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Structures with bridged cracks and weak interfaces

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Abstract

Structures with interfacial bridged cracks and weak interfaces are analyzed. In the frames of this approach is assumed that: a) the crack surfaces interact in some zones behind the crack tips (bridged zones); b) size of these zones can be comparable to the whole crack length; and c) the interface between different materials can be connected by mechanical ligaments and a relative displacement of initially adjacent materials may occur (non-ideal or weak interface). Several new problems for interface bridged cracks and interaction of bridged cracks with material interface were solved and analyzed by the direct boundary integral equations method.

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Keywords: interfacial cracks; crack bridged zone; weak interface; boundary elements

1. Introduction

Analysis of cracks growth in composite materials and in adhesive joints using models of a crack process zone (cohesive or bridged) includes the problem of displacements and stresses computation in crack process zones and in vicinity of the crack tips. In this paper the bridged crack approach will be considered as a crack process zone model. In this approach it is assumed that the stress intensity factors do not vanish at the crack tip. The problems of displacements and stresses analysis for bridged cracks have mainly considered for straight cracks in infinite homogeneous media. For bridged interfacial cracks between two semi-infinite homogeneous plates the problems were considered and solved firstly in (Goldstein and Perelmuter, 1999) by the singular integral-differential equations method (planar problem). Similar problem (but without analysis of stresses at the bridged zone) was considered for anisotropic materials in (Ni and Nemat-Nasser, 2000).

In the last two decades a number of papers were devoted to the application of the boundary integral equation (BIE) method to computation of the stress intensity factors for cracks on bi-material interfaces in finite size structures. In these papers interaction between crack surfaces was neither assumed nor considered, see review in (Hadjesfandiari and Dargush, 2011). Only few papers have been directed to analysis of straight bridged cracks in finite size structures. The extension of BIE approach for solving elasticity problems for curvilinear interfacial crack has been proposed and numerically implemented in (Perelmuter, 2013).

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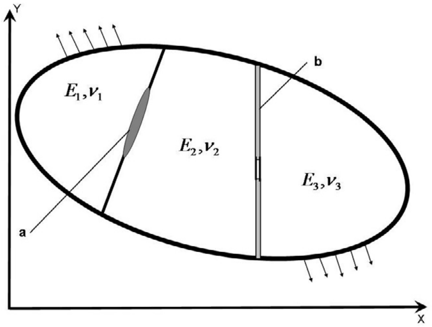

Fig. 1. Structure with three subregions: a) fully bridged crack on the ideal junction of subregions 1 and 2; b) weak interface between subregions 2 and 3 with internal region without ligaments.

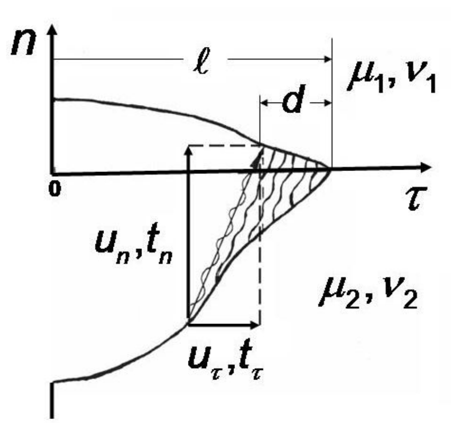

Fig. 2. ℓ is the interfacial bridged crack length, **d** is the bridged zone length.

The main objective of this paper is the extension of the BIE method to analysis of displacements, stresses and stress intensity factors for interfacial cracks with bridged zones and taking into account the non-ideal junction (weak interface) between different materials ahead of crack tips.

2. Models of bridged cracks and weak interfaces

In analysis of stresses and fracture parameters of adhesional joins two general approaches are used: a) cracks are placed between identical or different materials with the fracture process zone which can be comparable to the whole length of crack (Rose, 1987; Goldstein and Perelmuter, 1999); b) it is assumed that the process zone extends along the whole interface adhesional layer between materials (Antipov et al., 2001; Lenci, 2001). In the last case the definitions 'weak interface' or 'imperfect interface' are used and the first of these definition will be used in this paper. We will assume that both types of flaws, which might interact with each other (see Fig. 1), are inhered to materials interfaces.

2.1. Bridged crack model

In this paper we use the interfacial crack bridging concept (Perelmuter, 2016). It is assumed within this concept: a) there are the interfacial adhesion layer between jointed materials; b) a zone of weakened bonds in this layer is considered as an interfacial crack with distributed nonlinear spring-like bonds between the crack surfaces (bridged zone); c) fracture process is localized at the crack tip and inside the crack bridged zone, which might consist of several parts with different bonds properties.

The crack bridged zone modelling is based on the following assumptions: 1) distributed bridging tractions depending on a crack opening are imposed to faces of cracks at the bridged zone; 2) materials ahead of cracks tip are considered as linearly-elastic and it is assumed that these materials are deformed together with fibers (or adhesion layer) without loss of their continuity (infinitely thin interfacial layer, ideally junction). 3) the total stress intensity factor due to the external loading and the bridging tractions is not imposed to be zero. To describe mathematically the interaction between the crack surfaces, we assume that there exist bonds with nonlinear deformation law between the surfaces of the crack in the bridging zone as in (Goldstein and Perelmuter, 1999). The tractions in bonds between the interfacial crack faces are the result of the external loading action. These tractions have the normal t_n and tangential t_τ components even for the uniaxial tension case. The surfaces of the crack are loaded by the normal and tangential stresses corresponding to these tractions. The relations between bonds traction and displacements difference of the upper and lower crack faces in the crack bridged zone (the crack opening, see Fig. 2) are used in the following generalized form (Goldstein and Perelmuter, 1999)

$$t_{n,\tau}(x,\sigma) = \kappa_{n,\tau}(x,\sigma)\, u_{n,\tau}(x), \qquad \kappa_{n,\tau}(x,\sigma) = \varphi_{1,2}(x,\sigma)\frac{E_b}{H}, \quad \sigma = \sqrt{t_n^2 + t_\tau^2} \tag{1}$$

where $t_{n,\tau}$ and $u_{n,\tau}$ are the components of the traction vector and the crack opening in the local coordinate system connected with the normal n and tangential τ directions to the crack face, $\kappa_{n,\tau}(x,\sigma)$ are the stiffness of bonds depending on the distance from the crack tip and the traction vector modulus σ at the current point x of the crack bridged zone, $\varphi_{1,2}$ are dimensionless functions used for description of a nonuniform behavior of stiffness over the bridged zone, H is the length parameter proportionate to the bonding zone thickness, E_b is the bond effective elastic modulus.

Within the bridged model (in contrast to cohesive models) the total stress intensity factors (SIF) due to external loading and bonds tractions are not assumed equal to zero.

2.2. Weak interface model

In the frame of the weak interface model we assume: a) the process zone extends along the whole or part of interface layer between materials; b) there are no segments with ideal junction along of weak interface line; c) several parts of interface layer between material without ligaments (or with ligaments stiffness relatively less than in the adjacent segments) can be treated as cohesive cracks. The bond deformation law for ligaments in weak interface layer is used in the same form as for bridged cracks. The interface layer is not assumed to be infinitely thin, but for computational purposes it is substituted by the normal and shear stiffness of layer which are defined as in (1), where H and E_b are the layer thickness and its elastic modulus, both these parameters can vary along the interface layer.

3. Boundary element formulation

The modelling of bridged interfacial cracks and weak interface layers is based on the multi-domain BIE problem formulation. Within this approach, the direct boundary integral equations for elasticity problems are used for each homogeneous subregion of the structure and cracks are located between subregions (Blandford et al., 1981). The supplementary boundary conditions at interfacial boundaries (with ideal or weak contact) and at cracks bridged zones are introduced and used to eliminate additional variables on joint boundaries of subregions. This approach can be used for any finite size structures with an arbitrary external loading.

For elasticity problems without body forces the direct BIE for any homogeneous subregion of the 2D/3D structure is given by (Banerjee and Butterfield, 1981):

$$c_{ij}(p)\,u_i(p) = \int_{\Gamma} \left[G_{ij}(p,q)\,t_i(q) - T_{ij}(p,q)\,u_i(q)\right] d\Gamma(q), \quad i,j = 1,2 \tag{2}$$

here $c_{ij}(p)$ depends on the local geometry of boundary Γ (for a smooth boundary $c_{ij}(p) = 0.5\delta_{ij}$), $G_{ij}(p,q)$ and $T_{ij}(p,q)$ are Kelvin's fundamental solutions for displacements and tractions, respectively, $u_i(q)$ and $t_i(q)$ are displacements and traction over the boundary of the structure and the location of source and field points belonging to the subregion boundary Γ are defined by the coordinates of the points q and p.

The displacement continuity and the traction equilibrium supplementary relations are used at the interfacial boundaries of subregions with conditions of ideal contact in the following form

$$u_i^k(p) = u_i^n(p), \quad t_i^k(p) = -t_i^n(p), \quad i = 1,2 \tag{3}$$

where k and n are joint subregions numbers, $u_i(p)$ and $t_i(p)$ are displacements and traction components at the boundary point p.

The relationships between bonds tractions and the upper and lower crack surfaces displacements difference (the crack opening) at the crack bridged zone and along a weak interface layer is used in generalized form (1), where x corresponds to the coordinates of a curre4nt point p and $i = 1;2$ correspond to the tangential and normal directions to the interface zone

$$t_i(p,\sigma) = \kappa_i(p,\sigma)\Delta u_i(p), \quad \Delta u_i(p) = u_i^k(p) - u_i^n(p) \tag{4}$$

For numerical solution of the BIE (2) in two-dimensional case the boundaries of all subregions are subdivided into quadratic isoparametric elements. The quarter-point displacement and traction singular crack tip boundary elements are used (Perelmuter, 2013) for the interfacial crack displacements and stresses asymptotic modelling.

4. Numerical results

The algorithm of BIE system (2) solving for structures with bridged interfacial cracks and weak interfaces has been implemented into the computer code previously developed. In (Perelmuter, 2013) was given the comparison of the BIE results with the results obtained previously using by the singular integral-differential equations and it confirmed the presented BIE approach and implementation accuracy for bridged cracks. The stress intensity factors are computed on the base of the stress asymptotic field in the neighborhood of the crack tip ahead of the interfacial crack. The SIF modulus is defined as

$$K = \sqrt{K_I^2 + K_{II}^2}, \quad K_{I,II} = K_{I,II}^{ext} + K_{I,II}^{int} \tag{5}$$

where $K_{I,II}^{ext}$ and $K_{I,II}^{int}$ are the SIF caused by the external loads and bonds stresses; note that $K_{I,II}^{int} < 0$. SIF modulus can be found from the relation for stresses ahead of the crack tip, see details in (Perelmuter, 2013).

In the computational results for 2D problems with bridged interfacial cracks and weak interface which are presented below the bond deformation law was specified by the relationship (1) with assumption that $\varphi_{1,2}(q,\sigma) = \varphi_{1,2}$ are constants. Therefore, bonds stiffness also is constant over bridged/weak regions. The relative bond stiffness $\bar{\kappa}_{n,\tau}$ is defined as follows

$$\bar{\kappa}_{n,\tau} = \varphi_{1,2}\frac{E_b}{\kappa_0 H}, \quad \kappa_0 = \frac{E_k}{\ell} \tag{6}$$

where ℓ is the crack length, κ_0 is the technical stiffness for dimensionless purposes, E_k is an intrinsic elastic modulus (see below).

We considered the problem of the plate under an uniaxial uniform tension, with straight crack placed at $|x| \leq \ell,\ y = 0$ on the interface of two dissimilar elastic half-planes. Due to the problem symmetry only $1/2$ part of the full plate was considered, with the width W, and the height $2W$, it is shown in Fig. 3a, ($W/\ell = 5$), the origin point of the cartesian coordinate system is placed in the crack center, point 0. An uniform tension loading σ_0 is applied to the plate at $y = \pm W$. Poisson's ratios of subregions are $\nu_1 = \nu_2 = 0.3$ (plane strain conditions). We consider two types of the interface condition between subregions: 1) weak interface, relation (4), along the subregions junction line $\ell \leq x \leq W,\ y = 0$, the crack placed at $0 \leq x < \ell,\ y = 0$, crack is assumed without bonds and 2) bridged crack totally filled with bonds is placed at $0 \leq x < \ell,\ y = 0$, also relation (4), and conditions of ideal contact (3 is imposed along the subregions interface line $\ell \leq x \leq W,\ y = 0$. In the both cases is assumed that $\varphi_{1,2} = 1$ and the shear and normal stiffness of bonds are equal, see (6).

4.1. Weak interface

In this case we denote the modulus of the weak interface bonds E_b in (6) as E_w. The bonds modulus and intrinsic elastic modulus are assumed by the equal $E_w = E_k = E_1$. Thus, the relative stiffness of weak interface κ_w is defined as

$$\kappa_w = \bar{\kappa}_{n,\tau} = \frac{\ell}{H_w} \tag{7}$$

and the bonds stiffness variation defines by the variation of the parameter H_w.

For the case of weak interface condition along the subregions junction line the deformed states of structure relatively the bottom line $y = -W$ (the shaded area) for soft $\kappa_w = 0.5$ and hard $\kappa_w = 50$ bonds stiffness are shown in Fig. 3b where the relative gap between subregions is occurring for soft bonds stiffness. In Fig. 4 for relatively soft of weak interface bonds stiffness the stress σ_{yy} along the subregions junction line are shown. At the decreasing of bonds stiffness the normal stress distribution becomes more uniformly and the stress distribution becomes close to the case of an ideal contact between subregions at the increasing of bonds stiffness. This behavior of stresses qualitatively agrees with the asymptotic estimation given by (Yentov and Salganik, 1968).

Relative displacements along the crack region and weak interface line are shown in Fig. 5a,b, where u_0 is normal displacement at the center of interface crack without bonds. For relatively soft bonds there is noticeable gap between subregions along weak line, whereas for hard bonds the relative displacements are very close to the case of ideal junction of subregions.

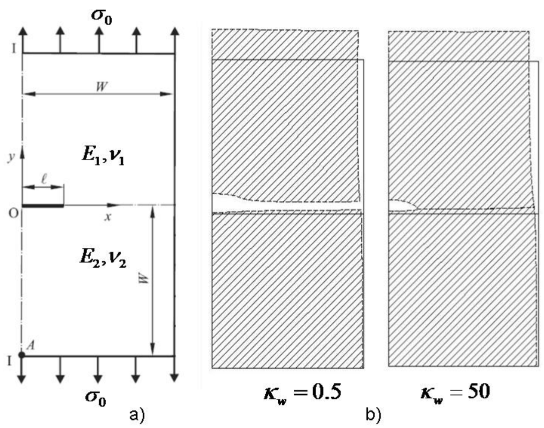

Fig. 3. (a) - Model of the plate for the BIE computations; (b) Scaled strain of plates with weak interface: soft ($\kappa_w = 0.5$) and hard ($\kappa_w = 50$) bonds, $E_2/E_1 = 5$.

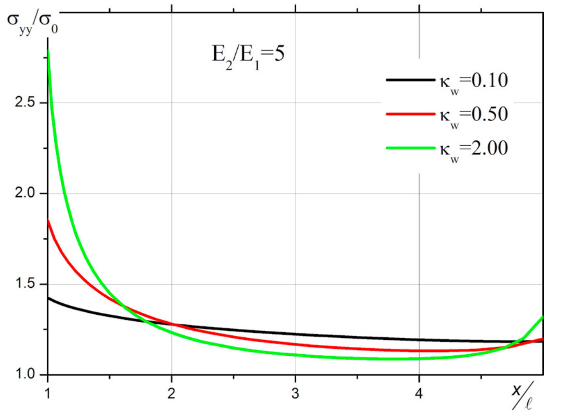

Fig. 4. Normal stresses distributions along weak interface, σ_{yy}

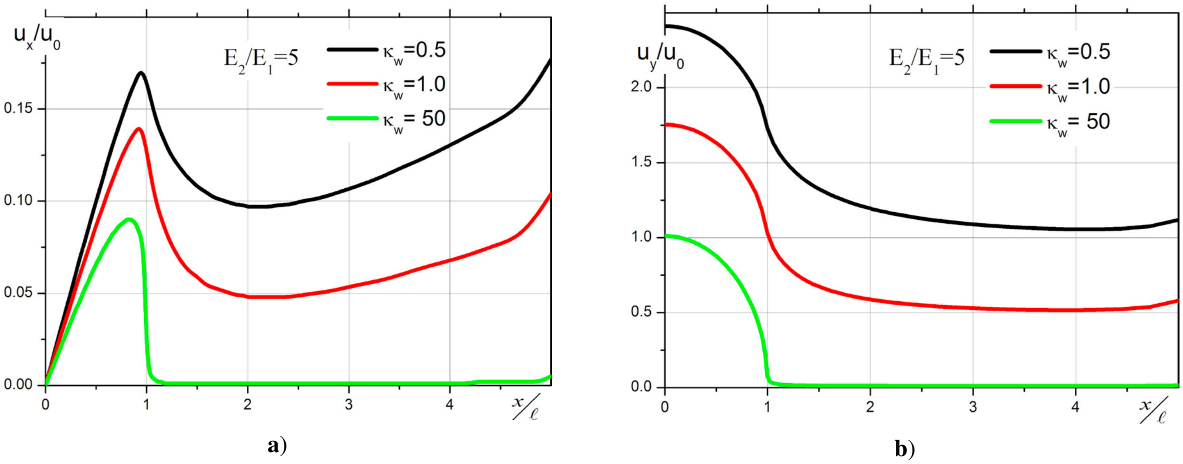

Fig. 5. Relative displacements along crack and weak interface layer: (a) shear displacements; (b) normal displacements.

4.2. Bridged crack

Bonds and intrinsic elastic modulus in this case are assumed as $E_b = E_k = E_1$ and, similar to relation (7), the relative stiffness of bonds for bridged crack κ_b is

$$\kappa_b = \frac{\ell}{H_b} \tag{8}$$

and bonds stiffness variation for bridged zone defines by the variation of the parameter H_b.

To analyze the effect of materials elastic modulus distinction on the SIF, we computed the SIF dependencies versus the relative bonds stiffness for different ratios of E_2/E_1, see Fig. 6. If the relative bonds stiffness is rather small then distinction of SIF for different rations of E_2/E_1 is insignificant, but for $\kappa_b \geq 0.5$ and for inhomogeneous plate the SIF module increases in comparison with homogeneous material by about 20%. This increase of the SIF is connected with shear stress which arises in bonds even under normal external tension, see Fig. 7. Shear stress is much less than normal ones and it depends on the relative bonds stiffness a little, therefore the difference in the SIF module for $E_2/E_1 = 5$ and $E_2/E_1 = 50$ is insignificant. In general, if $\kappa_b \geq 5$ then the effect of strengthening by bonds is stabilized. Note, also, that under external tension in the direction normal to the crack line, the shear bonds stiffness variation in the crack bridged zone has a little influence on the SIF module.

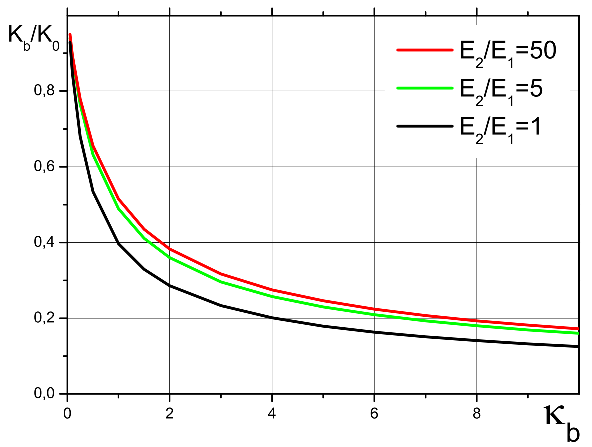

Fig. 6. SIF module vs relative stiffness of bonds at the crack bridged zone.

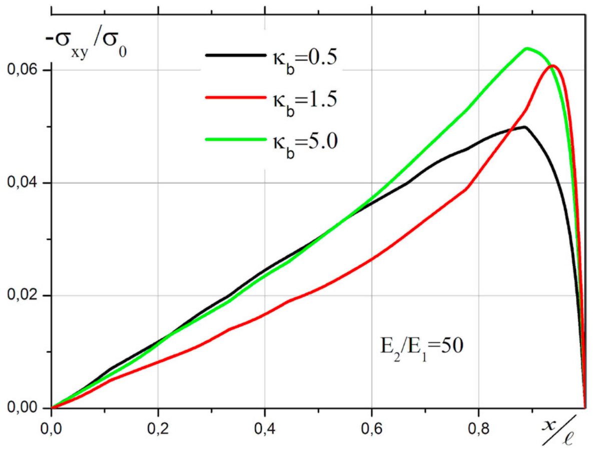

Fig. 7. Bonds stresses along the bridged crack: (a) shear stress; (b) normal stress.

5. Closing

Analysis of stresses distributions and SIF is the first part of cracks growth modelling in the framework of the weak interface or crack bridged zone models. This paper has demonstrated effectiveness of the boundary integral equation application for the solving stress problems with weak interface and bridged cracks. This approach can be used for structures of finite size with curvilinear bridged cracks and weak interfaces under mechanical loads, body forces, steady-state and transient thermal loading. Analysis of weak interface and/or bridged cracks growth can be incorporated into this BIE approach in the frame of the nonlocal criterion of bridged cracks growth (Goldstein and Perelmuter, 1999; Perelmuter, 2014).

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