surface entails a non-uniform character of electromagnetic field distribution at the Earth's surface irrespective of whether the primary field was uniform or not. Moreover, even in the presence of a non-conducting atmosphere, a galvanic mode is excited in the Earth whose contribution to the electric field proves frequently to be very substantial and sometimes dominant.

The tangential magnetic field at the surface of a laterally uniform medium may be represented in terms of electric field via the integral relation

$$\mathbf{n} \times \mathbf{H}_\tau^-(\mathbf{r}) = \mathbf{E}_\tau(\mathbf{r})/Z_0^- + \fint_s \hat{Y}(\mathbf{r} - \mathbf{r}')\{\mathbf{E}_\tau(\mathbf{r}') - \mathbf{E}_\tau(\mathbf{r})\}\, \mathrm{d}s' \tag{5}$$

This formula has been derived in slightly different form by Dawson and Weaver (1979) and McKirdy et al. (1985). Thus, the inequality

$$|\mathbf{E}_\tau(\mathbf{r})/Z_0^-| \gg |\fint_s \hat{Y}(\mathbf{r} - \mathbf{r}')\{\mathbf{E}_\tau(\mathbf{r}) - \mathbf{E}_\tau(\mathbf{r}')\}\, \mathrm{d}s'| \tag{5a}$$

gives the necessary and sufficient condition of T–C model applicability. The inequality shows that to answer the question when eqn. (5) may be reduced to (4) one has to examine the spatial behaviour of the admittance filter $\hat{Y}(\mathbf{r})$.

2. Spatial behaviour of the admittance filter

Separation of the electromagnetic field modes in the laterally uniform part of the section leads to the expression for tensorial kernel $\hat{Y}(\mathbf{r})$

$$\hat{Y}(\mathbf{r}) = (\mathbf{n} \times \mathbf{e}_r) \otimes (\mathbf{n} \times \mathbf{e}_r)\left(\frac{\mathrm{d}G^{\mathrm{i}}}{\mathrm{d}r} + \frac{G^{\mathrm{i}}(r)}{r}\right) + \mathbf{e}_r \otimes \mathbf{e}_r\left(\frac{G^{\mathrm{i}}(r)}{r} + \frac{\mathrm{d}G^{\mathrm{g}}}{\mathrm{d}r}\right) \tag{6}$$

Here, $\mathbf{e}_r = \mathbf{r}/r$; $\mathbf{a} \otimes \mathbf{b}$ denotes the tensorial product of vectors $\mathbf{a} = a_1\mathbf{e}_1 + a_2\mathbf{e}_2$ and $\mathbf{b} = b_1\mathbf{e}_1 + b_2\mathbf{e}_2$; $(\mathbf{a} \otimes \mathbf{b})_{ij} = a_i b_j$, where $\mathbf{e}_1$, $\mathbf{e}_2$ are the horizontal unit vectors of the Cartesian co-ordinate system. The functions $G^{\mathrm{i}}(r)$ and $G^{\mathrm{g}}(r)$ are determined at $r > 0$ in terms of the Fourier–Bessel integrals

$$G^{\mathrm{i}}(r) = \int_0^\infty \frac{\mathrm{d}}{\mathrm{d}k}\left(\frac{1}{Z_k^{\mathrm{i}}}\right) J_0(kr) \frac{\mathrm{d}k}{2\pi r} \tag{7}$$

$$G^{\mathrm{g}}(r) = \int_0^\infty \frac{\mathrm{d}}{\mathrm{d}k}\left(\frac{1}{Z_k^{\mathrm{g}}}\right) J_0(kr) \frac{\mathrm{d}k}{2\pi r} \tag{8}$$

where Z_k^{i}, Z_k^{g} are the spectral impedances of inductive and galvanic modes respectively.

The behaviour of kernels (7) and (8) may readily be studied if the underlying section is a uniform half-space. In this case, we have $Z_k^i = -i\omega\mu_0/\kappa_k$, $Z_k^g = \kappa_k/\sigma_0$, where $\kappa_k = (k^2 - i\omega\mu_0\sigma_0)^{1/2}$, and σ_0 is the conductivity of the medium. Calculation of the integrals (7) and (8) using the known results (Bateman and Erdelyi, 1953) yields

$$2\pi G^i(r) = (-i\omega\mu_0)^{-1}\exp(-\kappa_0 r)/r^2 \tag{9}$$

$$2\pi G^g(r) = -\mathring{Z}_0^{-1}\exp(-\kappa_0 r)/r \tag{10}$$

where $\mathring{Z}_0 = (-i\omega\mu_0/\sigma_0)^{1/2}$ is the impedance of the uniform half-space. Thus, we obtain $\hat{Y}(\mathbf{r}) \sim \exp(-\kappa_0 r)$ for a uniform half-space at $r \to \infty$. This result coincides with that of Dmitriev and Berdichevsky (1979).

The behaviour of the admittance kernel for a stratified section is more complicated. The known theorem (Tikhonov, 1959) provides the asymptotic expansions in powers of $1/r$ for integrals containing zero-order Bessel functions. The coefficients of the expansion are proportional to the coefficients in the power series expansion near the point $k = 0$ of the function which stands in the integrand before the Bessel function. For any stratified medium with $\sigma(+\infty) > 0$ the spectral impedances Z_k^i and Z_k^g are, however, even functions of k and, hence, all terms of the above-mentioned asymptotic expansion vanish. The only conclusion which may be drawn from the Tikhonov theorem in this case is that the functions $G^i(r)$ and $G^g(r)$ decrease at $r \to \infty$ more rapidly than r to any power. Although this conclusion is of great qualitative importance, it does not lead to any quantitative conclusions about spatial behaviour of the kernel $\hat{Y}(\mathbf{r})$.

Substitution of integral representation for the zero-order Bessel function (Bateman and Erdelyi, 1953)

$$J_0(x) = \frac{2}{\pi}\int_0^\infty \sin(x\cdot\cosh t)\,\mathrm{d}t$$

reduces (7) to the expression

$$2\pi r G^i(r) = \int_0^\infty \mathrm{d}t \int_{-\infty}^{\infty} \frac{\mathrm{d}}{\mathrm{d}k}\left(\frac{1}{Z_k^i}\right) \times \exp(ikr\cdot\cosh t)\,\mathrm{d}k/(\pi i) \tag{11}$$

With a view to further calculations, we have to examine the behaviour of the function $1/Z_k^i$ in the upper complex half-plane k. The values of spectral impedance $Z_k^i(z_2)$, $Z_k^i(z_1)$ at boundaries of the uniform layer $z_1 < z < z_2$ with conductivity σ are connected by the known relation

$$Z_k^i(z_1) = \frac{Z_k^i(z_2) - i\omega\mu_0\cdot\tanh(\kappa_k h^*)/\kappa_k}{1 + Z_k^i(z_2)\cdot(-i\omega\mu_0/\kappa_k)^{-1}\cdot\tanh(\kappa_k h^*}$$

where $h^* = z_2 - z_1$ and $\kappa_k = (k^2 - i\omega\mu_0\sigma)^{1/2}$. The function has a branch point $(i\omega\mu_0\sigma)^{1/2}$ in the upper half-plane. It can readily be verified, however, that the function $\tanh(\kappa_k h^*)/\kappa_k$ has no branch points. This means that $Z_k^i(z_1)$ has the same branch points as $Z_k^i(z_2)$ and, generally, the positions of the branch points of the function $Z_k^i(z)$ are independent of z (Tabarovsky, 1975). For example, if $\sigma(z)$ becomes constant at some depth, then function $Z_k^i(z)$ for any depth z has the single branching point $[i\omega\mu_0\sigma(+\infty)]^{1/2}$. It is assumed further that the examined section is underlain at some depth by the perfectly conducting basement. In this case, the function Z_k^i has no branch points at all and the internal integral in (11) reduces to the sum of the residues of the function

$$\frac{d}{dk}\left(\frac{1}{Z_k^i}\right) \exp(ikr\cdot\cosh t) \tag{12}$$

lying in the upper half-plane k. The fact that the impedance is a single-valued function means, in particular, that $\mathrm{d}(1/Z_k^i)/\,\mathrm{d}k$ cannot have a nonzero residue. Indeed, in the opposite case the function $1/Z_k^i$ may be represented in the vicinity of a singular point k^* by the sum of a single-valued function and the function $\mathrm{Res}\{\mathrm{d}(1/Z_k^i)/\,\mathrm{d}k, k^*\}\cdot\ln(k - k^*)$. This is in contradiction to the conclusion that the spectral admittance is a single-valued function in the k-plane. Therefore, we shall examine the second-order pole of the function $\mathrm{d}(1/Z_k^i)/\,\mathrm{d}k$. Obviously, this pole is a simple zero of the function Z_k^{i}. Designating the simple zeroes as k_m^{i}, we find that

$$\operatorname{Res}\left[\frac{\mathrm{d}}{\mathrm{d}k}\left(\frac{1}{Z_k^{\mathrm{i}}}\right)\exp(ikr\cdot\cosh t),\ k_m^{\mathrm{i}}\right]$$

$$-ir\cdot\cosh t\cdot\left[\frac{\mathrm{d}}{\mathrm{d}k}Z_k^{\mathrm{i}}\left(k_m^{\mathrm{i}}\right)\right]^{-1}$$

$$\times\exp(ik\cdot r\cdot\cosh t)$$

The same conclusion may be reached for the spectral admittance $1/Z_k^{\mathrm{g}}$. The integral representation of the McDonald functions (Bateman and Erdelyi, 1953)

$$K_\nu(z)=\int_0^\infty \exp(-z\cdot\cosh t)\cdot\cosh(\nu t)\,\mathrm{d}t$$

$(\mathrm{Re}\ z>0)$

can be used to reduce the kernels (7) and (8) to the form

$$G^{\mathrm{i}}(r)=\frac{1}{\pi i}\sum K_1\left(-ik_m^{\mathrm{i}}r\right)\left[\frac{\mathrm{d}}{\mathrm{d}k}Z_k^{\mathrm{i}}\left(k_m^{\mathrm{i}}\right)\right]^{-1} \quad (13)$$

$$G^{\mathrm{g}}(r)=\frac{1}{\pi i}\sum K_1\left(-ik_m^{\mathrm{g}}r\right)\left[\frac{\mathrm{d}}{\mathrm{d}k}Z_k^{\mathrm{g}}\left(k_m^{\mathrm{g}}\right)\right]^{-1} \quad (14)$$

where k_m^{i} and k_m^{g} designate the simples zeroes of the spectral impedances Z_k^{i} and Z_k^{g} in the upper half-plane respectively. It can readily be verified that, if the spectral impedances have higher-order zeroes, their contribution may be allowed for in the same manner as above. The resultant expressions for both of the kernels will be similar to (13) and (14). In practice, such representations may be used to calculate the admittance kernel at any value of r, if poles of spectral admittances in the upper complex half-plane k are known.*

In the general case, spectral impedance zeroes are rather difficult to specify. One may assert (Vyurkov and Dreizin, 1982) that the zeroes can be located only within the region $Im\ k>|\mathrm{Re}\ k|$. It may also be asserted that the zeroes with high m are located near $k_m^{\mathrm{i}}\approx k_m^{\mathrm{g}}\approx i\pi m/h$, where h is the depth of a perfectly conducting basement. The corresponding coefficients in (13) and (14) are $[-\omega\mu_0h^2/(\pi m)]^{-1}$ and $(i\pi m/\sigma_0)^{-1}$. Taking into account that the McDonald function has the asymptotic representation $K_\nu(z)\cong(2z/\pi)^{-1/2}\exp(-z)$ at high values of argument, one may state that series (13) and (14) converge approximately as a geometric progression with denominator $\exp(-\pi r/h)$.

The series expansions (13) and (14) make it possible to conclude that the admittance kernel decreases exponentially at $r\to\infty$. The distance L_i along which the kernel $G^{\mathrm{i}}(r)$ decreases, as well as the distance L_g along which the kernel $G^{\mathrm{g}}(r)$ decreases substantially, are defined by the spectral impedance zeroes k_m^{i} and k_m^{g} located most closely to the real axis. At small values of r, the kernels $G^{\mathrm{i}}(r)$ and $G^{\mathrm{g}}(r)$ of the stratified section behave in the same manner as the kernels of (9) and (10) which define the admittance kernel for a uniform half-space. In this case, σ_0 must be meant to be the value of conductivity on the roof of the underlying section.

3. Section with a high-resistive intermediate layer

Let us consider an example of a two-layered underlying medium consisting of a uniform layer with thickness h and the perfect conductor (Dmitriev, 1970). This model, on the one hand, belongs to the models which are essentially nonuniform in the vertical direction and, on the other hand, permits an analytic determination of the spectral impedances. It can easily be verified that the spectral impedances of the underlying section are equal to

$$Z_k^{\mathrm{i}}=-\frac{i\omega\mu_0}{\kappa_0}\tanh(\kappa_kh)\qquad Z_k^{\mathrm{g}}=\frac{\kappa_k}{\sigma_0}\tanh(\kappa_kh)$$

where $\kappa_k=(k^2-i\omega\mu_0\sigma_0)^{1/2}$ and σ_0 is the conductivity of the upper part of the underlying section. The spectral admittances have simple poles at the points $k_m=i\chi_m$, $m=1,2,\ldots$, where $\chi_m=(\kappa_0^2+\pi^2m^2/h^2)^{1/2}$, with residues $(m\pi/h)^2/(\omega\mu_0h\chi_{\mathrm{m}})$ for the induction mode and with the residues $\sigma_0/(i\chi_mh)$ for the galvanic one. Also, the galvanic mode spectral admittance has a pole at the point

* It should be noted that the formulae of the (13) and (14) type can be used to calculate the Hankel transform of any function $f(k)$ which has no singularities on the real axis k and is meromorphic in the upper complex k half-plane and if the $f(k)$ poles located in the upper half-plane are known. Such a procedure would have been stable at any value of r.

$k_0 = i\kappa_0$ with the residue $\sigma_0/(2i\kappa_0 h)$. All the poles are located in the first quarter of the complex k-plane at the upper half of the branch of the hyperbola Re $k \cdot$ Im $k = \omega\mu_0\sigma_0$, Re k < Im k. Thus, kernels $G^{\mathrm{i}}(r)$ and $G^{\mathrm{g}}(r)$ defining the admittance kernel $\hat{Y}(\mathbf{r})$ are of the form

$$G^{\mathrm{i}}(r) = \sum_{m=1}^{\infty} \frac{\pi\sigma_0}{\kappa_0^2 h^3} \frac{m^2}{\chi_m} K_1(\chi_m r) \qquad (15)$$

$$G^{\mathrm{g}}(r) = -\frac{\sigma_0}{2\pi\kappa_0 h} K_1(\kappa_0 r) - \sum_{m=1}^{\infty} \frac{\sigma_0}{\pi\chi_m h} K_1(\chi_m r) \qquad (16)$$

The expressions obtained make it possible to construct the admittance kernel at any value of r. At $r \gg h$ the induction kernel decays according to the law

$$G^{\mathrm{i}}(r) \cong \frac{\sigma_0}{\kappa_0^2 h^3} \cdot \sqrt{\frac{\pi}{2\chi_1^3 r}} \cdot \mathrm{e}^{-\chi_1 r} \qquad (17)$$

whereas the galvanic kernel attenuates as

$$G^{\mathrm{g}}(r) \cong -\frac{\sigma_0}{2\pi\kappa_0 h} K_1(\kappa_0 r) \qquad (18)$$

In the case of a poorly conducting intermediate layer, which is of importance for deep geoelectrics, $|\kappa_0| h \ll 1$, and, hence, $\chi_1 = \pi/h$. In this case, the induction part of the kernel $\hat{Y}(\mathbf{r})$ decays along the distance h/π. At the same time, the galvanic mode contribution is characterized by the distance $|L_{\mathrm{g}}| = |\kappa_0|^{-1}$, i.e., the galvanic part of the admittance kernel decays much more slowly than the induction one.

There exists a general type of underlying section, examined by Fainberg and Singer (1987), which admits also an analytic determination of the main asymptotic term in $G^{\mathrm{g}}(r)$. The underlying section of this type has a high-resistive upper part at depth $0 < z < h$ with conductivity $\sigma_0(z)$. The conductivity at depths $z > h$ varies according to the law $\sigma(z)$, with $\sigma \gg \sigma_0$. If the condition $\sqrt{\omega\mu_0\sigma_0}\,|\lambda_0| \ll 1$ is satisfied and the values of k are restricted by the inequality $k|\lambda_0| \ll 1$, where $|\lambda_0|$ is the effective depth of the induction mode penetration in the underlying medium, $\lambda_0 = Z_0^-/(-i\omega\mu_0)$, then

$$Z_k^{\mathrm{g}} = Z_0^- + \mathcal{J} k^2 \qquad (19)$$

where Z_0^- is the impedance of the underlying section for a plane wave and $\mathcal{J}$ is the transverse resistance of the section. The first term omitted in (19) has smallness of order $|k\lambda_0|^2$ with respect to $\mathcal{J}k^2$. It follows from (19) that the point $k_0 = i(Z_0^-/\mathcal{J})^{1/2}$ is the spectral impedance zero, which is nearest to the co-ordinate origin if $|k_0\lambda_0| \ll 1$; the next one should be located at 'distance' $|\lambda_0|^{-1}$ from the co-ordinate origin. Therefore, when $r \gg |\lambda_0|$

$$G^{g}(r) = -\frac{1}{2\pi\sqrt{T Z_0}} K_1(r/L_g)$$

where

$$L_{\mathrm{g}} = (\mathcal{J}/Z_0^-)^{1/2}$$

In the case of the two-layered underlying section with a perfectly conducting basement, we have $\mathcal{J} = h/\sigma_0$ and $Z_0^- = -i\omega\mu_0 h$; thus $L_{\mathrm{g}} = (-i\omega\mu_0\sigma_0)^{-1/2}$, which coincides with the earlier estimate.

As to induction mode, the analysis of typical deep MT-sounding sections shows that the induction part of the admittance filter decreases at distances of order $|\lambda_0|$.

It is of interest to estimate the values of λ_0 and L_{g} for a model with $h_1 = 50$ km, $\sigma_1 = 0.33 \times 10^{-3}$ S m^{-1}, $h_2 = 50$ km, $\sigma_2 = 0.33 \times 10^{-2}$ S m^{-1}, $\sigma_3 = 33$ S m^{-1}. At a period of 1 h, one obtains $\mathcal{J} = 1.6 \times 10^7$ Ω m^2, $|Z_0^-| = 2.2 \times 10^{-4}$ Ω and the galvanic mode admittance filter width $|L_{\mathrm{g}}|$ runs up to 850 km, while the induction mode one satisfies the inequality $|\lambda_0| < 100$ km.

4. Discussion

The analysis here gives us the opportunity to discuss conditions of T–C model applicability in situations when subsurface inhomogeneities are present.

Let us substitute $\mathbf{E}_\tau(\mathbf{r}')$ in (5) by its Taylor series expansion at the observation point $\mathbf{r}$

$$\mathbf{E}_\tau(\mathbf{r}') = \mathbf{E}_\tau(\mathbf{r}) + (\mathbf{R}\nabla_\tau)\mathbf{E}_\tau(\mathbf{r}) + \tfrac{1}{2} X^\alpha X^\beta \frac{\partial^2 \mathbf{E}_\tau}{\partial X^\alpha \partial X^\beta}(\mathbf{r}) + \qquad (22)$$

Here $R = \mathbf{r}' - \mathbf{r} = X^1\,\mathbf{e}_1 + X^2\,\mathbf{e}_2$, where $\mathbf{e}_1$, $\mathbf{e}_2$ are horizontal unit vectors of the Cartesian co-ordinate system; summation over α, $\beta := 1, 2$ is made in the second term of (22). Then (5) reduces to the estimate

$$\mathbf{n} \times \mathbf{H}_\tau^-(\mathbf{r}) = \mathbf{E}_\tau(\mathbf{r})/Z_0^- - \left(\frac{\mathrm{d}}{\mathrm{d}k^2}\frac{1}{Z_k^i}\right)_{k=0} (\mathbf{n} \times \nabla_\tau) [(\mathbf{n} \times \nabla_\tau)\mathbf{E}_\tau] - \left(\frac{\mathrm{d}}{\mathrm{d}k^2}\frac{1}{Z_k^g}\right)_{k=0} \nabla_\tau(\nabla_\tau \mathbf{E}_\tau) \tag{23}$$

The terms linear in $\mathbf{R}$ fail to make a contribution owing to the symmetry properties of the admittance tensor (6). The estimates made for simple sections show that in the case of the induction mode

$$\left|\frac{1}{Z_0^-}\frac{\mathrm{d}}{\mathrm{d}k^2}Z_k^i\right| \sim |\lambda_0|^2 \tag{24}$$

In the case of the galvanic mode, it follows from (19) that

$$\left|\frac{1}{Z_0^-}\frac{\mathrm{d}}{\mathrm{d}k^2}Z_k^g\right| = \left|\frac{\mathscr{I}}{Z_0^-}\right| = |L_g|^2 \tag{25}$$

To describe the spatial behaviour of the electric field let us define the parameter

$$L_f^2 = |\mathbf{E}_\tau(\mathbf{r})| / \max\{ |(\mathbf{n} \times \nabla_\tau) \cdot [(\mathbf{n} \times \nabla_\tau)\mathbf{E}_\tau]|, |\nabla_\tau[\nabla_\tau \mathbf{E}_\tau]| \}$$

This parameter characterizes the 'curvature' of the electric field distribution and may be named as the characteristic field scale (CFS). Then, as it follows from (23), the inequality (5a) is valid provided

$$L_f \gg \max(|\lambda_0|, |L_g|) \tag{26}$$

This means that if a horizontally uniform medium is excited by an arbitrary external field consisting of inductive and galvanic modes then the T–C model is applicable provided L_f exceeds the width $|L_g|$ of the admittance filter. For a typical case for deep MT-sounding, when the Earth is excited by the inductive mode only, the galvanic mode arises owing to subsurface inhomogeneities. As has been shown by Singer and Fainberg (1985) and Fainberg and Singer (1987), the galvanic part of the anomalous MT-field of a remote anomaly decays along the adjusting distance $r_A = \max\{|\lambda_0|, |\lambda_L|\}$, where

$$\lambda_L = \left[\mathscr{I}/(S^{-1} + Z_0^-)\right]^{1/2} \tag{27}$$

(Note, that in the particular case of an H-polarized field in the 2-D model consisting of a thin surface sheet underlain by a uniform $\mathscr{I}$-layer with thickness h and a perfectly conducting basement, $Z_0^- = -i\omega\mu_0 h$, and hence (27) reduces to a corresponding expression in Berdichevsky and Zhdanov (1981).)

This means that the domain of integration in (5) and (5a) may be bounded by $|\mathbf{r} - \mathbf{r}'| < r_A$. Thus, if

$$L_f \gg r_A \tag{28}$$

the T–C model is valid. It is important to note that the value λ_L, in contrast to L_g, is dependent on surface conductance $S(\mathbf{r})$. Since $|\lambda_L| < |L_g|$, the condition (28) must be used in practical magnetotellurics.

Finally, let us discuss the modern procedure used to suppress the effect of local subsurface inhomogeneities upon deep MT-sounding results with the low-pass filtration of experimental data in the space domain (for instance, averaging of the MT-field). To do this, we shall assume that the field changes along the distance $L_f < r_A$ due to the influence of local inhomogeneities. Let $\langle F \rangle_\mathbf{r}$ be the result of filtration of function F over domain $|\mathbf{r} - \mathbf{r}'| < L_A$, where $L_A \gg r_A$. According to (28), under a non-uniform thin sheet, filtered electric and magnetic fields satisfy the expression

$$\mathbf{n} \times \langle \mathbf{H}_\tau^- \rangle_\mathbf{r} = \langle \mathbf{E}_\tau \rangle_\mathbf{r} / Z_0^- \tag{29}$$

and can be used for the determination of the impedance of the underlying section. Using (2) and (29) for the filtered MT-field at the Earth's surface, one obtains

$$\mathbf{n} \times \langle \mathbf{H}_\tau \rangle_\mathbf{r} = \langle \mathbf{E}_\tau \rangle_\mathbf{r} \langle Y \rangle_\mathbf{r} + \mathbf{Q}(\mathbf{r}) \tag{30}$$

where

$$\langle Y \rangle_\mathbf{r} = 1/Z_0^- + \langle S \rangle_\mathbf{r}$$

$$\mathbf{Q}(\mathbf{r}) = \langle S\mathbf{E}_\tau \rangle_\mathbf{r} - \langle S \rangle_\mathbf{r} \langle \mathbf{E}_\tau \rangle_\mathbf{r}$$

When the filtration used is space averaging, $\mathbf{Q}(\mathbf{r})$ is the correlation function of conductance S and electric field $\mathbf{E}_{\tau}$. Thus, the application of the T–C model to an averaged MT-field leads to some bias depending on the contribution of $\mathbf{Q}(\mathbf{r})$ to the right-hand side of (30). In the case when the value of $\mathbf{Q}(\mathbf{r})$ may be estimated from experimental data, it can be used for correction of the bias.

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