Astronomy Letters, Vol. 24, No. 2, 1998, pp. 228–236. Translated from Pis'ma v Astronomicheskii Zhurnal, Vol. 24, Nos. 3–4, 1998, pp. 275–284.
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The Astronomical Timescale Based on the Orbital Motion of a Pulsar in a Binary System

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Received September 9, 1996; in final form, November 25, 1997

Abstract—Assuming a highly deterministic pulsar's orbital motion in a close binary system, we have studied the possibility of using the orbital rotation period for establishing a new astronomical scale of the ephemeris dynamical time, which would allow the determination with high accuracy (10^{-12}–10^{-15}) of time intervals on long time scales of the order of several tens of years. This is necessary, in particular, for searching for ultra-low-frequency stochastic background of relict gravitational waves in the frequency range 10^{-9}–10^{-12} Hz and for studying the limiting possibilities of testing alternative theories of gravity based on observations of binary pulsars. A theoretical analysis is done of the actually attainable stability of such a scale in the presence of stochastic fluctuations of pulsar pulse arrival times, which represent a mixture of "white" and "red" noise with a power-law dependence of the noise intensity on the Fourier frequency f^{-s} for $s = 1, 2, ..., 6$. It is shown that the dynamical timescale realized in the pulsar's orbital motion proves to be more stable on long time intervals than the kinematical timescale based on the pulsar's rotation.

INTRODUCTION

Neutron stars rotating with enormous kinetic energy ~10^{45} J were suggested as the objects keeping time intervals in the timescale called the pulsar timescale (PT) by Shabanova *et al.* (1979). Subsequent studies have shown that some pulsars, in particular millisecond, which represent virtually "giant low-friction flywheels," are comparable in rotation period stability with quantum frequency standards on time intervals ~10 yr (Ilyasov *et al.* 1989; Kaspi *et al.* 1994). Nevertheless, in spite of high attainable timing precision, the PT cannot be treated as autonomous and ideally uniform. Its nonuniformity is determined by the following factors:

(1) Random noise variations of the phase of the pulsar's rotation caused by unpredictable variations in the internal structure of the neutron star and/or its magnetosphere;

(2) Stochastic variations in the delay of pulsar's radio signals during their propagation through the nonuniform interstellar medium;

(3) Stochastic character of residual errors in determining the relative position of the observer and the pulsar.

These noise components cannot be fully eliminated even if we discriminate properly these effects from the observed phase of the pulsar's rotation and introduce corrections by means of special data-processing techniques (two-frequency timing; VLBI measurements of coordinates, parallaxes, and proper motions). The energy spectrum of stochastic variations in residuals of pulse arrival times (PATs) determines the functional nature of the time dependence of Allan variance (Rutman 1978), which describes the stability of pulsar's rotation on long observation intervals, as well as the limiting possibilities of unambiguous reproduction of the rotation phase which constitutes the basis for forming the kinematic pulsar timescale (Ilyasov *et al.* 1989; Guinot and Petit 1991).

The autonomy and significant improvement of the given scale can be provided by observing several reference pulsars (Ilyasov *et al.* 1989; Backer and Foster 1990), which gives, as a result, the group pulsar timescale by analogy, for example, with creating the international atomic timescale. Such an approach does not remove, however, from the agenda the question of searching for other physical periodic processes, more stable than the process of the pulsar's rotation. They could be used for solving fundamental astrophysical problems, for example, for high-precision tests of general relativity theory with the use of binary pulsars or for searching for the low-frequency stochastic background of gravitational waves with a frequency of $<10^{-8}$ Hz.

A radically new opportunity for solving the given problem can be provided by constructing and using the dynamical binary pulsar timescale (BPT) based on the well-established periodic process of pulsar's orbital motion in a close binary system. As is well known (see, for example, Damour and Taylor 1992), the formalism

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of relativistic celestial mechanics permits one to find with high accuracy pulsar's orbital parameters in a binary system under the assumption that the orbital motion is deterministic. It is necessary to point out that information about pulsar's orbital motion is gleaned by the terrestrial observer from sets of PATs containing noise variations caused by different factors: free precession of the neutron star, the readjustment of the star internal structure and magnetosphere, Tkachenko waves, the influence of the interstellar medium, etc. This raises the fundamental question: what restricts the limiting possibilities of reproducing the dynamical pulsar timescale in the presence of the noise components in PATs of reference pulsars located in close binary systems? This work is devoted predominantly to the solution of this question. The results of analysis have shown that the long-timing method enables one to estimate the value and the characteristic time of gravitational effects produced by nearby stars or planets traveling in the vicinity of the binary system, as well as by gravitational waves of different astrophysical origin falling on this system, including the stochastic background originated at the early stage of the Universe evolution (Mashkhun and Grishchuk 1980; Kopeikin 1997a).

Obviously, the proposed dynamical pulsar timescale (BPT) can be considered as a modified analog of the ephemeris timescale used in classical celestial mechanics, the practical implementation of which was primarily accomplished so far by means of observations of the lunar orbital motion about the Earth (Guinot 1989). It is evident that binary pulsars as the keepers of a new timescale would be greatly superior to the Moon–Earth system, the fact which allows us to expect higher homogeneity and stability of the new BPT scale.

DOUBLE PULSARS AS INTERVAL-KEEPING OBJETCS

The following characteristics are critically important for frequency standards:

(1) Frequency stability, i.e., the ability to keep constant the value of the unit reference interval of time;

(2) Reproducibility of the reference interval, i.e., the possibility to use it for measurements at various instants of time;

(3) Longevity of the object that keeps the standard frequency.

In this work, we consider the binary system consisting of a pulsar and its companion as a system capable of keeping the reference frequency. Matter accretion and tidal effects are assumed to be absent in this system. The pulsar and its companion star may be considered, therefore, as point gravitational masses unaffected by external forces. The evolution of the orbit of two point masses is predicted with high accuracy on the basis of the laws of relativistic celestial mechanics, which has been considerably advanced since the discovery of binary pulsar PSR B1913+16 (Hulse and Taylor 1975). The binary system similar to PSR B1913+16 is an ideal laboratory for studies in the field of fundamental metrology. Considered as a frequency standard, it is a sufficiently long-life object for practical use, because the time of its existence is determined by the rate of loss of the orbital energy caused by the emission of gravitational waves, which is in excess of $>10^6$ yr.

As we noted above, the noise components in the rotational motion of individual pulsars cannot be fully excluded from the results of observations of residual PAT deviations reduced to the barycenter of the Solar System. This hinders the formation of an autonomous pulsar's stable timescale (PT) based on the pulsar's rotation on a time interval of ~5–10 yr. At the same time, in a system consisting of two compact massive bodies (pulsar and its companion) there occurs a virtually deterministic orbital motion of gravitating masses, which is governed by the laws of relativistic celestial mechanics. This was verified by Taylor (1992) with high accuracy. Thus, the well-predicted periodicity of pulsar's orbital motion about the companion star can be used to realize a new dynamic timescale BPT, which is potentially more accurate on long time intervals than other timescales (Ilyasov *et al.* 1996). [1]

By analogy with the pulsar timescale, the BPT is constructed as a sequence of discrete time intervals in the barycentric coordinate system of the stellar system under consideration according to the following algorithm:

$$T = T_0 + P_b\left(N + \frac{1}{2}\dot{P}_b N^2\right). \qquad (1)$$

Here, T_0 is the initial epoch, N is the number of orbital periods counted off by the fictitious observer at the barycenter of the binary system, and P_b and $\dot{P}_b$ are, respectively, the pulsar's orbital period and its time derivative referred to the epoch T_0. This exact formula does not contain terms which depend on the higher-order derivatives of the theoretical period P_b, which vanish identically in the pulsar's coordinate system. This is a consequence of the relativistic theory of pulsar's orbital motion in the binary system (Damour 1987; Schäfer 1985; Kopeikin 1985; Kopeikin and Potapov 1994).

It is worth noting that in the framework of our assumptions the orbital period and its derivative, in contrast to the pulsar's rotation period and its derivative, do not contain the noise components. This reflects the deterministic character of motion of bodies in binary pulsars, such as B1913+16 or J1713+0747, and permits one to consider these bodies the astronomical standards, which are more stable over long time intervals than the time standard based on the pulsar's proper motion.

[1] This opportunity was first pointed out by Yu.P. Ilyasov in his report at the scientific session of the Council on the Russian Federal Program "Fundamental Metrology" held on March 24, 1994.

The accuracy of reproducing the unit time interval on the BPT timescale depends on systematic and random metrological errors in the determination of rotational and orbital parameters of binary pulsars. Systematic errors are due to the specific structure of the timing algorithm for binary pulsars (Kopeikin 1994, 1996; Damour and Taylor 1992; Bell and Bailes 1996) and may vary slowly with time. The residual random errors in measuring angular and orbital parameters of a binary pulsar within the assumptions made above arise at the stage of observational data processing by particular statistical methods of minimization of barycentric residuals of pulsar's PATs. We also note that the accelerated radial motion of the barycenter of the binary system, in combination with its tangential proper motion relative to the Solar System's barycenter, leads to the appearance of higher-order time derivatives in the observed orbital periods of the binary pulsar (Damour and Taylor 1992; Bell and Bailes 1996) and in the observed projection of its semimajor axis onto the line of sight (Kopeikin 1996).

As a result of observational data processing, the noise components which are present predominantly in the pulsar's rotational phase turn out to be superimposed, thereby masking the reliably determined, in the calculations, periodic dependence of PATs on pulsar's orbital motion relative to the barycenter of the binary system. In other words, in processing the data we are dealing with the linear combination of the deterministic signal to be inferred and the additive noise. The latter is due to measurement uncertainties and various stochastic effects of astrophysical nature. As a result, the spectral composition of the noise process includes the white noise component present in measurements and the linear set of the components of the low-frequency (red) noise of astrophysical origin and/or of the errors of ephemeris provision. Each of the components of the low-frequency noise has an inverse power-law frequency dependence and can be represented as h_s/f^s, $s \geq 1$, where f is the frequency in the Fourier transform of the time sequence of PAT residuals and h_s is the quantity characterizing the noise intensity. The presented model can be treated as a manifestation of shot-effect noise in the pulsar's PAT containing both the stationary and nonstationary component (Kopeikin 1997b). The ergodicity of this stochastic process is not assumed. The presence of low-frequency noise components gives rise to specific limitations on the possibility of determination of pulsar's rotational and orbital parameters and, consequently, affects the stability of the pulsar rotation rate and pulsar's dynamical BPT.

It is significant to emphasize the necessity and timeliness of working out the fundamental approach to the problem of testing the general relativity theory in the strong gravitational field of a binary pulsar, because the presence of ill-filtered red-noise components in PAT residuals leads to the fundamental limitation imposed on the accuracy of determining the mass function and relativistic effects, whose real estimates are of prime importance for successful development of relativistic astrophysics and theory of gravity. It should be noted that the works by J. Taylor and his colleagues on tests of general relativity through observations of the binary pulsar B1913+16 (Taylor and Weisberg 1989; Taylor 1992), which have come to be classical, were carried out under the assumption that only a white-noise component is present in the pulsar's PAT. This assumption has now a firm basis but presumably cannot be exploited with further increase of the observation time interval.

ALGORITHM FOR CONSTRUCTING THE BINARY PULSAR TIMESCALE AND ESTIMATION OF ITS STABILITY

The comprehensive study of the problem of stability of the pulsar's dynamic BPT presents significant mathematical difficulties. To simplify calculations and provide the possibilities for analytic calculations, we restricted the discussions to the case of a binary pulsar with zero-eccentricity orbit. This enables us to avoid the necessity of using the complex transcendent functions present in the description of the translational motion of the pulsar moving in an elliptical orbit. Below we present the results of calculations providing the estimates of stability of the BPT in the presence of low-frequency red noise. These estimates are more accurate than those obtained earlier by Bertotti *et al.* (1983) and Petit and Tavella (1996). The problem of the optimum estimate of signal parameters against the background of the low-frequency red noise is a known but inadequately advanced problem of statistical radio physics (Van Trees 1968; Tikhonov 1983). In the case we consider here, the signal is a linear combination of the pulsar's rotational phase represented by the polynomial of time and the periodic sinusoidal component which depends on the orbital phase. Taking into account the above assumption of the noise additivity, we can represent the functional expression for the signal to be analyzed in the form

$$\xi(t) = N(t, \lambda_i) + \epsilon(t), \quad 0 \leq t \leq \tau, \tag{2}$$

where $N(t, \lambda_i)$ is the function of time describing the deterministic multiparameter signal component, λ_i is the set of the estimated parameters (ν, $\dot{\nu}$, $\ddot{\nu}$, n, $\dot{n}$, x, $\dot{x}$, T_0, t_0) whose meaning will be explained in the subsequent discussion, $\epsilon(t)$ is the stochastic noise process whose correlation function is presumed to be known (Kopeikin 1997b), and τ is the time interval on which the measurements of the above parameters are carried out. We assume that observations are distributed uniformly with frequency m within one orbital period of the binary system. This makes it possible, in particular, to change the summation over observation points for integration over time, i.e., to consider the observational process as continuous.

The function $N(t, \lambda_i)$ is determined by the expressions

$$N(t, \lambda_i) = \nu T + \frac{1}{2}\dot{\nu}T^2 + \frac{1}{6}\ddot{\nu}T^3 + \ldots, \tag{3}$$

$$\begin{gathered} T + [x + \dot{x}(T - T_0)] \\ \times \sin[n(T - T_0) + \dot{n}(T - T_0)^2] = t - t_0, \end{gathered} \tag{4}$$

where ν is the pulsar rotation rate; $\dot{\nu}$, $\ddot{\nu}$ and n are the time derivatives of the rotation rate; n is the orbital mean motion ($n = 2\pi/P_b$, where P_b is the orbital period), $\dot{n}$ is the derivative of the orbital mean motion ($\dot{n} = -2\pi\dot{P}_b/P_b^2$); x is the projection onto the line of sight of the semimajor axis of the pulsar orbit expressed in light seconds; and $\dot{x}$ is the projection of the semimajor axis. By T we denote the time in the pulsar's coordinate system, t is the barycentric time of the Solar System, T_0 is the instant of time corresponding to the initial orbital phase λ, and t_0 is the initial epoch of observations (we recall that $t_0 \neq T_0$).

In order to investigate the stability of the PT and BPT, it is appropriate to introduce two new parameters $y = \delta\nu/\nu$ and $v = \delta n/n$, where $\delta\nu$ and δn are the differences between the real physical value of the parameter under consideration and its estimate obtained by the least square fitting (LSF) in the processing procedure of pulsar's PATs. The variances of residual estimates of the orbital and rotation rates (σ_v^2 and σ_y^2, respectively) are most demonstrative for comparing two pulsar's timescales. The mathematical expressions for the two variances are called true variations (Rutman 1978). They represent an idealized version of Allan variance used in the fundamental metrology to analyze the stability of frequency and time standards. In our problem of constructing the dynamical pulsar timescale, the expressions for σ_v and σ_y depend on the particular type of noise. Their explicit functional dependences on the noise amplitude h_s ($s = 0, 1, \ldots, 6$ and on the interval of observations τ are listed in Table 1 and plotted in Fig. 1. We should emphasize that σ_y is independent of pulsar's orbital parameters because it characterizes only the instability of the pulsar's rotation and has nothing to do with its orbital motion. On the other hand, the quantity σ_v depends in a natural way on the pulsar's orbital period P_b, the projection x of the orbit onto the line of sight, and the initial position λ of the pulsar in the orbit, because these quantities characterize the phase and amplitude of the sinusoidal function in formula (4), whose parameters we attempt to determine against the additive noise. The additional dependence of σ_v on the initial orbital phase λ is due to the fact that the pulsar's orbital frequency is one of the quadrature components of the sinusoidal functions, along with the first time derivative of the projection of the semimajor axis. The point is that while using the LSF, red noises with the

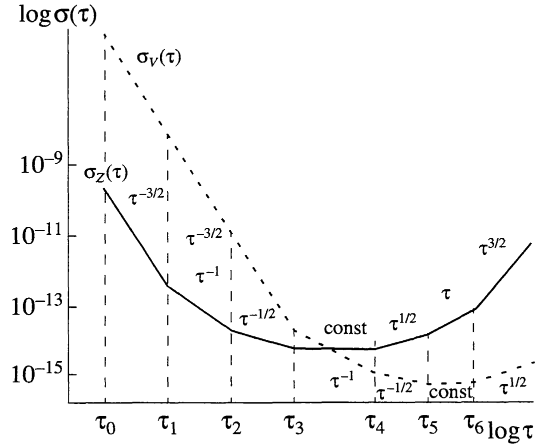

Fig. 1. Schematic behavior of the relative stability of pulsar's rotation rate characterized by the parameter σ_z (the solid line) and of pulsar's orbital frequency characterized by the parameter σ_v (the dashed line).

index $s \geq 2$ upset the balance in the accuracy of simultaneous determination of these quadrature components and lead to the relation between σ_v and the parameter λ. This relation is unacceptable from the physical point of view, indicating presumably the statistical inefficiency of the estimates obtained by the LSF in the presence of red noises. The determination of the effective estimates in the presence of red noises remains an as yet unresolved problem.

When deriving the formulas given in Table 1, the parameter λ was considered as concomitant (noninformative). For this reason, in the variance calculation we made the additional averaging over λ on the interval $[0, 2\pi]$ assuming the uniform probability density for this parameter.

The white phase noise caused by measurement errors and possibly by the correlated noises of the pulsar rotation phase is assumed to dominate on the sampling interval $[\tau_0, \tau_1]$. Various components of the red noise spectrum begin to manifest themselves on the intervals $\tau > \tau_1$. Their amplitudes h_s, as it follows from the experience of operation of quantum frequency standards, progressively decline with increasing the noise spectral index s (i.e., the exponent in the Fourier frequency f in the denominator of the red noise spectral density). For example, in the model situation considered in Fig. 1, the red flicker noise with the spectral density $1/f$ dominates in the range τ_1, τ_2, whereas in the range $\tau_2 < \tau < \tau_3$ the red noise of random walks with the spectral density $1/f^2$ prevails, etc. In general, the longer the observing interval, the more significant is the contribution of noise with large spectral index s. This is due to the fact that, according to our assumption, the noise with higher s has a lower amplitude h_s and therefore can provide significant contribution only over longer time intervals τ.

Table 1. Variances $\sigma_y(\tau)$, $\sigma_z(\tau)$, and $\sigma_V(\tau)$ for noises with various power spectra. The following notation is used: h_s ($s = 0, 1, 2, \ldots, 6$) is the intensity of the power spectrum component; P_b is the orbital period; x is the projection of the large semimajor axis of the orbit onto the line of sight; Δt is the time interval between the consecutive observations; $C_3, \ldots, C_6$ are the positive constants (~0.05–0.5), which depend on the nonstationary component of the noise model under consideration (Kopeikin 1997b)

$S(f)$	$\sigma_y^2(\tau)$	$\sigma_z^2(\tau)$	$\sigma_V^2(\tau)$
h_0	$\frac{3675}{16}\Delta t h_0 \tau^{-3}$	$\frac{2835}{16}\Delta t h_0 \tau^{-3}$	$\frac{75}{2\pi^2}\frac{P_b^2}{x^2}\Delta t h_0 \tau^{-3}$
h_1/f	$\frac{4851}{64}h_1\tau^{-2}$	$\frac{2499}{64}h_1\tau^{-2}$	$\frac{75}{4\pi^2}\frac{P_b^3}{x^2}h_1\tau^{-3}$
h_2/f^2	$\frac{1575}{416}h_2\tau^{-1}$	$\frac{441}{416}h_2\tau^{-1}$	$\frac{8925}{748\pi^4}\frac{P_b^4}{x^2}h_2\tau^{-3}$
h_3/f^3	$(C_3 + \ln\tau)h_3$	$\frac{819}{2560}h_3$	$\frac{15}{32\pi^4}\frac{P_b^4}{x^2}h_3\tau^{-2}$
h_4/f^4	$\left(C_4 - \frac{525}{18304}\right)h_4\tau$	$\frac{203}{18304}h_4\tau$	$\frac{25}{2288\pi^4}\frac{P_b^4}{x^2}h_4\tau^{-1}$
h_5/f^5	$\frac{1}{4}(C_5 + \ln\tau)h_5\tau^2$	$\frac{93}{20480}h_5\tau^2$	$\frac{5}{1792\pi^4}\frac{P_b^4}{x^2}h_5$
h_6/f^6	$\left(C_6 - \frac{581}{5601024}\right)h_6\tau^3$	$\frac{21}{77792}h_6\tau^3$	$\frac{5}{64064\pi^4}\frac{P_b^4}{x^2}h_6\tau$

It is of interest that the behavior of $\sigma_y(\tau)$ coincides basically with the shape of the curve for the so-called narrow-band variance of the frequency of time standards used in fundamental metrology in the presence of low-frequency noises (Rutman 1978). However, in the model examined here $\sigma_y(\tau)$ depends substantially on the nonstationary red-noise component with the spectral index $s \geq 3$ (Table 1). Since the nonstationary noise component cannot be modeled accurately, this dependence $\sigma_y(\tau)$ points to the need for working out another, more practical approach to the estimate of the variance characterizing instability of pulsar rotation. It is possible that the modified dispersion measure σ_z^2 suggested by Taylor (1992), which is related to the use of the formalism of transitional filtering functions in the spectral region is a quite constructive and fruitful step in this direction. In particular, Matsakis *et al.* (1997) obtained the expression for $\sigma_z(\tau)$ in which $\sigma_z(\tau)$ is proportional to the variation of the second derivative of the pulsar's rotation rate. We used our algorithm for calculating σ_z (see Table 1). It was found that the σ_z value does not depend on the nonstationary noise component and does not contain logarithmic trends. The same inference is also valid for the variance σ_V. In addition, our calculations show clearly that the instability of the pulsar's orbital phase $\sigma_V(\tau)$ is insensitive to the noises with spectral indices $s = 1, 2$ and, therefore, does not allow one to differentiate these noises from the white noise. On the other hand, such a behavior permits one to use the binary pulsar orbital phase as a new time standard, which is more stable at long time intervals. It is also worth noting that we included in Table 1 only the main terms in the expansions of the quantities σ_y, σ_z, and σ_V in inverse powers of τ. This means that these formulas are valid only for sufficiently long observation intervals.

As is evident from Fig. 1, the $\sigma_z(\tau)$ value of the pulsar's rotation rate begins to grow from the instant τ_4, whereas the value of $\sigma_V(\tau)$ of the orbital frequency proceeds to decrease until the noises with $s \geq 5$ begin to dominate. This result is quite general and does not depend on particular numerical values of the noise amplitude. As theoretical analysis shows, a minimum of the $\sigma_V(\tau)$ curve can be occasionally reached at a much later time than the similar minimum for $\sigma_z(\tau)$. The depth of the minimum for $\sigma_z(\tau)$ is determined by noise with the spectral index $s = 3$, generated by large-scale inhomogeneities of the interstellar medium (Blandford *et al.* 1984). The depth of the minimum for $\sigma_V(\tau)$ is determined by noise with spectral index $s \geq 5$, which is attributable to the existence of stochastic background of gravitational radiation generated by physical processes at the initial stage of the birth of the Universe

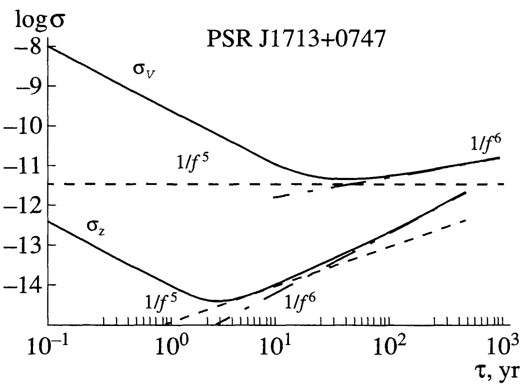

Fig. 2. The curves σ_z and σ_V calculated for pulsar J1713+0747 under the assumption that the orbital eccentricity $e = 0$. The minimum of the curve σ_V is reached on the interval $\tau = 20$ yr and is determined exclusively by the amplitude of noises of stochastic background gravitational radiation (the power spectrum of the form $1/f^5$) originated at the early stages of the evolution of the Universe. The noise amplitude is $h_5 = 2\Omega_g h^2 = 2 \times 10^{-8}$. The dashed and dot-dashed lines show the intensities of the noises of the form $1/f^5$ and $1/f^6$ and bound below the behavior of the curves σ_z and σ_V. The given pulsar is a suitable candidate for establishing the upper limit on the amplitude of background gravitational radiation, because the time interval on which minima of the curves σ_z and σ_V are reached is short enough to conduct experimental studies.

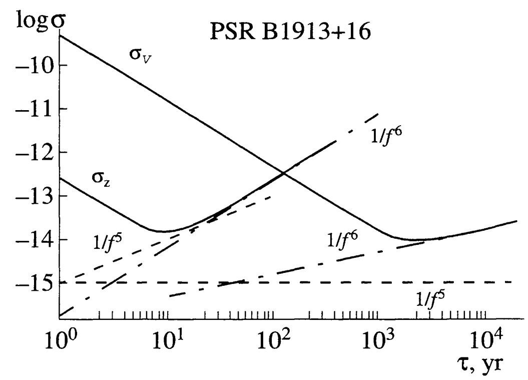

Fig. 3. The curves σ_z and σ_V calculated for pulsar B1913+16 without allowance for the ellipticity of its orbit. The minimum of the σ_V curve is reached much later than the minimum of σ_z. The dashed and dot-dashed lines show the intensities of the noises of the form $1/f^5$ and $1/f^6$ for σ_z and σ_V. This pulsar is not suited for searching for the amplitude of background gravitational radiation, because the minimum of σ_V is determined by the noise $1/f^6$, which begins to dominate much earlier than the noise of stochastic gravitational waves. This pulsar is a suitable candidate for keeping the scale of the dynamic ephemeris time, because the minimum of σ_V is rather deep and because it is reached over such a that long time interval that all other known time and frequency standards in this interval have a much worse stability.

(Mashkhun and Grishchuk 1980; Bertotti *et al.* 1983). The noise with index $s = 5$ might also arise, in principle, as a result of random fluctuations of the first derivative of the rate of the pulsar's rotation, although the appearance of such fluctuations is extremly unlikely (Cordes and Greenstein 1981) and will not be considered below. As an observational example, we can indicate the minimum of the $\sigma_z(\tau)$ curve for the single pulsar PSR B1937+21, which has a value of 10^{-14} for a two-year observation interval; its appearance is probably due to the dominating influence of instability of the rotational phase for the given pulsar (Kaspi *et al.* 1994).

It is of fundamental and practical importance that the value of the absolute minimum of $\sigma_V(\tau)$ can be calculated with certainty by neglecting noise with $s = 6$, because it is completely determined by the amplitude of stochastic gravitational-wave noise with spectral index $s = 5$, which is of fundamental cosmological origin. Specifically, the minimum of $\sigma_V(\tau)$ is determined by the formula

$$\sigma_V \simeq 2.4 \times 10^{-20} \sqrt{\Omega_g} P_b^2 x^{-1} \mathrm{h}, \qquad (5)$$

where Ω_g is the energy density of the stochastic background of gravitational waves in the log frequency range and h is the dimensionless Hubble constant (h = H/100 km s^{-1} Mpc^{-1}). It must be emphasized that the absolute minimum of $\sigma_V(\tau)$ depends not only on the value of the fundamental constants Ω_g and h, but also on the pulsar's orbital parameters. As indicated above, this is related to the fact that we determine parameters of pulsar's deterministic orbital motion, which obeys the sinusoidal law, against the background of the additive noise, which is exterior with respect to the sinusoid of interest. However, the real minimum of the $\sigma_V(\tau)$ curve depends on the amplitude of the noise with the spectral index $s = 6$. If the amplitude of the noise with the index $s = 6$ is lower than the amplitude of the stochastic background of gravitational waves, then the practically attainable minimum of the $\sigma_V(\tau)$ curve coincides with the absolute minimum calculated from formula (5). Otherwise, the minimum of the $\sigma_V(\tau)$ curve lies above the given absolute minimum. Our estimates show that the $\sigma_V(\tau)$ minimum may have the value 10^{-15}–10^{-16} on the assumption that $\Omega_g \leq 10^{-8}$ (Starobinskii 1979; Vilenkin 1981; Rubakov *et al.* 1982).

The result obtained can be used in two different ways. First, using pulsars with small orbital periods and small P_b/x ratios we can obtain the lowest possible value of the minimum of $\sigma_V(\tau)$ in order to provide the longest possible interval of BPT stability. Second, the presence of the minimum of $\sigma_V(\tau)$ makes it possible to search for (or to establish the upper limit on the amplitude of) stochastic gravitational radiation by using pulsars with large P_b/x ratios and choosing the optimum λ value that would permit us to measure equally well the frequency and amplitude of the orbital sinusoid. The large P_b/x value is necessary to obtain the highest possible value of the minimum of $\sigma_V(\tau)$, thus reducing the time interval required for its attainment. It should be

Table 2. Orbital parameters of some binary pulsars: P is the pulsar period; d is the distance to the pulsar; P_b is the pulsar orbital period, x is the projection of the semimajor axis of the orbit onto the line of sight in light seconds; e is the orbital eccentricity; and r is the amplitude of the Shapiro effect

PSR	P, ms	d, kpc	P_b, days	x, c	e	r, μs
J1713+0747	4.5701	1.1	67.82512988	32.342413	7.492×10^{-5}	>13.5
B1855+09	5.3621	0.9	12.32717119	9.2307802	2.168×10^{-5}	1.15
J0437–4715	5.7574	0.14	5.741042329	3.3666787	1.87×10^{-5}	–
J2317+1439	3.4452	1.89	2.459331464	2.3139483	5×10^{-07}	–
J1537+1155	37.9044	0.68	0.420737299	3.729468	0.2736779	6.70
J2130+1210C	30.5292	10	0.335282052	2.52	0.68141	–
B1913+16	59.0299	7.13	0.322997463	2.3417592	0.6171308	6.83

particularly emphasized that the measurement of the minimum of the $\sigma_V(\tau)$ curve presumably gives us a more accurate indicator of the upper limit of the amplitude of stochastic gravitational waves than the measurement of the respective slope of the $\sigma_z(\tau)$ curve. The point is that the minimum of the $\sigma_V(\tau)$ curve can be determined with higher confidence than the slope of the $\sigma_z(\tau)$ curve does, which depends on sufficiently large errors in the measurement of $\sigma_z(\tau)$ for long observation intervals (Kaspi *et al.* 1994; Thorsett and Dewey 1996).

To date, 46 binary pulsars have been discovered. By way of illustration, we present in Table 2 the parameters of some binary pulsars which can be used as detectors of the statistical background of gravitational waves (J1713+0747, B1855+09) or as BPT-keeping objects (B1913+16).

Another clear illustration of using the variance $\sigma_V(\tau)$ from the standpoint of fundamental applications is shown in Figs. 2 and 3, where we plotted $\sigma_y(\tau)$ and $\sigma_V(\tau)$ as a function of τ for J1713+0747 and B1913+16 with significantly differing orbital parameters. When constructing these figures, we disregarded for clarity the noises with spectral indices $s = 1, 2, 3, 4$, because they affect the variance only slightly in the time intervals considered here. Figure 2 demonstrates the behavior of $\sigma_z(\tau)$ showing the effect of the white noise [the initial part of the curve is taken from the work by Foster *et al.* (1996)]. We assume that $\Omega_g h^{-2} = 10^{-8}$, although it is quite probable that its actual value is much lower. The amplitude of the noise with $s = 6$ was chosen in such a way that this noise would begin to affect significantly the behavior of the $\sigma_z(\tau)$ curve after 20 yr of continuous observations. The $\sigma_z(\tau)$ function thus depicted can be recalculated into the corresponding function $\sigma_z(\tau)$, which is also shown in Fig. 2, by using the formulas in Table 1 and the values of the J1713+0747 orbital parameters listed in Table 2. A comparison of the curves in Fig. 2 shows that the binary pulsar J1713+0747 is a sufficiently good detector of stochastic gravitational radiation, as suggested by measurements of both σ_z and σ_V. It is significant that the use for this purpose of the $\sigma_V(\tau)$ curve should help avoid the uncertainty in the identification of the spectral character of the noise contained in pulsar's PAT residuals. For example, the mere existence of the minimum in the $\sigma_V(\tau)$ curve indicates immediately the presence of the noise with index $s \geq 5$. At the same time, the presence of a similar noise can be judged from the $\sigma_z(\tau)$ curve on condition that we have measured with rather good accuracy the slope of this curve, which is quite problematic. Figure 3 was constructed in a similar way. When plotting the $\sigma_z(\tau)$curve, we took the error in the parameter y equal to 10^{-13} on a time interval of 7 yr, according to the work by Taylor and Weisberg (1989). We also assumed that measurements in this time interval are dominated by the white noise. The $\sigma_V(\tau)$ curve was obtained by the corresponding recalculation using formulas in Table 1. The relative behavior of the curves $\sigma_z(\tau)$ and $\sigma_V(\tau)$ for pulsar B1913+16 in Fig. 3 differs from that for pulsar J1713+0747. The minimum of the $\sigma_V(\tau)$ curve equals approximately 10^{-14}; it lies deeper than the minimum of the curve $\sigma_z(\tau)$ and is reached after an extraordinarily long time span of ~2000 yr. Such a behavior of the variances of the rotational and orbital frequencies for B1913+16 does not permit one to obtain a satisfactory limit on the amplitude of stochastic gravitational waves, but makes this pulsar a highly reliable, stable standard of dynamic BPT over time intervals where random fluctuations of pulsar PAT residuals no longer represent the white noise and are due chiefly to the presence of the red noise with spectral index $s \geq 1$.

CONCLUSION

We have carried out the calculations of stability of the rotational and orbital timescales in binary pulsars. It is shown that in certain situations the scale of dynamic time (BPT) is far more stable than the kinematical timescale (PT). Nevertheless, the limiting uniformity of the BPT, even in the absence of intrinsic noises in the pulsar's orbital motion, is restricted by the presence of noise components in pulsar arrival times (PATs), although this limitation begins to show up much later than the time the instability of the pulsar's rotational phase begins to manifest itself. This property of the BPT gives hope that the use of the BPT in the future may provide a deeper insight into the laws of gravitation physics and must undoubtedly find practical applications in fundamental metrology.

The study of the problem of constructing and using the BPT for solving various problems of modern astronomy is connected intimately with the problem of the study of the noise origin in pulsar's PATs. The publication of the work by D'Allessandro *et al.* (1996), which involves a detailed discussion of the spectra of noises in PAT residuals for 18 single pulsars, is likely to give evidence for the renewed interest of observers to this problem.

The theoretical development of ideas related to the study of the behavior of the BPT can be continued along several lines. It is likely that we should examine first of all the binary systems with elliptical orbits with the maximum possible P_b^2/x ratio, which might improve the quality of the BPT and the accuracy of testing the general-relativity effects. The possible variations in the orbital elements of the binary system, which are caused by unpredictable external factors such as passages of massive bodies near the binary pulsar or gravitational waves with periods close to the orbital period, should also be taken into account.

It is easy to show that the binary systems with sufficiently low m_p/m_c ratio (m_p and m_c are, respectively, the masses of the pulsar and its companion), large semimajor axis a_p, small period P_b, and with $\sin i \simeq 1$, i.e., large x, are preferable for constructing the BPT. We would like to emphasize the significance of the regular long-term timing of the set of reference binary pulsars at the leading radio astronomical observatories that have at their disposal radio telescopes with large effective area providing a high signal-to-noise ratio and the limiting accuracy of measurements of pulsar's PATs.

ACKNOWLEDGMENTS

The authors are indebted to N.S. Kardashev and V.A. Brumberg, who carefully examined the results of this study and made some useful comments. The authors are also grateful to T. Fukushima and G. Petit for helpful discussions, to D.N. Matsakis for the opportunity to familiarize with the paper (Matsakis *et al.* 1997) prior to its publication, and to S.N. Bagaev and L.K. Isaev for assistance in making this study within the "Astrocomplex" project, which is part of the Russian interdisciplinary scientific and technical program "Fundamental Metrology." S.M. Kopeikin is indebted to the administration and personnel of the National Astronomical Observatory of Japan (Mitaka, Tokio), where this work was initiated, for long-term constant support, as well as to G. Neugebauer and G. Schäfer for financial support (grant no. B501-96060) at the stage of completing this paper. Yu.P. Ilyasov and A.E. Rodin are grateful to the INTAS for partial financial support (grant no. 94-3097). The authors express their gratitude to the referees of the "Astronomical Letters" for valuable comments and advice, which led to significant improvement of the paper, and to V.A. Potapov for help in the preparation of the paper.

REFERENCES

Backer, D.C. and Foster, R.S., *Astrophys. J.*, 1990, vol. 361, p. 300.

Bell, J.F. and Bailes, M., *Astrophys. J.*, 1996, vol. 456, p. L33.

Bertotti, B., Carr, B.J., and Rees, M.J., *Mon. Not. R. Astron. Soc.*, 1983, vol. 203, p. 945.

Blandford, R., Narayan, R., and Romani, R.W., *J. Astrophys. Astron.*, 1984, vol. 5, p. 369.

Cordes, J.M. and Greenstein, G., *Astrophys. J.*, 1981, vol. 245, p. 1060.

D'Allessandro, F., Deshpande, A.A., and McCulloch, P.M., *J. Astrophys. Astron.*, 1996 (in press).

Damour, T. and Taylor, J.H., *Phys. Rev. D*, 1992, vol. 45, p. 1840.

Damour, T., *300 Years of Gravitation*, Hawking, S.W. and Israel, W., Eds., Cambridge: Cambridge Univ., 1987, p. 128.

Foster, R.S., Camilo, F., and Wolszczan, A., *7th MG Meeting*, Jantzen, R.T. and Mac Keiser, G., Eds., Singapore: World Scientific, 1996, p. 1209.

Guinot, B. and Petit, G., *Astron. Astrophys.*, 1991, vol. 248, p. 292.

Guinot, B., *Reference Frames*, Kovalevski, J., Mueller, I.I., and Kolaczek, B., Eds., Dordrecht: Kluwer, 1989, p. 351.

Hulse, R.A. and Taylor, J.H., *Astrophys. J.*, 1975, vol. 195, p. L51.

Ilyasov, Yu.P., Kardashev, N.S., Pobedonoscev, K.A., and Poperechenko, B.A., *Acoustoelectronics, Frequency Control & Signal Generation*, Proc. Intern. Symp., Moscow: MPEI Publ., 1996, p. 204.

Ilyasov, Yu.P., Kuz'min, A.D., Shabanova, T.V., and Shitov, Yu.P., *Pul'sary* (Pulsars), Trudy FIAN, 1989, vol. 199, p. 149.

Kaspi, V.M., Taylor, J.H., and Ryba, M.F., *Astrophys. J.*, 1994, vol. 428, p. 713.

Kopeikin, S.M. and Potapov, V.A., *Astron. Zh.*, 1994, vol. 71, p. 120.

Kopeikin, S.M., *Astron. Zh.*, 1985, vol. 62, p. 889.

Kopeikin, S.M., *Astrophys. J.*, 1994, vol. 437, p. L67.

Kopeikin, S.M., *Astrophys. J.*, 1996, vol. 437, p. L93.

Kopeikin, S.M., *Mon. Not. R. Astron. Soc.*, 1997b, vol. 288, p. 129.

Kopeikin, S.M., *Phys. Rev. D*, 1997a (in press).

Mashoon, B. and Grishchuk, L.P., *Astrophys. J.*, 1980, vol. 236, p. 990.

Matsakis, D.N., Taylor, J.H., and Eubanks, T.M., *Astron. Astrophys.*, 1997 (in press).

Petit, G. and Tavella, P., *Astron. Astrophys.*, 1996, vol. 308, p. 290.

Rubakov, V., Sazhin, M.V., and Veryaskin, A., *Phys. Lett.*, 1982, vol. 115B, p. 189.

Rutman, J., *Proc. IEEE*, 1978, vol. 66, p. 1048.

Schäfer, G., *Ann. Phys.*, 1985, vol. 161, p. 8.

Shabanova, T.V., Il'in, V.G., Ilyasov, Yu.P., *et al.*, *Izmer. Tekhn.*, 1979, no. 10, p. 73.

Starobinsky, A.A., *Pis'ma Zh. Eksper. Teor. Fiz.*, 1979, vol. 30, p. 682.

Taylor, J.H. and Weisberg, J.M., *Astrophys. J.*, 1989, vol. 345, p. 434.

Taylor, J.H., *Phil. Trans. R. Soc. Lond.*, 1992, vol. A341, p. 117.

Thorsett, S.E. and Dewey, R.J., *Phys. Rev. D*, 1996, vol. 53, p. 3468.

Tikhonov, V.I., *Optimal'nyi priem signalov* (Optimal Signal Reception), Moscow: Radio i Svyaz', 1983.

Van Trees, H.L., *Detection, Estimation, and Modulation Theory*, New York: J. Willey & Sons, 1968.

Translated by A. Kozlenkov