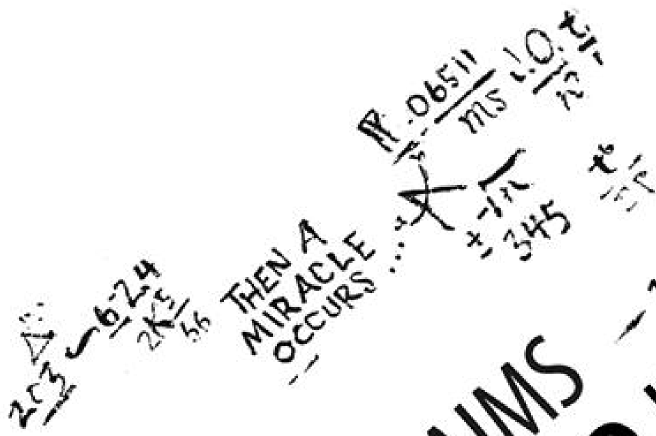

MATHEMATICAL AIMS BEYOND JUSTIFICATION

— 10-11 December 2015 —

WORKSHOP - PHILOSOPHY OF MATHEMATICS

Rubens Auditorium,
Royal Flemish Academy of
Belgium for Science and the Arts,
Hertogsstraat 1, Brussels, Belgium

Invited Speakers

Jeremy Avigad (Carnegie Mellon University)
Alan Baker (Swarthmore College)
Ronny Desmet (Vrije Universiteit Brussel)
Jean Paul Van Bendegem (Vrije Universiteit Brussel)

Contributed speakers:

Josephine Salverda
Yannick Van den Abbeel
William D'Allessandro
Victor Gijsbers
Aziz F. Zambak
Flavio Baracco
Sorin Bangu
Fiona Doherty
Line E. Andersen & Henrik Kragh Sørensen
Vladislav Shaposhnikov

INFORMATION:
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BSLPS CLWF KVAB fwo

Workshop aim and scope

Welcome to this philosophy of mathematics workshop of which the central theme is, as the title indicates, *Mathematical Aims beyond Justification*.

Questions concerning the justification of mathematical knowledge have always played a crucial role in philosophy of mathematics. There is, however, a growing consensus that working mathematicians are not only interested in the justification of mathematical results, but are also driven by other goals.

Consequently, if philosophers of mathematics want to provide and discuss an account of mathematical practice, it is essential to get a grip on topics that go beyond the nature of justification. Such topics can, among others, include the nature and role of mathematical explanation, mathematical understanding, mathematical creativity, mathematical discovery, mathematical beauty and mathematical experimentation.

The objective of this workshop is to reflect on, evaluate and understand what mathematicians look for beside justification.

Organization and Support

The main organizer of this event is the Centre for Logic and Philosophy of Science (CLWF) of the Vrije Universiteit Brussel. Ever since it was founded in 1998, this research centre always had a clear interest in philosophy of mathematics. As a result, the centre has (co-)organized various events on philosophy of mathematics in Brussels, including: *Philosophy of Mathematical Practices Conference* (2002, 2007), *Mathematics as/in Culture* (2003), *Mathematics in Education* (2004), *Philosophical and Psychological Perspectives on Number* (2006), *First International Meeting of the Association for the Philosophy of Mathematical Practice* (2010), *International Workshop on Logic and Philosophy of Mathematical Practices* (2014).

This workshop is a continuation of the tradition of the CLWF to reflect on questions within philosophy of mathematics, with a special involvement with the study of mathematical practices. It is closely related to and funded by the strategic research project of the CLWF, entitled *Logic and Philosophy of Mathematical Practices*, and the FWO-aspirant project of CLWF-member Joachim Frans, which will lead to a PhD on the explanatory value of mathematical proofs and visualisations.

In addition to the CLWF, this workshop has also been made possible with the support of the Belgian Society for Logic and Philosophy of Science (BSLPS) and the Royal Flemish Academy of Belgium for Sciences and Arts.

Scientific Committee.

Jeremy Avigad, Alan Baker, Ronny Desmet, Joachim Frans, Karen Francois, Brendan Larvor, Jean Paul Van Bendegem, Bart Van Kerkhove

Organizing Committee.

Jeremy Avigad, Alan Baker, Sven Delariviere, Ronny Desmet, Joachim Frans, Karen François, Yacin Hamami, Brendan Larvor, Colin Rittberg, Jean Paul Van Bendegem, Bart Van Kerkhove, Nigel Vinckier

Location

The workshop will be held at the **Palace of Academies** (Hertoggstraat/Rue Ducalle 1, 1000 Brussels), situated beside the Brussels Royal Park at the very heart of Brussels. The Palace of the Academies is the home of the Royal Flemish Academy of Sciences and the Arts, and serves as a meeting place for scientists, artists and researchers from both home and abroad. During our event, we will meet at the **Rubens auditorium**, which is located on the ground floor of the Palace.

Directions

From railway station **Brussel-Noord/Bruxelles Nord**: For participants who arrive at the Brussels North railway station, it is advised to take the exit labelled "Centrum" and walk straight up the street to the subway stop Rogier. Then take metro line 2 or 6 and get off at stop Troon/Throne.

From railway station **Brussel-Central/Bruxelles-Central**: For participants who arrive at Brussels Central railway station, it is a 15 minute walk through Ravenstijn Gallery and Royal Park to get to the Palace of the Academies.

From railway station **Brussel-Zuid/Bruxelles Midi**: For participants who arrive at the Brussels South railway station, it is advised to take the subway line 2 or 6 and get off at stop Troon/Throne.


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Koninklijk Paleis van Brussel
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Rue Royale
Rue Ducale
Rue de la Loi
Rue Zinner
Bd du Régent
Av. des Arts
Rue des Sols
Mont des Arts
Rue Ravenstein
Pl. des Palais
Pl. Royale
Rue du Musée
Rue de Namur
Rue Montoyer
R20
Rue de la Régence
Rue Brederode
Pl. du Trône
Av. Marnix
Rue de Luxembourg
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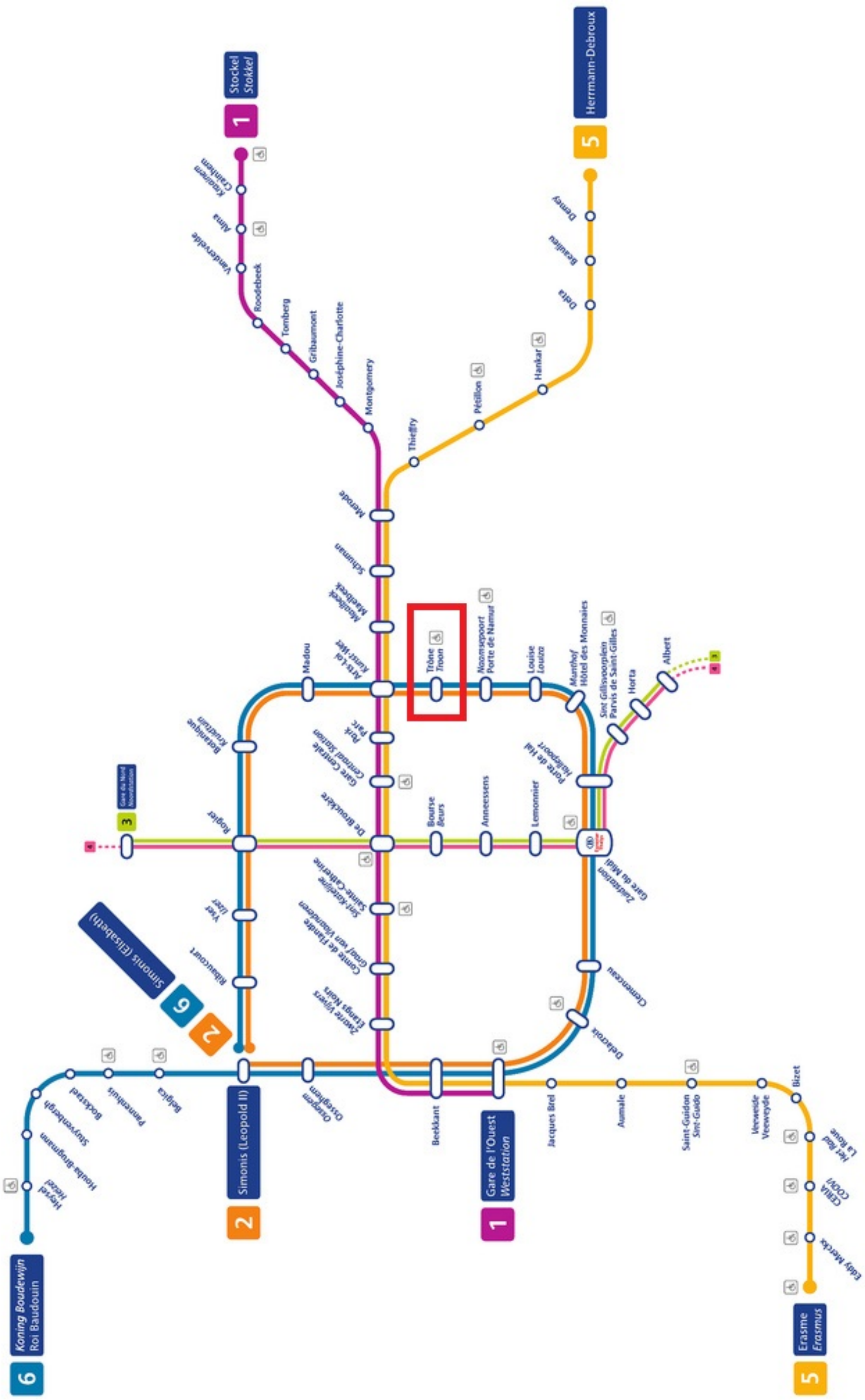
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Naamsepoort Porte de Namur
Louise Louiza
Munthof Hôtel des Monnaies
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Osseghem Ossegem
Beekkant
1 Gare de l'Ouest Weststation
Jacques Brel
Aumale
Saint-Guidon Sint-Guido
Veeweyde Veeweide
Bizet
La Roue Het Rad
CERIA COOVI
Eddy Merckx
5 Erasme Erasmus
6 Koning Boudewijn Roi Baudouin
Heysel Heizel
Houba-Brugmann
Stuyvenbergh
Bockstael
Pannenhuis
Belgica

Workshop dinner

The workshop dinner takes place on Thursday evening at 19.30, at **Taverne du Passage**. This restaurant is located at Koninginnegalerij/Galerie de la Reine, which is a shopping arcade that can be entered from Rue d'Arenberg, Rue des Bouchers or Rue de la Montagne. We will organize a walk from the location of the workshop to the restaurant at 19.00.

Proceedings

We will publish a selection of the papers as a special issue of Logique et Analyse, edited by Joachim Frans and Bart Van Kerkhove. All speakers that are interested are asked to submit a final version of their papers by

March 15th, 2016.

A standard refereeing process will apply. The special issue will appear either at the end of 2016 or the beginning of 2017.

Schedule

THURSDAY 10 DECEMBER

09:00 – 09:30	*Registration + Coffee*
09:30 – 09:45	*Opening*
09:45 – 10:45	Alan Baker *Three aspects of mathematical explanation: Bertrand's Postulate*
10:45 – 11:15	*Break*
11:15 – 11:50	Josephine Salverda *Characterizing properties and explanation in mathematics*
11:50 – 12:25	Yannick Van den Abbeel *Mathematical practices and the development of eighteenth-century mechanics. Conflicting values, philosophical obstacles and methodological ideals*
12:25 – 13:00	William D'Allessandro *Intertheoretic reduction and explanation in mathematics*
13:00 – 14:30	*Lunch Break*
14:30 – 15:30	Ronny Desmet *The beauty of the Kochen-Specker Theorem*
15:30 – 16:00	*Break*
16:00 – 16:35	Victor Gijsbers *Mathematical explanation as interventionist quasi-explanation*
16:35 – 17.10	Aziz F. Zambak *Mathematical ontology for knowledge representation*

FRIDAY 11 DECEMBER

09:00 – 09:30 *Registration + Coffee*

09:30 – 10:30 Jean Paul Van Bendegem
Proofs, narratives, rhetorics, and style

10:30 – 11:00 *Break*

11:00 – 11:35 Flavio Baracco
Explanatory proofs in mathematics: Noneism, someism, and allism

11:35 – 12:10 Sorin Bangu
What is distinctive about distinctively mathematical scientific explanation?

12:10 – 12:45 Fiona Doherty
The method is message: mathematical truth and the Frege-Hilbert controversy

12:45 – 14:15 *Lunch Break*

14:15 – 15:15 Jeremy Avigad [Session co-organized by the BSLPS]
Dirichlet's theorem on primes in an arithmetic progression

15:15 – 15:45 *Break*

15:45 – 16:20 Line E. Andersen & Henrik Kragh Sørensen
The role of trust in mathematical knowledge: Facing unsurveyability in proofs

16:20 – 16:55 Vladislav Shaposhnikov
Cathedral builders: The sublime in mathematics

Abstracts

THURSDAY 10 DECEMBER

09:45 – 10:45 *Three aspects of mathematical explanation: Bertrand's Postulate*

Alan Baker (Swarthmore College)

Bertrand's Postulate states that there is always a prime between n and $2n$. It was proved by Chebyshev in 1850, but the first elementary proof was not discovered until 1932, by Paul Erds, in his first published paper. Erds believed that this latter proof was a proof from The Book, in which God maintains the perfect proofs for mathematical theorems, and the proof is included in Aigner and Ziegler's 2010 volume, Proofs from THE BOOK, which is a partial attempt to collect together 'Book-worthy' proofs from various mathematical fields.

My focus in this paper is on explanatory aspects of Erdös's proof. It seems plausible that a Book proof of Bertrand's Postulate should also explain why this result holds. However, despite its beauty and its use of elementary methods, the Erdös proof has two features that are in potential tension with explanatoriness. Firstly, the general proof only works for n ¿ 4000, which means that the conjecture needs to be checked on a case-by-case basis for small n. However, proving that a conjecture holds over some finite range by checking all the cases in that range is not normally taken to be explanatory. Secondly, the key lemma of the main proof is proved by induction. Following an important 2009 paper by Marc Lange, there has been considerable debate in the philosophy of mathematics literature over whether proofs by induction are ever genuinely explanatory. I discuss how these two aspects of the proof interact, and what improvements are possible.

In the final part of the paper, I look at how Bertrand's Postulate may play a role in the mathematical explanation of a physical phenomenon. The phenomenon in question concerns the (much discussed) prime periods of certain cicada species. Appeal to Bertrand's Postulate allows the mathematical explanation to be generalized, by guaranteeing the existence of prime-numbered periods in a given ecological range. I argue that the explanatoriness or otherwise of Erdös's proof is irrelevant to the status of the overall scientific explanation.

11:15 – 11:50 *Characterizing properties and explanation in mathematics*

Josephine Salverda (UCL, London)

One aim mathematicians may have, beyond merely piling up proofs of theorems, is to find explanatory proofs. But how can we understand what it means for a proof to be explanatory? One important attempt to answer this question has been Mark Steiner's account of mathematical explanation, which appeals to characterizing properties in proofs. Steiner's account has met with a number of objections, but these often involve looking for counterexamples, which can descend into trading intuitions. I think it is interesting instead to try to make sense of the remarks Steiner makes about his own examples, which e.g. Hafner and Mancosu find 'very puzzling indeed'.

In this paper, I propose a way to make sense of Steiner's remarks, arguing that my reading makes room for both an ontic and epistemic component of mathematical explanation.

I focus on two examples from Steiner's paper: (1) the sum of integers from 1 to n, and (2) the irrationality of $\sqrt{2}$. In case (1), Steiner considers three different proofs but does not explicitly give a characterizing property for the proofs he takes to be explanatory. I propose a suitable characterizing property 'being the sum of an arithmetic sequence in N' and show that this fits Steiner's description of the property as a 'symmetry' property. I also show how my suggestion fits Steiner's remark that explanation is a relation between an array of proofs and theorems. I argue that explanatoriness tracks generalizability for Steiner, and show how the proofs Steiner takes to be explanatory are generalizable.

In case (2), Steiner considers a proof that makes use of unique prime factorization. This proof generalizes to cover the irrationality of the square root of any prime. I present Tennenbaum's geometric proof of the irrationality of $\sqrt{2}$, which I suggest is also explanatory and yet generalizes to a lesser extent. That is, generalizability admits of degree. This creates a dilemma for Steiner: either his account cannot accommodate the geometric proof, or, if it can, Steiner should allow for degrees of explanatoriness. Yet Steiner explicitly denies that his concept of explanation admits of degree.

In response to this dilemma, I propose a fresh reading of generalizability: 'A proof is generalizable in case the same argument applied to any other object with the same characterizing property results in a proof of the (suitably modified) proposition which is now the conclusion'. On this reading, although one characterizing property may be instantiated more times than another, generalizability does not admit of degree.

Although this may not map exactly onto use of the term 'generalizability' in mathematics, I claim my reading provides an interesting new approach. Mathematical explanation, I suggest, has an ontic or objective component: a proof either makes use of a characterizing property in Steiner's sense, or it does not. And mathematical explanation also has an epistemic component: two proofs may present the same characterizing property in different and differently accessible ways. A reader of the proof may find one presentation more illuminating than another, depending on her cognitive background and skills.

11:50 – 12:25 *Mathematical practices and the development of eighteenth-century mechanics. Conflicting values, philosophical obstacles and methodological ideals*

Yannick Van den Abbeel (VUB, Brussels)

Whereas Newton developed his physics under the strong belief that the use of geometry was the proper way to "achieve a science of nature supported by the highest evidence," about one century later Lagrange said in his Analytical Mechanics that "the reader will find no figures in this work but only algebraic operations, subject to a regular and uniform rule of procedure." What motivated this radical change in mathematical practice? It is often claimed that the transition of Newtonian to Lagrangian mechanics was just a straightforward translation of Newton's geometrical demonstrations into the language of the calculus. In my presentation, I shall offer a somewhat more broad picture. More concretely, I shall show how mathematical practices in eighteenth-century mechanics were often guided by certain values, philosophical concerns and methodological ideals.

(1) To better understand the importance of mathematical values I shall begin by discussing the views of Newton and Leibniz. I shall show that Newton in his work often referred to aesthetic values such as elegance and conciseness. Leibniz, on the other hand, valued the mechanical and algorithmic character of his calculus and the possibility to reason independently of geometrical interpretation. The values which oriented Newton and Leibniz in two different directions were also accepted by their followers in the eighteenth century and gave rise to two different schools: the British Newtonian and the Continental Leibnizian school. Focussing on some of the key figures of these different schools, I shall show that the way a problem was solved mathematically was often contingent upon the values which were shared by the mathematical community.

(2) In the second part of my presentation I shall look more closely at the relation between mathematical practices and philosophical issues. Galileo initiated the seventeenth-century belief that physics was an enterprise which allowed the nature of things to be penetrated via geometry. However, the complications of accepting the infinitely small in nature forced eighteenth-century physicists to put aside any meaningful ontological claims about the infinite. I will argue that this lowering of philosophical ambitions was closely related to certain shifts in mathematical practice and promoted the widespread use of a formal calculus independent of geometrical-ontological concerns.

(3) The last point I want to discuss in my presentation is the role of methodological ideals such as unification and reductionism in eighteenth-century mechanics. Whereas the geo-

metrical approach required a different strategy for each problem, the calculus harvested in itself the potential of being an unified tool to solve all problems in mechanics. Lagrange even admitted that the goal of his lifelong research program was to find one general analytic principle from which the equations of motion of any system could be deduced. To accomplish this he appealed to a strong form of reductionism. For Lagrange mechanics was reduced to geometry, geometry was reduced to analysis, and analysis was reduced to algebra. Once again, my goal will be to highlight how physicists had certain trans-mathematical aims in mind which guided their mathematical practice.

12:25 – 13:00 *Intertheoretic reduction and explanation in mathematics*

William D'Allesandro (University of Illinois-Chicago)

Over the past five decades or so, intertheoretic reduction has been a central issue in both general philosophy of science and in the philosophies of the various special sciences. In that time, the nature, properties and varieties of intertheoretic reduction have been thoroughly explored. Much attention has also been paid to the link between reduction and explanation, the metaphysical basis of reducibility, the relationship between reduction and other forms of theory succession, and other such issues. This, of course, is as it should be. Theory reduction is an intrinsically important phenomenon, and by better understanding it we stand to gain valuable insights about the metaphysics, epistemology, psychology and practice of the empirical sciences.

Intertheoretic reduction has long been acknowledged to occur in pure mathematics also. (Most everyone is familiar with the idea that we can "reduce mathematics to set theory", for instance.) But relatively little attention has been paid to the above sorts of questions as they arise in the mathematical context. This is unfortunate and rather unaccountable, since the answers to such questions might well prove as illuminating for the philosophy of mathematics as their counterparts have for the philosophy of science.

The present paper attempts to address this imbalance. Of course, correcting it completely would require the efforts of a substantial research program, so my contribution here will necessarily focus on a small piece of the picture. My topic has to do with reduction and explanation in mathematics. I plan to argue for the following three claims: (1) Intertheoretic reduction in mathematics should be understood in broadly Nagelian terms. That is, it should be understood as an essentially linguistic phenomenon that need not involve intertheoretic identities, composition relations or other metaphysical connections. (2) Intertheoretic reductions are relatively common and natural in mathematics, just as they are in empirical sciences. In particular, foundational-style reductions, involving e.g. set theory or category theory, are far from the only examples. (3) Unlike what appears to be the case in the empirical context, where a successful reduction is virtually always an explanatory achievement, only some mathematical reductions are explanatory. (I illustrate the second and third claims with an examination of two cases – first, the reduction of classical algebraic geometry to Grothendieck's theory of schemes, and second, the reduction of arithmetic to set theory. The first case furnishes an example of a non-foundational but explanatory reduction; the second case, I argue, fails to be explanatory, though it's of value in other respects.)

14:30 – 15:30 *The beauty of the Kochen-Specker Theorem*

Ronny Desmet (VUB, Brussels)

Gian-Carlo Rota wrote: "The beauty of a theorem is best observed when the theorem is presented as the crown jewel within the context of a theory." The aim of this lecture is twofold. Its first part is an account of Alfred North Whitehead's theory of beauty to show that his idea of the background dependency of beauty philosophically substantiates Rota's aphorism. Its second part is an introduction to the Kochen-Specker Theorem, one of the most famous mathematical theorems of Quantum Mechanics, in a way to make the audience experience the truthfulness of Rota's aphorism. The lecture also has a very interesting spin-off: the notion of 'beauty' in Whitehead's philosophy and the notion of 'spin' in the Kochen-Specker Theorem illustrate the notion of properties that do not exist prior to the aesthetic or measurement experience, but emerge from the actual process of experience.

16:00 – 16:35 *Mathematical explanation as interventionist quasi-explanation*

Victor Gijsbers (Leiden University)

Explanations in mathematics are not yet well understood; at the very least, our theories about them are far less advanced than our theories about causal explanation. It would therefore be interesting to try to apply insights from the latter literature to the former problem. In this paper, I use the most influential current theory of causal explanation – Woodward's interventionism – to argue that our intuitions about mathematical explanations mimic the intuitions we have about causal explanations; but I then go on to show that mathematical explanations turn out to be subjective in a way that causal explanations are not, and that they are therefore better thought of as quasi-explanations.

To make my argument, I start out from Steiner's well-known theory of mathematical explanation. I argue that his theory is both too vague and too strict, but that its core intuition can be assimilated in a highly natural way to the formal framework of the interventionist theory of explanation. The main choice one has to make during this assimilation is whether or not to introduce a concept of 'mathematical intervention' as an analogue to the causal interventions of Woodward. I argue that a quasi-interventionist theory of mathematical explanation that dispenses with the idea of mathematical interventions is unable to handle certain explanatory asymmetries reminiscent of the traditional flagpole-and-shadow problem; e.g., the fact that we can explain why N^2 is countable by using the countability of N and Cantor's pairing function, but that we cannot explain why N is countable by performing a reverse argument on the countability of N^2.

I then develop a fully interventionist theory of mathematical explanation, which includes the idea of mathematical interventions as interventions on the processes of choosing axioms and constructing mathematical objects. This theory is found to handle the asymmetries in a satisfactory way. However, the nature of these interventions depends on the mental make-up of the mathematician, not on the objective structure of mathematics itself. I conclude that the intrinsic subjectivity which thus seems to be at the heart of mathematical explanation makes it best to see mathematical explanations as quasi-explanations that fulfil a purely pragmatic role.

16:35 – 17:10 *Mathematical ontology for knowledge representation*

Aziz F. Zambak (Middle East Technical University)

Data and information are exponentially growing in certain disciplines such as bioinformatics, finance, education, natural science, weather science, life science, physics, astronomy, law and social science. Knowledge representation is a field of Artificial Intelligence and it is related to many topics such as the frame problem, semantic web, conceptual mapping, big data, automated problem solving, theorem proving and NLP. The main goal of knowledge representation is to find a proper way of representing data and information in a form that provides computers to build a decision support system and solve complex problems in above mentioned topics. There are many challenging issues in knowledge representation such as modality, data structures, markup languages, classification reasoners etc. However, the real challenge for knowledge representation is the integration of data in a way that allows for inferring novel information. This presentation aims at showing that knowledge representation includes many formal methods/techniques (such as XML, OWL, DL and RDF) that depend on set theoretic principles. We defend the idea that we need a mathematical ontology, depending on the category theory, in order to overcome the real challenge for knowledge representation. I will propose GISONT (Global Inference System Ontology) as a mathematical ontology for knowledge representation. There will be five main parts in this presentation. In the first part, I will show that the formal methods/techniques, used in knowledge representation, have certain limitations due to their set theoretic principles. In the second part, I will very briefly introduce the main characteristics of category theory and describe the essential differences between set theory and category theory. In the third part, I will propose the main principles of a new markup language, called Lexicographic

Markup Language (LML), based on category theory. In the fourth part, I will describe some category theoretic operations as novel tools for improving OWLs capability. In the final part, I will discuss certain possible applications of GISONT into various topics in knowledge representation.

FRIDAY 11 DECEMBER

9:30 – 10:30 *Proofs, narratives, rhetorics, and style*

Jean Paul Van Bendegem (VUB, Brussels)

11:00 – 11:35 *Explanatory proofs in mathematics: Noneism, Someism, and Allism*

Flavio Baracco (State University of Milan)

In recent times philosophers of mathematics have generated great interest in explanations in mathematics. They have focused their attention on mathematical practice and searched for special cases that seem to own some kind of explanatory power. The talk aims to discuss this issue. We firstly point out the vagueness of the notion of explanation and the no easy problem of evidence. Hence we identify two main views in this debate, namely noneism and someism: the first is the view that no proof is explanatory, whereas the second is the view that some proofs are explanatory while others are not. Then we deal with this second view focusing on Frans and Weber's account. The authors suggest that, at least in geometry, some proofs seem to apply a mechanistic model of explanation and for this reason they can be thought as explanatory. We believe that such model is interesting, although it needs few clarifications. Hence, we try to clarify their account and we further point out some criticisms by means of a test case on a reductio proof. The authors explicitly claim that reductio proof are not explanatory at all, whereas we show that their model is able to capture at least one reductio proof. Hence, we outline our final assessment on someism which turns out to be rather sceptical. We argue that someism is a troublesome view whereas another view that we call allism seems to work better, i.e. the view that all proofs are explanatory, at least in some sense. We show the plausibility of this view and its advantages over someism, and finally we aim to reintrepreted Frans and Weber's account as an allistic model able to reveal some kind of explanatory degree in a proof, at least in geometry. This attempt suggests that allism might be a fruitful view and we believe that future investigations with respect to this view might shed light on this troublesome topic.

11:35 – 12:10 *What is distinctive about distinctively mathematical scientific explanations?*

Sorin Bangu (University of Bergen)

The question as to whether there can be mathematical explanations of physical phenomena has received a lot of attention lately. This interest is in part fuelled by the role of these explanations in a larger metaphysical project: if such explanations make indispensable use of mathematics, then, presumably, (a certain version of) the so-called Indispensability argument for mathematical realism deserves more credibility (Colyvan 2001, Baker 2005, Saatsi 2011, Pincock 2012, Bangu 2013). But the query has been found intriguing even by many philosophers whose main focus is not indispensability (Batterman 2010, Bueno and French 2012, Skow 2013, Lange 2013) and this is the direction I take in this paper: I will examine the very idea of a physical fact receiving a mathematical explanation. My interest, however, will be restricted to a recent insightful paper by Marc Lange (2013), in which he sets out to characterize "certain scientific explanations of physical facts" (p. 485) as 'distinctively mathematical' (explanations), or DMEs for short. He conducts the investigation by analyzing several examples of DMEs[1] , and here is one of them: the mathematical truth that twenty-three cannot be divided evenly by three explains why Mother will always fail to evenly distribute twenty-three strawberries among her three children. This is a case of an "explanation[ ] that [is] mathematical in a way that intuitively differs profoundly from" what Lange calls "ordinary scientific explanations employing mathematics." (p. 486) OEMs hereafter. Baker's well-known explanation of the primeness of the life-cycles of cicadas (2005) is for Lange an example of the latter.

According to Lange, unlike the OEMs, the DMEs "do not exploit [the] causal powers" (p. 497) of the entities and structures involved in the scientific context. Although I'm generally in agreement with several of the ideas Lange articulates in the paper, I will argue that the dichotomy he proposes, between DMEs and OEMs, doesn't in fact exist: I claim that both DMEs and OEMs are ultimately on a par (hence the answer to the question in the title is 'Nothing'.) The argument leading to this thesis proceeds in several steps, beginning with (i) a discussion of what it means to 'exploit' causal powers as opposed to presupposing a causal background, then advances to (ii) an argument for the idea (overlooked by Lange, I think) that both DMEs and OEMs presuppose such a background, and finishes with the point that (iii) although the causal background in the two cases may differ in saliency, this is a pragmatic (not epistemic) factor, and thus should not count as decisive in establishing the distinction central to Lange's view. Along the way, I will devote some space to discussing the distinction between physical v. mathematical facts, the senses of explanation relevant in this context, as well as the role of causal considerations in explanation more generally.

Selected bibliography

Lange, M. [2013] 'What Makes a Scientific Explanation Distinctively Mathematical?' BJPS 64: 485 – 511.

[1]Lange presents three or four more examples, including Kitcher's knot (1989) and Pincock's Koenigsberg bridges (2007).

12:10 – 12:45 *The method is message: mathematical truth and the Frege-Hilbert controversy*

Fiona Doherty (University of Cambridge)

One of the richest exchanges exploring the nature of mathematical knowledge and truth in the 19th century is the correspondence between Frege and Hilbert concerning Hilbert's seminal Foundations of Arithmetic. In this paper I provide an exposition of, and draw a moral from, the Frege-Hilbert controversy. The moral is that our understanding of the nature of mathematical knowledge and truth is deeply interconnected with the methodology we employ in mathematical practice: Our conception of mathematical truth and knowledge constrains and directs the methodology we deem legitimate, and a fruitful methodology informs and affects our understanding of mathematical truth. This observation is important because it provides a concrete connection between the philosophical investigation of mathematics and its practice by mathematicians. It follows, for one, that philosophical inquiry into the nature of mathematical knowledge and truth will be enriched by attention to the methods mathematicians have shown to be most fruitful. Further, the moral explains the broad consensus mathematicians report on their intuitions regarding the subject of mathematical truth. As such, the mathematician will likewise do well to turn to the philosopher in order to understand, and even adapt, the philosophical presumptions encoded in the methodology they inherit and advance.

For Hilbert, the new methodology he presents in Foundations was a revolution; for Frege, it was a complete failure. Hilbert departed from Euclid's canonical understanding of an axiom as a foundational truth and instead employed as axioms formal sentences whose primitive geometric terms were indeterminate in meaning. This enabled the primitive terms to be re-interpretable in distinct background theories and allowed for the methodology Hilbert employs in his consistency and independence proofs.

For Frege, axioms were to be understood as propositions expressed by determinate sentences and established as true prior to their employment as axioms. As such, Hilbert's schematic sentences incapable of being true or false were an abomination, incapable of playing the foundational role of an axiom or of defining the primitive geometric terms. Hilbert, by contrast, understood the subject-matter of mathematics to be any collection of idealized objects which satisfy the relevant structure expressed by his axioms. The consistency results established by way of Hilbert's reinterpretation method could not establish consistency as Frege understood it, since the reinterpreted sentences merely expressed distinct propositions.

I will argue that Frege and Hilbert's antithetical understandings of the nature of mathematical knowledge and truth is what results in their stark disagreement about the effectiveness of Hilbert's methodology. Having done this, I will extract the moral from their controversy.

Hilbert's methodology has proved most fruitful in modern mathematics, but if the moral is correct, then employing these methods of proof already commits us to a certain conception of the subject of mathematical knowledge and truth, one which is in completely opposition to a Fregean conception. In particular, it commits us to the view that the subject-matter of mathematics is not as Euclid thought it was reality, or at least, a fixed collection of concepts and objects but higher-order concepts which characterise structures across infinitely many abstract systems.

14:15 – 15:15 *Dirichlet's theorem on primes in an arithmetic progression*

Jeremy Avigad (Carnegie Mellon University)

In 1837, Peter Gustav Lejeune Dirichlet proved that there are infinitely many primes in any arithmetic progression in which the terms do not all have a common factor. This beautiful and important result was seminal in the use of analytic methods in number theory.

Contemporary presentations of Dirichlet's proof are manifestly higher-order. To prove the theorem for an arithmetic progression with common difference k, one considers the set of "Dirichlet characters modulo k," which are certain types of functions from the integers to the complex numbers. One defines the "Dirichlet L-series" L(s, χ), where s is a complex number and chi is a character modulo k. One then sums certain expressions involving the L-series over the set of characters.

This way of thinking about characters, which involves treating functions as objects just like the natural numbers, was not available in the middle of the nineteenth century. Subsequent presentations of Dirichlet's theorem from Dedekind to Landau show a gradual evolution towards the contemporary viewpoint, shedding light on the development of modern mathematical method.

15:45 – 16:20 *The role of trust in mathematical knowlege: Facing unserveyability in proofs*

Line E. Anderson & Henrik Kragh Sørensen (Aarhus University)

This paper outlines an account of the role of trust in mathematical knowledge. At the same time, it illustrates how the philosophy of mathematics in practice can learn from the philosophy of science in practice. Inspired by the work on trust in social epistemology and philosophy of science in practice, we thus turn to the role of trust in mathematical practice.

While the stereotypical view of mathematical research still features the isolated professor in his office, working away on a blackboard, reality is moving further and further away from it. Often mathematical research articles are written in collaborations, which can be vast and span multiple expertises, and, as a consequence, the proofs of some key results in contemporary mathematics cannot be read, comprehended, internalized, and evaluated by any individual within a reasonable time span. These and other forms of unsurveyability make it necessary for mathematicians to trust other mathematicians both when doing this type of collaborative work themselves and when using its outcomes in their own individual work.

This observation implies that we should include trust in our epistemologies of mathematics. Thus encouraged to develop a philosophical account of the role of trust in mathematical knowledge, we turn to analyze mathematicians' own standards as to when an individual can make a claim to mathematical knowledge on the basis of trust. To do so, we draw upon recent in depth interviews with research mathematicians performed by philosopher Eva MllerHill (2011) and additional empirical evidence. Combining this with John Hardwig's (1985, 1991) classical account of the role of trust in knowledge, we then develop an account of the role of trust in mathematical knowledge, thus showing how Hardwig's account can be molded to fit mathematicians' practice, and how it can thereby be of use to them.

16:20 – 16:55 *Cathedral builders: The sublime in mathematics*

Vladislav Shaposhnikov (Lomonosov Moscow State University)

1. The topic of mathematical aims and goals is not so widely discussed as the kindred one on scientific aims and goals. Nevertheless, there is a wide range of specific aims pursued by mathematicians as mathematicians. Moreover external goals differ from multilevel internal ones. External goals are immediate objectives of mathematician's efforts. They are equation solving, classification, generalization, proving and so on. Among internal goals one can find systematization, unification, explanation and justification. On the next level one can ask why we are looking for say justification, what kind of justification is appropriate for our purposes and what are these purposes. Finally one can try to uncover the ultimate internal goals of the mathematical activity. It is sensible to stop just before we lose the specificity of mathematical activity and get into the field of general human goals.

2. The same external goal may be motivated by diverse internal goals. One may recall an old parable, the parable of the three stonecutters working at the construction of a cathedral. Their external goal was exactly the same: to cut stones giving them the shape required; but being asked what they were doing they gave different answers. The first one said: "I am making a living". The second one: "I am doing the best job of stonecutting in the entire county". The third one: "I am building a cathedral!" Their internal goals turned out to be quite diverse.

I would like to interpret the parable as representing three attitudes to doing mathematics: pragmatic, aesthetic and theological. According to the first attitude mathematics is something very useful in science and everyday life, no wonder it helps mathematicians to earn their living. According to the second one a pure mathematician inhabits a sort of ivory tower and practices a kind of art for art's sake. These two attitudes are widely recognized, while theological one is less known. I dare interpret the metaphor of 'building a cathedral' as the theological attitude to doing mathematics according to another famous metaphor describing a cathedral as 'theology in stone'. The third attitude is possible owing to the phenomenon of the mathematical sublime.

3. The sublime and beauty are closely connected and often confused. The sublime forms a religious stratum within aesthetic values. Expression 'scientific sublime' usually refers to the image of natural science in Romanticism. The 'sublime' in that context means a feeling on its way from terror to awe. If we try to apply it to contemporary science it will require something at the frontiers of knowledge. There are some contemporary attempts at working with microbiological sublime' as well as 'computational' or 'digital sublime'.

'Mathematical sublime' has been used almost exclusively in the Kant's sense. I use this expression in the sense of the experience of the sublime evoked especially by doing mathematics. It does not require something at the forefront of mathematics, and is almost free from straightforward sense of horror, but still is constituted by "the informing of the infinite into the finite" (as Schelling defined the sublimity). I am going to discuss different variants and examples of the mathematical sublime: mathematical perfection; mathematical infinite; absolute certainty of proofs, etc. My thesis is that the experience of mathematical sublime serves as one of the irreducible ultimate goals of mathematical activity.