
ELEMENTARY PARTICLES AND FIELDS

Theory

Macroscopic Model of Radio Emission from Extensive Air Showers

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Abstract—A macroscopic model of radio emission from extensive air showers is developed. This model is appropriate for calculating this radio emission at frequencies below 100 MHz. It is constructed on the basis of an analysis of the radiation integral and is verified by comparing field observables predicted by the model with the respective results obtained within the microscopic approach to calculating radio emission from extensive air showers.

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1. INTRODUCTION

Two macroscopic effects accompanying the development of extensive air showers and leading to the generation of coherent (at frequencies below about 100 MHz) radio emission were first indicated by G.A. Askar'yan [1]. These are the accumulation of an electron excess and the polarization of a shower in the geomagnetic field.¹⁾ The idea of employing radio emission from extensive air showers as a basis of a new method for detecting ultrahigh-energy ($\geq 10^{17}$ eV) cosmic rays was also put forth in [1].

The first experimental measurement of radio emission from extensive air showers at a frequency of 44 MHz was performed by Jelley and his coauthors [2] in 1965. Further investigations, which had continued up to the mid-1970s, confirmed that the new procedure for detecting cosmic rays on the basis of radio emission from extensive air showers [3, 4] was quite promising, but, in those years, they did not lead to a decisive breakthrough, mainly because of the absence of precise calculations of radio emission from extensive air showers, as well as because of technical difficulties associated with suppressing noises. After a pause that lasted nearly 40 years, experimental (and, before long, theoretical as well) investigations of radio emission from extensive air showers have once again gained momentum. The facilities CODALEMA [5] (in the frequency range 20–80 MHz) and LOPES [6] (in the frequency range 40–80 MHz), which record radio pulses from extensive air showers, are operating at the present time. The exploitation of a set of AERA

radio antennas [7] will begin in the near future within the Pierre Auger project [8]. Also, several antennas are operating in the composition of the Tunka facility [9], and the plans for the future include an extension of the system.

The first theoretical investigations of radio emission from extensive air showers were performed as far back as the 1960s and 1970s on the basis of simple macroscopic models of a shower that were of a heuristic character [10–12]. Owing to the development of computational facilities, a microscopic approach to calculating radio emission [13–15] where one simulates extensive air showers by the Monte Carlo method [16] and calculates emission from each charged particle came to the fore in the past few years (the first estimations within this scheme were performed in [17]). Within the approach in question, the problem of calculating radio emission from a “known” extensive air shower (direct problem) can be thought to be solved in principle at the present time. At the same time, there are at least three reasons for which it is desirable to complete the study initiated in [10], where the authors made the first ever attempt at constructing a macroscopic model of radio emission from extensive air showers.

The main advantage of the microscopic approach based on a Monte Carlo simulation of extensive air showers consists in the completeness of the description of a shower in Monte Carlo codes [18]. However, the total number of particles in an extensive air shower grows with increasing E_0/E_{cut} , where E_0 is the primary-particle energy and E_{cut} is the minimum threshold energy to which one can trace the history of secondary particles; as a result, the time of the respective Monte Carlo simulation becomes

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¹⁾Askar'yan wrote in [1] that it was V.I. Goldansky who told him about the latter mechanism (private communication).

longer.²⁾ The use of powerful computational clusters solves this problem only partly. In order to perform calculations in the region of ultrahigh energies, one employs hybrid schemes [19] or resorts to a weighted simulation (the thinning algorithm is the most popular method in simulating extensive air showers [20]), artificial fluctuations appearing in the development of extensive air showers in the last case [21]. On the contrary, the macroscopic approach, which also reduces considerably the total time of the simulation of radio emission from extensive air showers (by several orders of magnitude) owing to optimizing the scheme for calculating fields, does not affect the Monte Carlo shower-simulation scheme as such and is therefore preferable.

The second reason is that, within the microscopic approach, one usually performs summation where the result is known a priori. Indeed, there are many charged particles in the coherent region per each Fresnel zone into which one can break down a shower [in observing radiation propagating at small angles with respect to the axis of the extensive air shower being considered, the dimensions of these zones are several wavelength (λ) units in the direction orthogonal to the shower axis but may exceed λ by several orders of magnitude in the direction along the axis]. Moreover, the radio emission in question may be due exclusively to the excess of electrons in a shower [1] and the polarization of extensive air showers by the geomagnetic field [1, 10], so that the calculation of radio emission from each particle would be redundant.

Finally, the third reason is the following. The first important steps toward creating a macroscopic model of radio emission from extensive air showers were made in [22–24]. However, there is still no such model in the proper sense—a model from which one could not exclude anything without substantially increasing the error in the results and which is suitable to calculating radio emission at frequencies of about 100 MHz and below to a precision commensurate with that which is reached in the microscopic approach. In addition to simplifying the calculations and rendering them faster, the significance of developing such a model is to pinpoint the extensive-air-shower parameters to which radio emission is sensitive (and

to what degree). Within the microscopic approach, it is much more difficult to solve this problem, because, there, summation over an enormous number of particles that is necessary for obtaining the final result blurs the connections in question. One can find them only by the trial-and-error method in performing a statistical analysis [15, 25].

In all publications concerning the subject that are known to us [10, 11, 12, 26], the work relied on the following strategies: one first formulates a shower model and then calculates relevant fields. For example, Kahn and Lerche [10] represented a shower (on the basis of general considerations) as a special configuration of three macroscopic sources (electron excess, transverse current, and dipole), whereupon they substituted these sources into the wave equation and solved it. Castagnoli and his coauthors [11] and Fujii and Nishimura [12] pursued further studies on the basis of the model proposed in [10]. Moreover, Kahn and Lerche [10] indicated that it is necessary to take into account radiation that arises owing to the change in the aforementioned field sources with depth. Calculations for radio emission that start from an extensive-air-shower model preliminarily formulated on the basis of general considerations are a possible way to tackle the problem in question, but this approach complicates the originally simple picture. Indeed, there are only two physical reasons behind a charge asymmetry in extensive air showers over the radiation-wavelength scale [1]; therefore, the abundance of radio-emission mechanisms considered in [10, 11, 12, 26] suggests that some of them are corrections introduced in order to improve a simplified model. In this case, it seems more reasonable to go over from involved to simple concepts rather than moving in the opposite direction.

In the present study, a macroscopic model of radio emission from extensive air showers is constructed on the basis of a detailed analysis of the radiation integral. It is indicated in [27] that, for the electric field generated by a set of electric currents $\mathbf{j}$, the Fourier component of its strength at a frequency ω , $\mathbf{E}_\omega$, can be represented in the form

$$\mathbf{E}_\omega(\mathbf{x}) = \frac{i\omega}{4\pi\epsilon_0 c^2} \times \int \mathbf{j}_\perp(\mathbf{x}') \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{r} \left(1 - \frac{1}{i\mathbf{k}\cdot\mathbf{r}}\right) d\mathbf{x}', \quad (1)$$

where ϵ_0 is the dielectric constant of a vacuum, c is the speed of light in a vacuum, $\mathbf{k} = n(\omega/c)\mathbf{e}_r$ is the wave vector directed from the element $d\mathbf{x}'$ of the emitting system to the point $\mathbf{x}$ at which the signal is received, n is the air refraction index, $\mathbf{j}_\perp = -[\mathbf{e}_r \times [\mathbf{e}_r \times \mathbf{j}_\omega]]$ is the projection of the Fourier component of

²⁾This explains why it was impossible to calculate precisely radio emission from extensive air showers in the 1960s and 1970s, when one could perform a full-scale Monte Carlo simulation of showers only for very low energies such that the corresponding number of particles at the shower-development maximum was about ten. The lower threshold for primary energies at which the detection of radio pulses from extensive air showers is possible is $E_0 \simeq 5 \times 10^{16}$ eV [5, 6], which corresponds to showers where the number of charged particles at the maximum of development is about 10^8 .

the current $\mathbf{j}$ onto the direction orthogonal to the observation direction $\mathbf{e}_r$, and r is the distance between the points $d\mathbf{x}'$ and $\mathbf{x}$. In (1), there are no dipoles or derivatives with respect to the depth at the present stage. The aforementioned analysis consists in the following.

In expression (1), an extensive air shower is represented in the form of the current $\mathbf{j}$, which is related to the particle-number (N) distribution by the equation $\mathbf{j}^{(i)} = e^{(i)} N^{(i)} \mathbf{u}$, where $e^{(i)}$ is the charge of an i -type particle and $\mathbf{u}$ is the particle velocity. In general, N is a multidimensional function. It can be found either by solving integro-differential equations of cascade theory [28, 29] or by performing a Monte Carlo simulation of extensive air showers [16, 18]. However, the function $\mathbf{j}$ appears in the integrand on the right-hand side of (1). Moreover, we are interested not in the entire wavelength interval but in that part of it where the radiation being studied is coherent (in the sense that the radiation wavelength satisfies the condition $\lambda > a$, where a is the average distance between particles of extensive air showers). Therefore, it is natural to expect that the true dimensionality of the problem of calculating the field in the coherent region is smaller than the dimensionality of the function $\mathbf{j}$ taken for the general case. The question is that of what value this dimensionality takes, or, in other words, what integrals of the function $\mathbf{j}$ one does in fact need in order to calculate the field to a precision not poorer than the preset precision. Finding these integrals amounts to constructing a macroscopic model of radio emission from extensive air showers.

The field strength obtained within the microscopic approach [15] is a precise result that one obtains on the basis of expression (1) and with which one can compare the results of a macroscopic calculation of radio emission (in that interval of primary energies of extensive air showers where this is possible). Thus, the fact that we know the result for the radiation integral in (1) provides an easier way to solve the problem at hand. In particular, this permits readily and fast testing the validity of assumptions on the true dimensionality of the integral in (1) that are made in constructing a macroscopic model of radio emission from extensive air showers.

In the present study, a Monte Carlo simulation of extensive air showers is performed on the basis of the CORSIKA 6.960 code [18]. Unless otherwise stated, it is implied in the following that (i) protons are used as primary particles and (ii) extensive air showers are simulated without employing the thinning option [21]. The threshold energy in simulating particles took the value of $E_{\text{cut}} = 100$ keV for electrons and photons and the value of 50 MeV for muons and hadrons. The strength and direction of the Earth's magnetic field correspond to these parameter

values at the locus of the LOPES experiment [6]. The density and optical properties of the atmosphere are reproduced with a step of 10 g/cm² in altitude, and the air refraction index n is then represented in the form

$$n(h) = 1 + \eta_0 \rho(h)/\rho(0),$$

where $\eta_0 = 3 \times 10^{-4}$ [30] and $\rho(h)$ and $\rho(0)$ are the air density at an altitude h and at sea level, respectively. All statements about the properties of radio emission refer to the range of frequencies below 100 MHz.

2. BASIC RELATIONS AND PRELIMINARY COMMENTS

Since the results obtained within the microscopic approach to calculating radio emission from extensive air showers are taken in the present study for a reference, it is reasonable to touch briefly upon this approach (a more detailed account of it can be found in [15]). After that, we will present basis formulas of the macroscopic approach, which were substantiated in [23] and which are used as starting point in the present study.

The microscopic approach consists in calculating radiation at the level of individual shower particles. At the present time, a simulation of the development of showers in a medium relies as a rule on the Monte Carlo method [16], within which the motion of particles from one point to another occurs according to the law $\boldsymbol{\xi}(t) = \boldsymbol{\xi}_0 + \mathbf{u}(t - t_0)$ at a constant velocity $\mathbf{u}$ along a given track. As a result, the problem reduces to a straightforward calculation of the field of a charge e moving at a constant velocity along a straight line. In the Fraunhofer approximation, the field obtained for this source from expression (1) has the form [15]

$$\mathbf{E}_\omega^{(0)}(\mathbf{x}) = \frac{e}{8\pi^2 \varepsilon_0 c} \frac{e^{ikR}}{R} e^{i\omega(t_0 - n\mathbf{e}_R \cdot \boldsymbol{\xi}_0/c)} \times \left(\frac{e^{i\omega\Delta t(1 - n\mathbf{e}_R \cdot \boldsymbol{\beta})} - 1}{1 - n\mathbf{e}_R \cdot \boldsymbol{\beta}} \right) \boldsymbol{\beta}_\perp, \quad (2)$$

$$\mathbf{E}_\omega(\mathbf{x}) = \left(1 - \frac{1}{i\mathbf{k} \cdot \mathbf{R}} \right) \mathbf{E}_\omega^{(0)}(\mathbf{x}), \quad (3)$$

where $\mathbf{E}_\omega^{(0)}(\mathbf{x})$ and $\mathbf{E}_\omega(\mathbf{x})$ stand for radiation calculated, respectively, without and with allowance for the nonwave part of the field; Δt is the time it takes for a particle to move at a constant velocity $\boldsymbol{\beta} = \mathbf{u}/c$ over the track being considered; $\boldsymbol{\beta}_\perp = -[\mathbf{e}_R \times [\mathbf{e}_R \times \boldsymbol{\beta}]]$; $\mathbf{e}_R = \mathbf{R}/R$; and R is the distance between the particle in question at the instant t_0 and the observation point $\mathbf{x}$ ($R = |\mathbf{x} - \boldsymbol{\xi}_0|$). The total radio-emission field from the whole extensive air shower being considered is the sum of expressions of the type in (2) [or (3)].

If we take the square of expression (2) and perform integration over all radiation angles, then, under the condition $\omega\Delta t(1 - n\beta) \gg 1$, we arrive at the well-known Frank–Tamm formula [31, 32], which serves as a basis for calculating optical radiation from extensive air showers.

Although the method described above gives no way to perform calculations at high primary energies, it is of great methodological value, because, within it, one makes a minimum number of assumptions on field sources; this is precisely the reason why we take for a reference the results obtained on the basis of this approach. In view of this, it is of paramount importance to verify the microscopic approach as such. The dissertation of Nehls [33], which appeared in 2008, contains individual lateral distributions of radio emission from extensive air showers as obtained at LOPES facility [6] (with extensive-air-shower energies and arrival directions reconstructed on the basis of KASCADE-Grande data [34] for each event being indicated). By way of example, a comparison of calculated and measured lateral distributions of radio emission from extensive air showers is illustrated in Fig. 1, where LOPES data are presented in the form of groups of events characterized by zenith angles of $\theta = 35^\circ \pm 3^\circ$ in the northern part of the celestial sphere ($\varphi = 0^\circ \pm 45^\circ$), while the calculated curves correspond to the arrival-direction angles of $\theta = 35^\circ$ and $\varphi = 0^\circ$. In view of relatively low statistics (eight events) and large errors in measuring the radio-emission field, it is rather difficult to estimate the degree of agreement between the experimental and calculated lateral distributions. At the same time, we can be sure that the calculated and experimental data in Fig. 1 are not at odds.

The macroscopic scheme is based on the same radiation integral in (1) as the microscopic approach. Following [23], we assume that, in the frequency range of interest (frequencies below 100 MHz), one can disregard the distributions of the current $\mathbf{j}(\mathbf{v}')$ with respect to energy and angles of particle motion with respect to the extensive-air-shower axis. Further, we assume that the showers in question are indefinitely narrow and then find from (1) that

$$\mathbf{E}_\omega(\mathbf{x}) = \frac{i\omega}{8\pi^2\epsilon_0 c^2} \int \int \frac{\mathbf{j}_\perp(t, \mathbf{r}_\perp)}{r} \times \left(1 - \frac{1}{i\mathbf{k} \cdot \mathbf{r}}\right) e^{i\omega t(1 - n\mathbf{k} \cdot \mathbf{s}/k)} \times e^{-i\mathbf{k} \cdot \mathbf{r}_\perp} e^{-i\mathbf{k} \cdot \boldsymbol{\varsigma}(t, \mathbf{r}_\perp)} d\mathbf{r}_\perp dt, \quad (4)$$

$$\mathbf{j}_\perp(t, \mathbf{r}_\perp) = -\frac{1}{k^2} [\mathbf{k} \times [\mathbf{k} \times \mathbf{j}(t, \mathbf{r}_\perp)]], \quad (5)$$

where t is the time, $d\mathbf{r}_\perp = r_\perp dr_\perp d\phi$, $r_\perp$ and ϕ are the polar coordinates of the current element $\mathbf{j}(t, \mathbf{r}_\perp) d\mathbf{r}_\perp$, $\mathbf{s}$

is a unit vector parallel to the shower axis, and $\boldsymbol{\varsigma}(t, \mathbf{r}_\perp)$ is the shower-front shape function. Calculating the field reduces to numerically integrating expression (4) with known source functions calculated and recorded in the course of a Monte Carlo simulation of extensive air showers. Since the number of Fresnel zones in a shower is independent of the primary energy E_0 , the higher the energy E_0 , the greater the gain in time with respect to calculating radiation from individual particles of extensive air showers. In [23], it was shown that expression (4) is appropriate for calculating radio emission from extensive air showers at frequencies below 100 MHz and distances of at least 300 to 400 m to the same degree of precision as in the microscopic approach.

3. REDUCTION OF THE DIMENSIONALITY OF THE RADIATION INTEGRAL

For the ensuing analysis, we will need formulas that describe the source functions $\boldsymbol{\varsigma}(t, \mathbf{r}_\perp)$ and $\mathbf{j}(t, \mathbf{r}_\perp)$. Since they emerge from a Monte Carlo simulation of extensive air showers, it is necessary to perform an approximation of the results of the simulation. We are interested only in the electron–positron component of extensive air showers, since the contribution of the remaining components (consisting of muons and hadrons) to the observed radio-emission field is small (the largest contribution here comes from muons; its magnitude decreases with increasing primary energy E_0 and, at $E_0 = 10^{15}$ eV, becomes as small as 1 to 2%).

In the course of the simulation of extensive air showers, the quantity $\boldsymbol{\varsigma}(t, \mathbf{r}_\perp)$ is calculated as the difference between the real particle position and the imaginary plane that is orthogonal to the shower axis and which moves at a speed c . An analysis shows that the shape of the shower front possesses a symmetry that is sufficiently high for the disregard of the dependence on ϕ to be legitimate and is accurately reproducible by the quadratic form (see Fig. 2)

$$\varsigma(t, r_\perp) = \varsigma_0(t) + \varsigma_1(t)r_\perp + \varsigma_2(t)r_\perp^2, \quad (6)$$

where $\varsigma_0(t) < 1$ m and $\varsigma_1(t)$ is $\varsigma_1(t) \simeq 0.1$ at the maximum of extensive-air-shower development and changes moderately with time (within 20 to 30%).

The azimuthal-angle-integrated lateral distribution of the current density (for electrons) is shown in Fig. 3, where two of the three projections are on display. These are $\mathbf{j}_{SA}(t, r_\perp)$, which is parallel to the shower axis $\mathbf{s}$, and $\mathbf{j}_{LF}(t, r_\perp) \propto [\mathbf{s} \times \mathbf{B}]$, where $\mathbf{B}$ is the induction vector of the Earth's magnetic field. The component proportional to $[\mathbf{j}_{SA} \times \mathbf{j}_{LF}]$ makes virtually no contribution to radio emission, and we do not

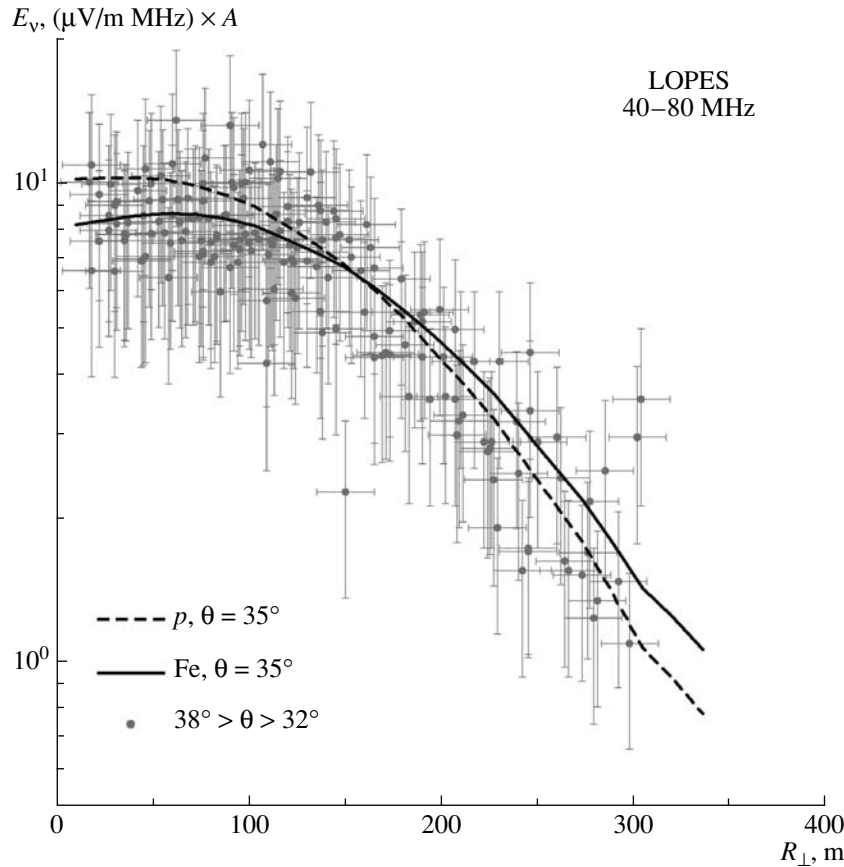


Fig. 1. Strength of the radio-emission field as a function of the distance from the axis of the extensive air shower being considered. The LOPES experimental data [6] (points) are contrasted against the results of a simulation of radio emission at a frequency of 60 MHz from extensive air showers (microscopic calculations of the field) generated by primary protons (p) and iron nuclei (Fe) of energy $E_0 = 5 \times 10^{17}$ eV (curves). The factor A is $A = A_0(\sin \alpha_B \cdot E_0/(10^{17} \text{ eV}))^{-1}$, where $A_0 = 1$ for experimental data (the average energy of extensive air showers in the quoted events is $E_0 = 4.7 \times 10^{17}$ eV) and $A_0 = 2\pi$ for the results of the calculations and α_B is the angle between the extensive-air-shower axis and the geomagnetic-field direction. The calculated curves correspond to average values of the field. The number of simulated showers is ten (the thinning level is 10^{-6} and the maximum statistical level is 10^3 [21]).

consider it for this reason. If we take the current summed over all charged particles,

$$\mathbf{J}_{\text{SA}} = \sum_i \mathbf{j}_{\text{SA}}^{(i)}, \quad \mathbf{J}_{\text{LF}} = \sum_i \mathbf{j}_{\text{LF}}^{(i)},$$

where i is the number of the particle sort, then the component $\mathbf{J}_{\text{SA}}$ stems from the excess of electrons (about 25% at the maximum of the shower development), while the component $\mathbf{J}_{\text{LF}}$ is due to a systematic separation (drift) of electrons and positrons in the geomagnetic field. It is interesting to note that, although J_{SA} is one order of magnitude larger (at angles between the shower axis and the geomagnetic field in excess of 20°) than the current J_{LF} , it is precisely the latter that determines almost completely radio emission from extensive air showers. This is because radio emission occurring at small angles to the extensive-air-shower axis is detected at characteristic observation distances (several hundred me-

ters from the shower axis). According to expressions (4) and (5), radiation from the current $\mathbf{J}_{\text{SA}}$ in the “forward” direction is suppressed, while radiation from the current $\mathbf{J}_{\text{LF}}$ is concentrated in the direction of extensive-air-shower propagation.

From Fig. 3, it can be seen that the lateral distributions of the two current projections, $\mathbf{j}_{\text{SA}}(t, r_\perp)$ and $\mathbf{j}_{\text{LF}}(t, r_\perp)$, can be approximated by a formula of the Nishimura–Kamata type; that is,

$$j(t, r_\perp) \propto \left(\frac{r_\perp}{r_0(t)} \right)^{\mu(t)-1} \left(1 + \frac{r_\perp}{r_0(t)} \right)^{-\alpha(t)}, \quad (7)$$

where $r_0(t)$, $\mu(t)$, and $\alpha(t)$ are depth-dependent parameters. The characteristic values of r_0 around the maximum of the development of vertical extensive air showers are $\simeq 30$ m (for $j_{\text{SA}}(t, r_\perp)$) and $\simeq 50$ m (for $j_{\text{LF}}(t, r_\perp)$). At almost all (with the exception of the earliest) stages of the development of extensive air

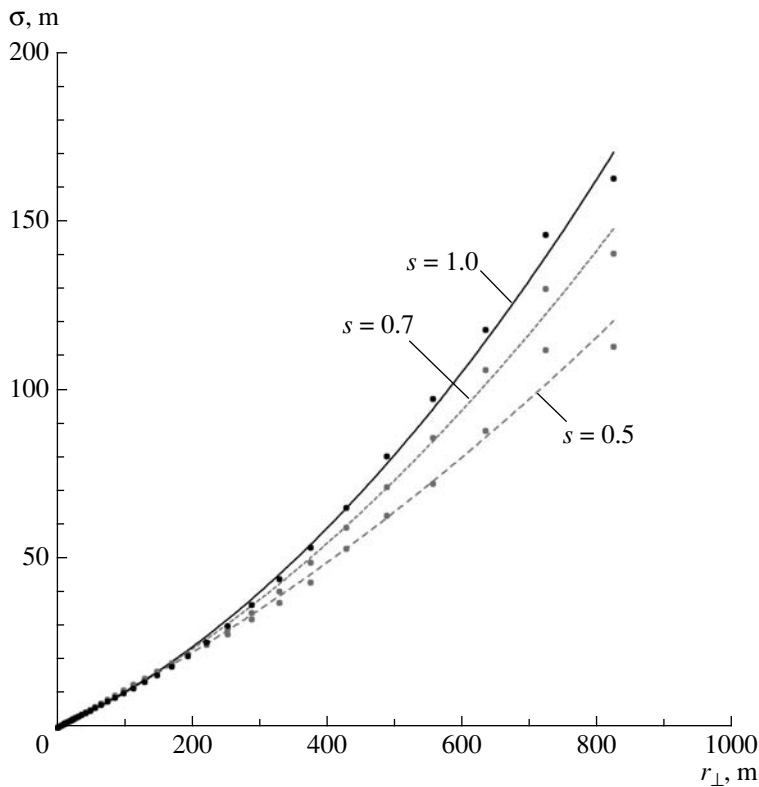


Fig. 2. Azimuthal-angle-averaged shape of the front of an individual vertical extensive air shower with energy $E_0 = 10^{16}$ eV at various stages of its development (they are characterized by the parameter $s = 3t/(t + 2t_{\max})$, where t is the depth of the atmosphere and $t_{\max}$ is the shower-maximum depth). The values indicated for s correspond to the altitude levels of 0.5–11.5, 0.7–9.1, and 1.0–5.8 km. The displayed points are the results of a Monte Carlo simulation, while the curves represent a quadratic-form approximation in the form (6).

showers, the parameter $\mu(t)$ falls within the range of $0 < \mu(t) < 1$. The parameter $\alpha(t)$ is positive at all depths. As an alternative to the approximation in (7), we can consider an approximation in the exponential form

$$j(t, r_{\perp}) \propto \frac{e^{-r_{\perp}/a(t)}}{b(t) + r_{\perp}}, \quad (8)$$

where $b(t)/a(t) \leq 0.1$. The function in (8) does not describe the behavior of the current over the range of distances that extends up to 1 km, but we do not need this. For our purposes, it is sufficient that formula (8) is able to describe the lateral distribution within the first 100 to 200 m. An advantage of this approximation above that in (7) is that, as we will see below, the formula obtained on its basis for the radio-emission field is especially simple.

We will now proceed to consider expression (4) and focus our attention on integration with respect to the transverse size of extensive air showers. Disregarding the nonwave part (it is proportional to $1/r^2$), we obtain

$$\mathbf{E}_{\omega}(\mathbf{x}) = -\frac{i\omega}{8\pi^2\epsilon_0 c^2} \frac{1}{k^2} \quad (9)$$

$$\times \int [\mathbf{k}_0 \times [\mathbf{k}_0 \times \mathbf{J}(\omega, t)]] \frac{e^{i\omega t(1-n\mathbf{k}_0 \cdot \mathbf{s}/k)}}{R_0} dt,$$

$$\mathbf{J}(\omega, t) = \int \mathbf{j}(t, \mathbf{r}_{\perp}) e^{-i\mathbf{k} \cdot \mathbf{r}_{\perp}} e^{-i\mathbf{k} \cdot \boldsymbol{\zeta}(t, \mathbf{r}_{\perp})} d\mathbf{r}_{\perp}, \quad (10)$$

$$\begin{aligned} & \mathbf{k} \cdot \mathbf{r}_{\perp} + \mathbf{k} \cdot \boldsymbol{\zeta}(t, \mathbf{r}_{\perp}) \\ & = k \cos(\phi - \psi)(r_{\perp} \sin \theta + \zeta(t, r_{\perp}) \cos \theta), \end{aligned} \quad (11)$$

where R_0 is the distance between the extensive-air-shower center (at the instant t) and the point $\mathbf{x}$, ψ is the polar angle of the ground-based observation point $\mathbf{x}$, and θ is the angle between the shower axis $\mathbf{s}$ and the observation direction $\mathbf{k}$. In (9), it was adopted that $1/r \approx 1/R_0$ and $\mathbf{k}_0 (=k\mathbf{R}_0/R_0) \approx \mathbf{k} (=k\mathbf{r}/r)$. We will disregard the current azimuthal asymmetry, which stems from the presence of an electron excess and from the polarization of extensive air showers in the geomagnetic field and, at the level of individual showers, from fluctuations in shower development.

Considering that

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-ix \cos \phi} d\phi = J_0(x),$$

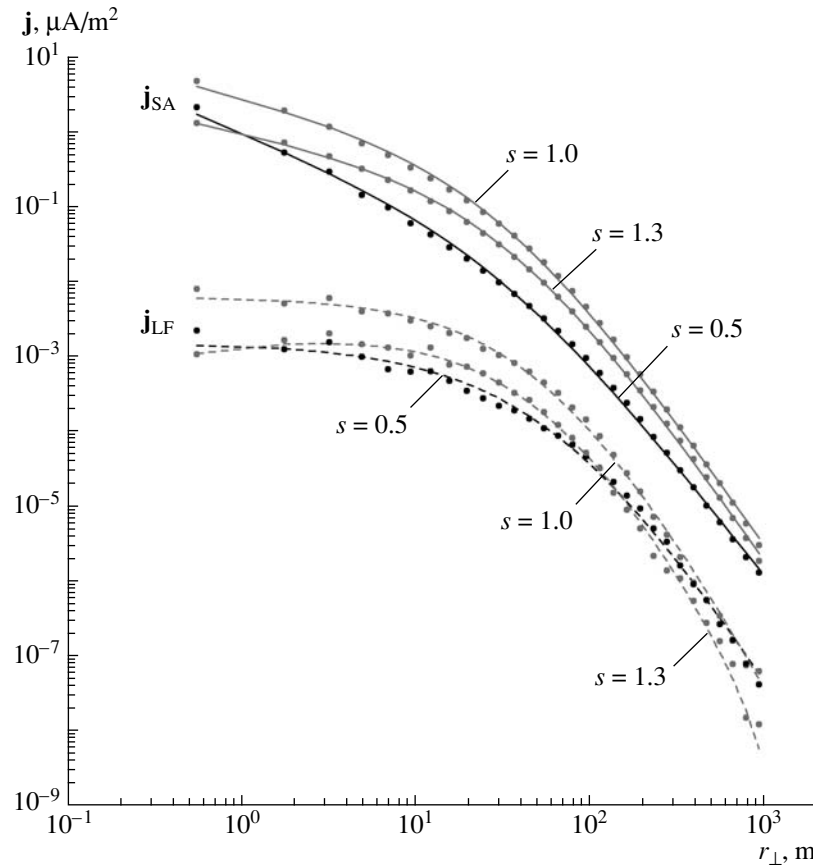


Fig. 3. Azimuthal-angle-integrated lateral distribution of the electron-current density in an individual vertical shower with energy $E_0 = 10^{16}$ eV at various stages of its development. The values indicated for s correspond to the altitude levels of 0.5–11.5, 1.0–5.8, and 1.3–2.8 km. Shown in the figure are the projections of the current onto the shower axis ($\mathbf{j}_{SA}$) and onto the Lorentz force direction ($\mathbf{j}_{LF}$), which is proportional to $[\mathbf{s} \times \mathbf{B}]$, which is approximately the west–east direction. The displayed points stand for the results of a Monte Carlo simulation, while the curves represent an approximation in the form (7).

where $J_0(x)$ is a Bessel function of order zero [35], we find from expressions (10) and (11) that

$$\mathbf{J}(\omega, t) = \int J_0\left(\frac{n\omega}{c}(r_\perp \sin \theta + \varsigma(t, r_\perp) \cos \theta)\right) \mathbf{j}(t, r_\perp) r_\perp dr_\perp. \quad (12)$$

The first term in parentheses on the right-hand side of (12) ($n\omega r_\perp \sin \theta / c$) describes interference because of the transverse size of extensive air showers, while the second term ($n\omega \varsigma(t, r_\perp) \cos \theta / c$) owes its existence to the curvature of the shower front. In the observation of the radio-emission field at distances $R_\perp$ of zero to several hundred meters from the extensive-air-shower axis, the phase difference arises primarily because of a nonzero curvature of the shower front. At frequencies lower than 300 MHz, we can set

$$J_0\left(\frac{n\omega}{c}(r_\perp \sin \theta + \varsigma(t, r_\perp) \cos \theta)\right) \quad (13)$$

$$\approx J_0\left(\frac{n\omega \tilde{\varsigma}_1(t)}{c} r_\perp\right),$$

where $\tilde{\varsigma}_1(t) = \sin \theta + \varsigma_1(t) \cos \theta$.

The integral in (12) can be calculated by using the method of Fresnel zones [31]. According to this method, we have

$$\mathbf{J}(\omega, t) = \int_0^\infty (\dots) dr_\perp = F(\omega) \int_0^{r_\perp^{(1)}} (\dots) dr_\perp, \quad (14)$$

where $r_\perp^{(1)}$ is the first zero of the respective Bessel function [35],

$$J_0\left(\frac{n\omega \tilde{\varsigma}_1(t)}{c} r_\perp^{(1)}\right) = 0, \quad (15)$$

which corresponds to

$$\frac{n\omega \tilde{\varsigma}_1(t)}{c} r_\perp^{(1)} \approx 2.4.$$

At small observation distances $R_\perp$ from the shower axis (first few hundred meters), the quantity $\tilde{\zeta}_1(t)$ changes only slightly with depth, and the size of the first Fresnel zone is $r_\perp^{(1)} \approx (3.5-4.5)\lambda$. The factor $F(\omega)$ is to be determined. One can readily show that it possesses the following properties: $F(\omega)$ decreases monotonically in the interval $[0, \infty)$ and ranges between $F(0) = 1$ and $F(\infty) \approx 5/8$, its derivative at the origin being $F'(0) = 0$. Further, it only remains to find the first corrections to the limiting values of $F(\omega)$ and to match the resulting asymptotic expressions. It is not easy to do this for $\omega \rightarrow \infty$. In practice, however, the properties described above prove to be quite sufficient for reconstructing the sought function to a precision of a few percent. For a trial function, we can take a function of the form

$$F(\omega) = \frac{5 \left(a/r_\perp^{(1)} \right)^2 + 1}{8 \left(a/r_\perp^{(1)} \right)^2 + 1}, \quad (16)$$

where a is the root-mean-square radius of the lateral distribution of the current. The calculation of the lateral distribution by formula (4) in the case of a complete integration with respect to $r_\perp$ (from 0 to 1 km) and in the case of integration in accordance with expressions (14) and (16) leads to nearly identical results (at frequencies in the range 10–100 MHz, the discrepancy does not exceed 2 to 3%).

The method of Fresnel zones does not lead (in an explicit form) to the reduction of the dimensionality of the original integral, but it clearly demonstrates advantages of employing (in qualitatively analyzing radio emission from extensive air showers) the concept of Fresnel zones. For example, it follows from expression (14) that, if $r_\perp^{(1)}$ exceeds the characteristic width of the lateral distribution of current a , then the entire extensive air shower (at a given depth) emits coherently. Otherwise, radio emission from extensive air showers is similar to emission from a bunch whose transverse dimension is approximately equal to the dimension of the first Fresnel zone, $r_\perp^{(1)}$, so that, for radio emission from extensive air showers, one can introduce the effective emission radius defined as the size of the region important for the formation of the field at the point where one receives the signal being considered. As the frequency grows, this radius decreases, so that radio emission from extensive air showers becomes ever more similar to radiation from a pointlike source. It follows that all corrections to the field in the form (9) that stem from taking into account transverse dimensions of extensive air showers at the level of the factor $1/r$ in (4) contain (in addition to the factor a/R_0) the factor $r_\perp^{(1)}/R_0$; that is, they decrease with increasing frequency.

We now revisit expression (12), representing it in the form

$$\begin{aligned} & \int J_0 \left(\frac{n\omega\tilde{\zeta}_1(t)}{c} r_\perp \right) \mathbf{j}(t, r_\perp) r_\perp dr_\perp \\ &= \Phi(\omega, t) \int \mathbf{j}(t, r_\perp) r_\perp dr_\perp, \end{aligned} \quad (17)$$

where $\Phi(\omega, t) = \Phi_\alpha(\omega, t)$ ($\alpha = 1, 2, 3$) is the interference factor for each of the current components $\mathbf{j}$. Expanding the respective Bessel function in a series [35], we obtain

$$\begin{aligned} \Phi_\alpha(\omega, t) &= 1 - \frac{1}{4} \left(\frac{n\omega\tilde{\zeta}_1}{c} \right)^2 \frac{\langle r_\perp^2 \rangle}{\langle r_\perp^0 \rangle} \\ &+ \frac{1}{64} \left(\frac{n\omega\tilde{\zeta}_1}{c} \right)^4 \frac{\langle r_\perp^4 \rangle}{\langle r_\perp^0 \rangle} - \dots, \end{aligned} \quad (18)$$

$$\langle r_\perp^n \rangle = \int j_\alpha(t, r_\perp) r_\perp^n r_\perp dr_\perp. \quad (19)$$

In the frequency range studied in the experiments reported in [5, 6] (20–80 MHz), the series in (18) converges slowly (at the maximum of vertical showers, we have

$$\frac{\langle r_\perp^2 \rangle}{\langle r_\perp^0 \rangle} \sim 3 \times 10^3 \text{ m}^2, \quad \frac{\langle r_\perp^4 \rangle}{\langle r_\perp^0 \rangle} \sim 10^7 \text{ m}^4,$$

while, for the frequencies in those experiments, $n\omega\tilde{\zeta}_1/c \sim 0.1 \text{ m}^{-1}$). It follows that, although the factor $\Phi(\omega, t)$ in (18) is expressed in terms of the moments of the current lateral distribution (that is, integrals that depend only on the depth), it is desirable to find a different, more compact, representation of $\Phi(\omega, t)$; for this, it is necessary to employ a more concrete expression for the lateral distribution of the current in a shower.

First of all, we will focus on the current component j_{LF} generated by the Earth's magnetic field, since it is precisely this component that makes a dominant contribution to the radio-emission field. Employing the approximation in (8) and setting $b(t) = 0$, we obtain

$$\Phi_{\text{LF}}(\omega, t) \approx \left(1 + \frac{n^2 \omega^2 \tilde{\zeta}_1^2(t)}{c^2} a^2(t) \right)^{-1/2}. \quad (20)$$

The parameter $a(t)$ can formally be expressed in terms of a momentum in (19) of any order—in general, not uniquely. From (8) [at $b(t) = 0$], we obtain

$$\begin{aligned} a(t) &= \left(\frac{1}{n!} \frac{\langle r_\perp^n \rangle}{\langle r_\perp^0 \rangle} \right)^{1/n} \\ \text{or } a(t) &= \frac{1}{n} \frac{\langle r_\perp^{n+1} \rangle}{\langle r_\perp^n \rangle}, \end{aligned} \quad (21)$$

where $n \geq 1$. The emerging “ambiguity” can be removed by requiring that, for

$$\frac{n\omega\tilde{\zeta}_1}{c}a(t) \ll 1, \quad (22)$$

the term that follows unity in the expansion of $\Phi_{\text{LF}}(\omega, t)$ in $n\omega\tilde{\zeta}_1 a(t)/c$ in (20) coincide with the analogous term in the expansion in (18), which obviously has a general character. For this, it is necessary to choose the first version in (21) at $n = 2$; that is,

$$\Phi_{\text{LF}}(\omega, t) \approx \left(1 + \frac{n^2\omega^2\tilde{\zeta}_1^2 \langle r_{\perp}^2 \rangle}{2c^2 \langle r_{\perp}^0 \rangle}\right)^{-1/2}. \quad (23)$$

An analysis shows that, in this case, the error in calculating $\Phi_{\text{LF}}(\omega, t)$ because of the disregard of the parameter $b(t)$ in (8) proves to be about 3%. With allowance for expression (23), the field then has the form

$$\begin{aligned} \mathbf{E}_{\omega}(\mathbf{x}) &\approx -\frac{i\omega}{8\pi^2\varepsilon_0 c^2} \frac{1}{k^2} \\ &\times \int \left(1 + \frac{n^2\omega^2\tilde{\zeta}_1^2 \langle r_{\perp}^2 \rangle}{2c^2 \langle r_{\perp}^0 \rangle}\right)^{-1/2} \\ &\times [\mathbf{k}_0 \times [\mathbf{k}_0 \times \langle \mathbf{r}_{\perp}^0 \rangle]] \frac{e^{i\omega t(1-n\mathbf{k}_0 \cdot \mathbf{s}/k)}}{R_0} dt, \\ \langle \mathbf{r}_{\perp}^0 \rangle &= \int \mathbf{j}_{\text{LF}}(t, r_{\perp}) r_{\perp} dr_{\perp}. \end{aligned} \quad (24)$$

Proceeding to consider the frequency spectrum of radio emission, we find from (24) that, under conditions of full coherence, the field in question is in direct proportion to the frequency ω . Specifically, we have

$$\begin{aligned} \mathbf{E}_{\omega}(\mathbf{x}) &\approx -\frac{i\omega}{8\pi^2\varepsilon_0 c^2} \frac{1}{k^2} \\ &\times \int [\mathbf{k}_0 \times [\mathbf{k}_0 \times \langle \mathbf{r}_{\perp}^0 \rangle]] \frac{1}{R_0} dt. \end{aligned} \quad (25)$$

(We note, however, that, at frequencies below 1 MHz, the inclusion of the nonwave part leads to the breakdown of this law; that is, $\mathbf{E}_{\omega} \propto \omega^{\alpha}$, where $0 < \alpha < 1$.) As the frequency grows and at moderate distances from the shower axis, the condition of coherence in the direction orthogonal to the shower axis [see the strong inequality in (22)] becomes invalid, with the result that, for

$$\eta(\omega, t) = \frac{n\omega\tilde{\zeta}_1 \langle r_{\perp}^2 \rangle^{1/2}}{c \langle r_{\perp}^0 \rangle^{1/2}} > 1, \quad (26)$$

the spectrum flattens

$$\mathbf{E}_{\omega}(\mathbf{x}) \approx -\frac{i\sqrt{2}}{8\pi^2\varepsilon_0 c n} \frac{1}{k^2} \quad (27)$$

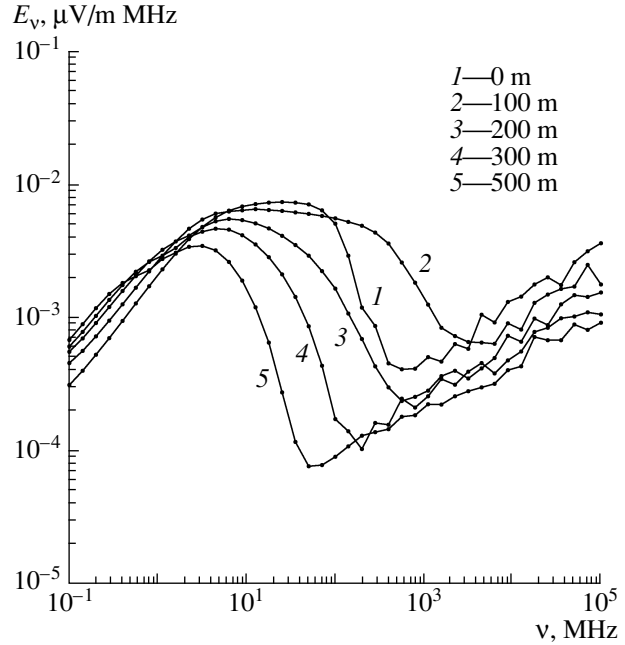


Fig. 4. Frequency spectrum of the strength of the total field of radio emission (microscopic calculation) from a vertical extensive air shower of energy $E_0 = 10^{15}$ eV at various distances from the shower axis. The field was averaged over ten showers.

$$\times \int \frac{1}{\tilde{\zeta}_1(t)} \frac{\langle r_{\perp}^0 \rangle^{1/2}}{\langle r_{\perp}^2 \rangle^{1/2}} [\mathbf{k}_0 \times [\mathbf{k}_0 \times \langle \mathbf{r}_{\perp}^0 \rangle]] \frac{1}{R_0} dt.$$

In the case of the breakdown of coherence in the longitudinal direction, the phase factor in (24) oscillates, which entails a decrease in the field strength as the frequency grows (see Fig. 4).

We now proceed to consider the current component j_{SA} parallel to the extensive-air-shower axis, employing this time an approximation of the form (7). In this case, it is difficult to calculate precisely the corresponding interference factor $\Phi_{\text{SA}}(\omega, t)$, but one can readily obtain expressions for it in the limiting cases of small and large $\eta(\omega, t)$ (26). For $\eta(\omega, t) \ll 1$, the factor $\Phi_{\text{SA}}(\omega, t)$ is determined by the expansion in (18). In the region where $\eta(\omega, t) \gg 1$, the behavior of j_{SA} only within the first Fresnel zone, whose dimension is smaller than r_0 , is of importance for calculating the integral appearing on the left-hand side of (17); therefore, a dominant contribution to it comes from the first factor in (7)—that which is proportional to $(r_{\perp}/r_0)^{\mu-1}$. Denoting by $h(r_{\perp})$ the second factor in (7), we can obtain, in this case, the expansion [36]

$$\begin{aligned} \Phi_{\text{SA}}(\omega, t) &\simeq 2^{\mu} \left(\frac{n\omega\tilde{\zeta}_1 r_0}{c} \right)^{-(1+\mu)} \\ &\times \left(c_0 + \sum_{m=1}^{\infty} c_m \frac{h^{(m)}(0)}{m!} \left(\frac{n\omega\tilde{\zeta}_1 r_0}{c} \right)^{-m} \right), \end{aligned} \quad (28)$$

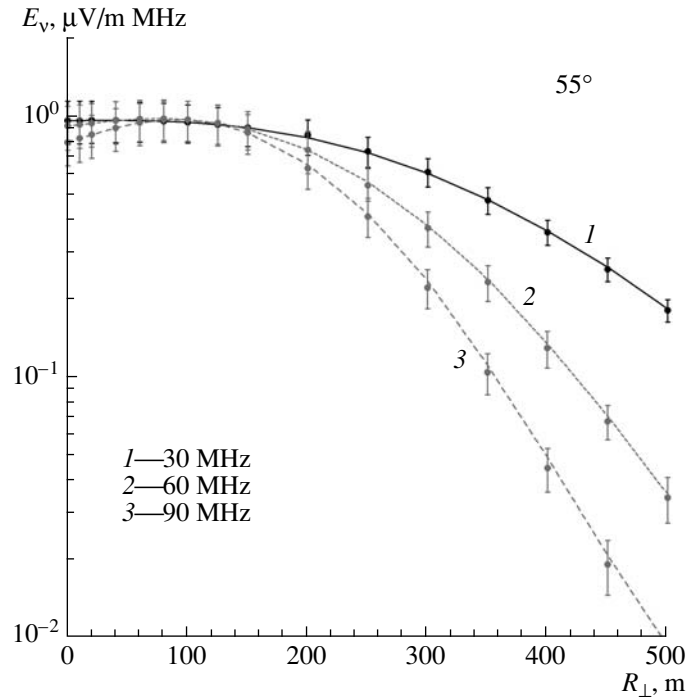


Fig. 5. Lateral distribution (in the direction to the geographical north) of the strength of the total field of radio emission from an extensive air shower having an energy of $E_0 = 10^{17}$ eV and arriving from the north at an angle of 55° . The results of the microscopic calculation (curves) are contrasted against their counterparts from the macroscopic calculation by formula (24) (points). The statistical sample used included ten showers (the thinning level was 10^{-6} , and the maximum statistical weight was 10^3 [21]).

$$c_m = \frac{\Gamma((1+m+\mu)/2)}{\Gamma(1-(1+m+\mu)/2)}, \quad (29)$$

$$h^{(m)}(0) = (-1)^m \prod_{l=0}^m (\alpha + l),$$

which is valid for $\mu(t) < 1/2^3$ and where $\Gamma(x)$ is the Euler gamma function. One can see that the parameter $\alpha(t)$, which determines [in the combination $\mu(t) + \alpha(t)$] the behavior of $j_{SA}(t, r_\perp)$ at large distances ($r_\perp \geq r_0$), appears in $\Phi_{SA}(\omega, t)$ as a small (for $\omega \rightarrow \infty$) correction and does not enter into the first, leading, term of the expansion in (28). Thus, we have

$$\lim_{\omega \rightarrow 0} \Phi_{SA}(\omega, t) = 1 - \frac{1}{4} \left(\frac{n\omega\tilde{\zeta}_1}{c} \right)^2 \frac{\langle r_\perp^2 \rangle}{\langle r_\perp^0 \rangle} + O(\omega^4), \quad (30)$$

³⁾This restriction follows from the requirement that the integral $\int_0^\infty x^\mu J_0(x) dx$ be finite, but we can readily remove it if, in (12), we avoid disregarding the quadratic term (which is of importance at large distances $r_\perp$) in the function in (6), which describes the shape of the extensive-air-shower front. Thus, the case in point is the convergence of the integral $\int_0^\infty x^\mu J_0(x^2) dx$.

$$\lim_{\omega \rightarrow \infty} \Phi_{SA}(\omega, t) = f(\mu) \left(\frac{n\omega\tilde{\zeta}_1 r_0}{c} \right)^{-(1+\mu)} + O(\omega^{-(2+\mu)}), \quad f(\mu) = 2^\mu c_0(\mu). \quad (31)$$

These asymptotic expressions can be matched with each other (see for example, [37]). The character of the function $\Phi_{SA}(\omega, t)$ is not expected to be strongly different from the character of the function $\Phi_{LF}(\omega, t)$. At low frequencies, these two functions have the same asymptotic behavior, while, at high frequencies and $\mu(t) \rightarrow 0$, the factor $\Phi_{SA}(\omega, t)$ obviously goes over to $\Phi_{LF}(\omega, t)$. As a possible version, one can therefore take expression (23) as a starting point; that is,

$$\Phi_{SA}(\omega, t) \approx \left(1 + \frac{\eta^2(\omega, t)}{2(1+\mu)} \right) \times \left(1 + \frac{\eta^2(\omega, t)}{1 + \eta^2(\omega, t)} \right) \times \left(2^{-(1+\mu)/2} f(\mu) - 1 \right) \right)^{-(1+\mu)/2}, \quad (32)$$

where $\eta(\omega, t) = n\omega\tilde{\zeta}_1 r_0/c$ and $r_0^2 = \langle r_\perp^2 \rangle / \langle r_\perp^0 \rangle$. Expression (32) reproduces the asymptotic behavior of $\Phi_{SA}(\omega, t)$ in the first- and zero-order approximation

for $\omega \rightarrow 0$ and $\omega \rightarrow \infty$, respectively, and one can refine it, if necessary.

The lateral distributions of the total field of radio emission from extensive air showers are shown in Fig. 5 according to calculations performed within the microscopic approach and by formula (24) [the interference factor in (32) was used to calculate radiation from the component j_{SA}]. One can see that only at large distances from the extensive-air-shower axis are there discrepancies. We also note that, as might have been expected, the replacement of expression (23) by expression (32) has virtually no effect on the result in calculating the field. This is because expressions (7) and (8) approximate equally well the lateral distributions of the current components at distances in the range $r_{\perp} < 100\text{--}200$ m, a precise knowledge of the behavior at longer distances being immaterial for calculating the radiation integral (see Fig. 3).

The interference factor $\Phi(\omega, t)$ was calculated in [38–40] under various assumptions on the lateral distribution of the source density. In the last two studies, use was made of a Gaussian function, $\propto \exp(-r_{\perp}^2/a^2)$, which does not describe any component $\mathbf{j}(t, r_{\perp})$ (see Fig. 3), and the resulting factor $\Phi(\omega, t) \propto \exp(-\omega^2/\omega_0^2)$, where ω_0 is a constant, leads (for $\omega > \omega_0$) to a decrease in the field by several orders of magnitude in response to only a severalfold increase in ω , but, as one can see from Fig. 4, this is incorrect. Askaryan's study reported in [38] is of interest since the function in (8) at $b(t) = 0$ was considered there and since expression (20) for $\Phi(\omega, t)$ was derived in that way but (and this is of importance) with a different coefficient of ω : instead of $\zeta_1(t) = \sin \theta + \zeta_1(t) \cos \theta$, $\sin \theta$ was obtained in [38]. We have already indicated that it is a nonzero curvature of the shower-particle front rather than the transverse size of extensive air showers that makes a dominant contribution to the change in the phase of radiation at short distances from the shower axis. A direct calculation reveals that, in the frequency range 10–100 MHz, the disregard of the front shape leads to an overestimation of the field by a factor of five to ten.

4. DISCUSSION OF THE RESULTS

Our quest for a minimization of the volume of information about extensive air showers that is used in calculating radio emission is associated with the fact that, in solving the inverse problem, one can hope for extracting only this volume of information from experimental data. We have found that, in order to calculate the field of radio emission from extensive air showers, it is sufficient to know moments of the current lateral distribution that are of order not higher than two. There are arguments in support of the statement

that this result does not stem from approximating the current lateral distribution by specific functions [expressions (7) and (8)], but that it expresses a feature peculiar to the integral of radiation from extensive air showers in the radio-frequency range.

From a general theory, it is well known that, under specific conditions, the set of moments of a function determines it completely. Following [41], we formulate these conditions, putting aside for the time being the question of the procedure for reconstructing a specific function by its moments. Let us consider the integral

$$\langle x^n \rangle = \int_0^{\infty} x^n f(x) x dx, \quad (33)$$

where we imply by x the distance from the shower axis and by $f(x)$ the source density normalized in such a way that $\langle x^0 \rangle = 1$. We assume that the function $f(x)x$ defined in the interval $x \in [0, \infty)$ is continuous, decreases monotonically, and does not change sign.⁴⁾ According to [41], the moments in (33) determine the function $f(x)$ unambiguously if

$$\lim_{n \rightarrow \infty} \frac{\langle x^n \rangle^{1/n}}{n} < \infty. \quad (34)$$

If the function $f(x)$ is concentrated within a finite interval of length Δx , then the condition in (34) obviously holds since $\langle x^n \rangle \leq \Delta x^n$ and, hence, $\langle x^n \rangle^{1/n}/n \leq \Delta x/n$. Since any real shower has finite dimensions, there is always an interval $\Delta r_{\perp}$ beyond which there are no particles, so that $f(r_{\perp}) = 0$ for all $r_{\perp} > \Delta r_{\perp}$ [in this sense, one must correct expressions (7) and (8) or simply cut them off at large distances from the axis].

There are several methods for reconstructing the function $f(x)$ from its moments [29]. The method that was proposed by Chartres [42] and in which the Mellin transformation is used to reconstruct $f(x)$ is the most appropriate for our purposes. This method does not require introducing a weight function [29] and relies on the most general assumptions on the function $f(x)$ (it must be continuous and decrease

⁴⁾From Fig. 3, one can see that, in the case of $f(x)x \equiv j_{\alpha}(t, r_{\perp})r_{\perp}$, the function $f(x)x$ does not decrease monotonically in the interval $[0, \infty)$: there is a point x_0 at which $f(x)x$ reaches a maximum. However, this is not critical since we can break down the interval $[0, \infty)$ into two subintervals $[0, x_0)$ and $[x_0, \infty)$. After the linear change of variable $x \rightarrow x' = x_0 - x$ in the first interval, we arrive at considering two functions that satisfy all of the aforementioned requirements.

monotonically). In [42], the direct Mellin transformation is defined as follows:

$$h(s) = \int_0^{\infty} x^{2s} f(x) x dx. \quad (35)$$

The inverse transformation is

$$f(x) = \frac{1}{2\pi i} \int_{s_0-i\infty}^{s_0+i\infty} x^{-2(s+1)} h(s) ds. \quad (36)$$

For integral values of $s = n$, the direct Mellin transformation determines the moments of $f(x)$: $h(n) = \langle x^{2n} \rangle$. Setting $h(s) = \exp(h_0(s))$ and employing the saddle-point method [35] to calculate the integral in (36), we arrive at

$$f(x) = \left(\frac{2}{\pi}\right)^{1/2} \frac{x^{-2(s+1)} h(s)}{(h_0''(s))^{1/2}}, \quad (37)$$

$$h_0'(s) = 2 \ln x. \quad (38)$$

Expression (37), which is defined for $x > 0$, establishes a direct relationship between the value of the function $f(x)$ at the point x and the moment $h(s) = \langle x^{2s} \rangle$ whose order is determined from relation (38), the value of $f(x)$ being expressed precisely in terms of that moment which makes a dominant contribution to the function at the point x (for details, see [42]). One can readily show that the last circumstance does not arise from employing the Chartres method as such but has a general character: low- and high-order moments form the function under study at, respectively, small and large values of its argument.

To summarize, we indicate that, in the limit of low frequencies [$\eta \rightarrow 0$; see the condition in (26)], only the first (zero-order) moment $\langle r_{\perp}^0 \rangle$ [see (25)] survives in the radiation integral and that, for $\eta \rightarrow \infty$, in which case the size of the first Fresnel zone, $r_{\perp}^{(1)}$, is much smaller than the characteristic radius of the source lateral distribution [$\simeq (\langle r_{\perp}^2 \rangle / \langle r_{\perp}^0 \rangle)^{1/2}$], the calculation of the radio-emission field requires knowing the behavior of the lateral distribution only within this zone (see the method of Fresnel zones), the moment $\langle r_{\perp}^n \rangle$ corresponding to the distance $r_{\perp} = r_{\perp}^{(1)}$ (in the sense specified in discussing the Chartres method). As the frequency increases, the order of this moment decreases, tending to zero. All of the foregoing is true, irrespective of the form of the source lateral distribution.

It follows that, if we introduce a function $S(\eta)$ of the argument η , defining it as the maximum order of the moment $\langle r_{\perp}^n \rangle$ (where n is arbitrary), that must be known in calculating the field of radio emission

from extensive air showers to a preset precision, then this function must possess the following properties:

$$\lim_{\eta \rightarrow 0} S(\eta) = 0, \quad \lim_{\eta \rightarrow \infty} S(\eta) = 0, \quad (39)$$

$$S'(\eta) > 0 \text{ at } \eta \ll 1, \quad (40)$$

$$S'(\eta) < 0 \text{ at } \eta \gg 1.$$

Since the function $S(\eta)$ is positive and finite at all $\eta \in [0, \infty)$, it follows from the properties in (39) and (40) that there exists a point $\eta = \eta_0$ at which $S(\eta)$ reaches a maximum. By virtue of the definition of the function $S(\eta)$, the region of the maximum is in principle the most informative in solving the inverse problem. One can expect that $S(\eta_0) \simeq 2$, η_0 being $\eta_0 \sim 1$. Indeed, one can see from the general expansion in (18) that it is precisely the relation $r_{\perp}^{(1)} = (\langle r_{\perp}^2 \rangle / \langle r_{\perp}^0 \rangle)^{1/2}$ (which corresponds to $\eta \approx 2.4$) that determines the boundary beyond which one of the asymptotic expressions becomes dominant (depending on whether the argument η increases or decreases); therefore, $S(\eta)$ begins decreasing.

5. CONCLUSIONS

From the fact that the quantity $\mathbf{j}(\mathbf{x}')$ appears in the radiation integral (1), by no means does it follow that, in solving the inverse problem, one can reconstruct this quantity as comprehensively as it is specified in solving the direct problem. The analysis presented in this article revealed what integrals $\mathbf{j}(\mathbf{x}')$ must be known for adequately calculating the radio-emission field and, hence, what information about an extensive air shower radio emission from it carries. The former leads to the reduction of the computation time, while the latter brings to an end the discussion on the potential of the radio method as a tool for studying extensive air showers.

An analysis of the original integral in (1) led us to its rather simple modification in (24), which, as was shown, is appropriate for quite precisely calculating the field at frequencies below about 100 MHz. From (24), it follows that, in order to calculate this field, it is necessary to know only three depth-dependent features of a shower. These are the vector of the total current (orthogonal to the shower axis), the root-mean-square radius of the lateral distribution of this current, and one parameter that characterizes the shape of the extensive-air-shower front.

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