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Effect of Planar Waveguide Geometry on the Formation of Optical Bullets

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Abstract—The formation of optical bullets in propagating light beam pulses inside a planar waveguide with quadratic nonlinearity is studied. Cases of a waveguide with saturation and an unlimited waveguide with a quadratic profile of the change in the refractive index are considered.

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INTRODUCTION

Two- and three-component solitons in a medium with quadratic nonlinearity have been studied since 1974, when the possibility of their existence was first demonstrated [1]. The first investigations were on (1 + 1)D cases. Multicomponent multidimensional solitons with quadratic nonlinearity are of interest because of their stability and lower excitation threshold, compared to solitons with cubic nonlinearity [2].

The stability of light beams at quadratic nonlinearity was studied either analytically, using the variational approach, or numerically [3, 4]. Space–time solitons were shown to be stable under conditions where pulses at the first and second harmonics propagate in the anomalous dispersion mode. In [5, 6], we developed a detailed theory of breathing light bullets propagating in a medium with anomalous dispersion.

To form completely localized waves in the normal dispersion mode, we must compensate for linear diffraction effects, and for dispersion and nonlinear diffraction effects. The extraordinary ability of waveguides to maintain stable soliton structures is well known [3]. Waveguide geometry can play a focusing or defocusing role and compete with the tendency of stretching.

In this work, we analyze the possibility of the formation and stable propagation of light bullets in a plane waveguide with quadratic nonlinearity. Considerable attention is given to the normal dispersion mode at both frequencies corresponding to the visible frequency range of the radiation.

As is well known, the propagation of an optical beam-pulse in a medium with cubic nonlinearity can be accompanied by a collapse of space–time, the nature of which depends on the input parameters. One example of a nonlinear mechanism that can stop a collapse and contribute to the generation of stable space–time solitons (so-called light bullets) is the use of a medium with saturating nonlinearity [1].

Compared to solitons at cubic nonlinearity, multidimensional solitons at quadratic nonlinearity are characterized by much higher stability and a low threshold of generation. The main restrictions on the possibility of forming optical bullets in a homogeneous medium are related to the type of dispersion determining whether the medium is focusing or defocusing.

The properties of a focusing waveguide, along with the competition between nonlinearity, dispersion, and diffraction, were shown in [2] to play an important role in transitioning to waveguide geometry. For a waveguide with a parabolic refractive index profile, an approximate analytical solution in the form of a two-component optical bullet was obtained for both normal and anomalous dispersion. However, this solution can be applied to narrow beams, since the parabolic function grows asymptotically without limit. Real waveguides considerably alter the refractive index in the center, while the waveguide index does not change notably at the periphery. This must be taken into account when the size of the beam is comparable to that of the waveguide.

STATEMENT OF THE PROBLEM

In this work, saturation is introduced into the geometry of a waveguide with a quadratic nonlinearity. The transverse profile of the refractive index remains parabolic near the waveguide's center and reaches a constant value at the periphery. This profile corresponds to real situations better than a parabolic profile. Under the combined effects of diffraction and dispersion, the pulsed generation mode of the second harmonic in this waveguide is described by the following system of equations for the complex amplitudes of fundamental frequency ψ_1 and second harmonic ψ_2:

$$i\left[\frac{\partial\psi_1}{\partial z}+\left(\varepsilon_1\alpha_1\frac{x^2}{1+x^2/a_1^2}+\delta\right)\frac{\partial\psi_1}{\partial\tau}\right]+\frac{\beta_2^{(1)}}{2}\frac{\partial^2\psi_1}{\partial\tau^2} - \gamma_1\psi_1^*\psi_2 e^{i\Delta kz}=\varepsilon_1 q_1\frac{x^2}{1+x^2/a_1^2}\psi_1+\frac{c}{2n_1\omega}\frac{\partial^2\psi_1}{\partial x^2}, \quad (1)$$

$$i\left[\frac{\partial\psi_2}{\partial z}+\left(\varepsilon_2\alpha_2\frac{x^2}{1+x^2/a_2^2}-\delta\right)\frac{\partial\psi_2}{\partial\tau}\right]+\frac{\beta_2^{(2)}}{2}\frac{\partial^2\psi_2}{\partial\tau^2} - \gamma_2\psi_1^2 e^{-i\Delta kz}=2\varepsilon_2 q_2\frac{x^2}{1+x^2/a_2^2}\psi_2+\frac{c}{4n_2\omega}\frac{\partial^2\psi_2}{\partial x^2}. \quad (2)$$

Here, $n_{1,2}$ are the refractive indices of the first and second harmonics in the waveguide's center; $\beta_2^{(1,2)}$ are the coefficients of group velocity dispersion (GVD), which depend on refractive index $n_{1,2}(\omega)$; $\tau = t-\frac{1}{2}\left(\frac{1}{\upsilon_{g1}}+\frac{1}{\upsilon_{g2}}\right)z$; $\gamma_1=\frac{4\pi\omega}{cn_1}\chi_2(-\omega;2\omega)$ and $\gamma_2=\frac{4\pi\omega}{cn_2}\chi_2(\omega;\omega)$ are coefficients of quadratic nonlinearity proportional to the second-order susceptibility; $\Delta k = 2\omega(n_1-n_2)/c$ is phase detuning; c is the speed of light in a vacuum; ω is the carrier frequency of the first harmonic; and $\delta=\frac{1}{2}\left(\frac{1}{\upsilon_{g1}}-\frac{1}{\upsilon_{g2}}\right)$ is group velocity detuning. Coefficients $\alpha_{1,2}=\frac{n_{1,2}^2-1}{2cn_{1,2}}$ and $q_{1,2}=\omega\alpha_{1,2}$ are responsible for the waveguide's properties; $a_{1,2}$ are the characteristic dimensions of the waveguide's inhomogeneity; $\varepsilon_{1,2}=+1$ corresponds to a defocusing waveguide; and $\varepsilon_{1,2}=-1$ corresponds to a focusing waveguide.

Below, we assume that characteristic dimensions a_1 and a_2 of transverse waveguide inhomogeneity generally differ for the fundamental frequency and the second harmonic. Let us consider the case of equal phase and group velocities: $\Delta k=0$, $n_1=n_2=n$, $\delta=0$, $\upsilon_1=\upsilon_2=\upsilon$, $\tau=t-z/\upsilon$. Dispersion coefficients are considered to be linked by relations $\beta_2^{(2)}=2\beta_2^{(1)}$. A soliton solution of the form $\psi_{1,2\,0}=E\cosh^{-2}(x/r_x)$ is known to exist in the case of one dimension [7]. In the case of many dimensions, a soliton solution without a waveguide exists only upon anomalous dispersion ($\beta_2^{(1,2)}<0$). With a normal dispersion ($\beta_2^{(1,2)}>0$), a compensating effect (e.g., a waveguide) is needed for the formation of an optical bullet. Let us consider cases where a waveguide has a refractive index profile that is parabolic with saturation.

NUMERICAL MODELING

In numerical modeling, Eqs. (*1*) and (*2*) were made dimensionless: $\psi_1=B_1A_0$, $\psi_2=B_2A_0$, $z=\bar{z}l_{\rm nl}$, $x=\bar{x}R_0$, $a_{1,2}=\overline{a_{1,2}}R_0$, $\tau=\bar{\tau}\tau_0$, $\Delta\bar{k}=\Delta kL_{\rm nl}$, $\delta=\frac{\bar{\delta}\tau_0}{L_{\rm nl}}$, $\alpha_{1,2}=\bar{\alpha}_{1,2}\tau_0L_{\rm nl}^{-1}$, and $q_{1,2}=\bar{q}_{1,2}L_{\rm nl}^{-1}$, where A_0 is the peak amplitude of the fundamental harmonic. We now introduce characteristic lengths $L_{\rm dis}=\frac{2\tau_0^2}{\left|\beta_2^{(1)}\right|}$ of dispersion spreading and $L_{\rm D}=\frac{n_1\omega}{c}R_0^2$ of diffraction, plus nonlinear $L_{\rm nl}=\frac{1}{\gamma_1A_0}$. The dimensionless coefficients are $D_{aj}=\frac{R_0^2}{a_j^2}\overline{\alpha}_j{\rm sgn}(\varepsilon_j)$, $D_{qj}=\frac{R_0^2}{a_j^2}\overline{q}_j{\rm sgn}(\varepsilon_j)$, $\bar{\gamma}=\frac{\gamma_2}{\gamma_1}$, $D_{\tau1}={\rm sgn}\left(\beta_2^{(1)}\right)\frac{L_{\rm nl}}{L_{\rm dis}}$, $D_{\tau2}={\rm sgn}\left(\beta_2^{(2)}\right)\left|\frac{\beta_2^{(2)}}{\beta_2^{(1)}}\right|\frac{L_{\rm nl}}{L_{\rm dis}}$, $D_{x1}=\frac{L_{\rm nl}}{L_{\rm D}}$, and $D_{x2}=\frac{n_1}{n_2}\frac{L_{\rm nl}}{L_{\rm D}}$.

We thus arrive at the dimensionless system of equations

$$i\left[\frac{\partial B_1}{\partial\bar{z}}+\left(D_{a1}\frac{\bar{x}^2}{1+\bar{x}^2/\bar{a}_1^2}+\bar{\delta}\right)\frac{\partial B_1}{\partial\bar{\tau}}\right]+D_{\tau1}\frac{\partial^2B_1}{\partial\bar{\tau}^2} - B_1^*B_2\exp(i\Delta\bar{k}\bar{z})=D_{q1}\frac{\bar{x}^2}{1+\bar{x}^2/\bar{a}_1^2}B_1+\frac{1}{2}D_{x1}\frac{\partial^2B_1}{\partial\bar{x}^2}, \quad (3)$$

$$i\left[\frac{\partial B_2}{\partial\bar{z}}+\left(D_{a2}\frac{\bar{x}^2}{1+\bar{x}^2/\bar{a}_2^2}-\bar{\delta}\right)\frac{\partial B_2}{\partial\bar{\tau}}\right]+D_{\tau2}\frac{\partial^2B_2}{\partial\bar{\tau}^2} - \bar{\gamma}B_1^2\exp(-i\Delta\bar{k}\bar{z})=D_{q2}\frac{\bar{x}^2}{1+\bar{x}^2/\bar{a}_2^2}B_2+\frac{1}{4}D_{x2}\frac{\partial^2B_2}{\partial\bar{x}^2}. \quad (4)$$

Finally, we omit the bar above the variable notation.

Let us consider the generation of bullets by a powerful primary frequency beam. In accordance with [8], we use the initial condition

$$\psi_{10}=E_1\cosh^{-2}(x/r_x)\cosh^{-2}(\tau/r_t) \text{ and } \psi_{20}=0. \quad (5)$$

We first observed a pulsating optical bullet inside a parabolic waveguide. In our equations, this corresponds to when $a_{1,2}\to\infty$. Figure 1 shows the amplitude distribution of the first and second harmonics at central sections xz and tz. We used dimensionless parameters $D_{\tau1}=0.025$, $D_{\tau2}=0.05$ (normal dispersion), $D_{q1}=-1$, $D_{q2}=-2$ (focusing waveguide), $D_{x1}=0.3$, $D_{x2}=0.3$, and $\alpha_1=\alpha_2=-0.01$. The sec-

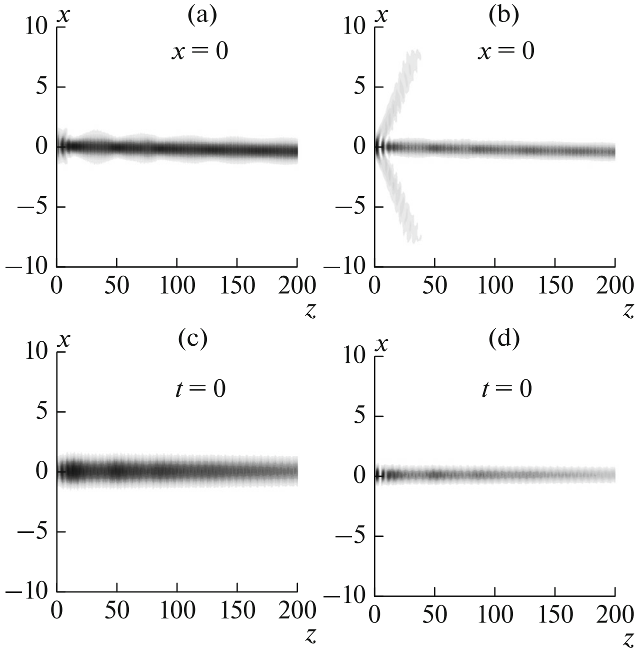

Fig. 1. Distribution of intensity at the central sections (a, b) tz, (c, d) xz of beams of the (a, c) first and (b, d) second harmonics inside a parabolic waveguide ($a_{1,2} \to \infty$).

ond harmonic arising at the beginning is seen to disintegrate into parts over time, while the main part remains at the center and another two lag behind or advance the center. The remaining part at the center, while interacting with the fundamental frequency, forms an optical bullet that propagates in a pulsating mode.

The parabolic waveguide was then replaced with a waveguide with saturation under the same conditions. Figure 2 shows the amplitude distributions of the first and second harmonics at central sections xz and tz. The parameter of waveguide saturation was $a_{1,2} = 1$ (i.e., equal to the initial beam width). As can be seen in Fig. 1, however, the bullet that formed was smaller than the input beam, so the waveguide profile was close to parabolic for most of the bullet. In contrast to a parabolic waveguide, an optical bullet without pulsation is in this case formed.

Figure 3 shows the amplitude distributions of the first and second harmonics at central sections xz and tz for smaller parameters of waveguide saturation $a_{1,2} = 0.75$. As can be seen in the figure, most of the input beam energy is dissipated, and we can no longer talk about an optical bullet. As waveguide parameters $a_{1,2}$ decline, the beam decays at shorter longitudinal distances z. Part of the energy from the tails is transferred to the periphery, reducing the intensity at the beam's center, and an optical bullet does not form. This shows that in order to obtain real optical bullets with widths comparable to that of the waveguide, we must increase the nonlinearity (i.e., the intensity of an incident wave). From a physical viewpoint, this is quite obvious, since the medium becomes almost homogeneous if $x > a_{1,2}$. Bullet formation is in this

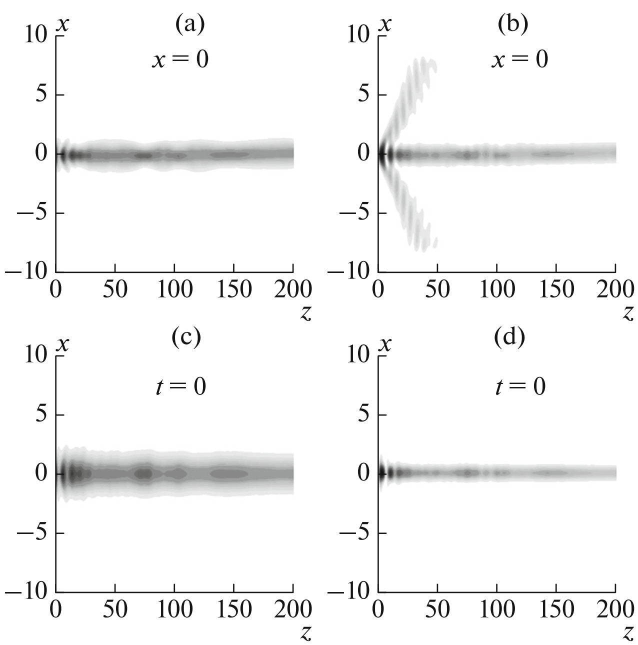

Fig. 2. Distribution of ntensity at central sections (a, b) tz, (c, d) xz of beams of the (a, c) first and (b, d) second harmonics in a waveguide with radius of saturation $a_{1,2} = 1$.

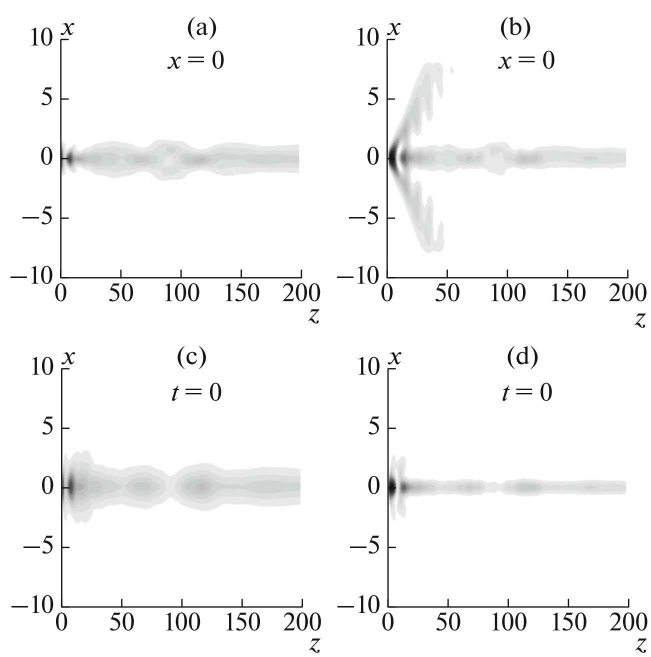

Fig. 3. Intensity distribution at the central sections (a, b) tz and (c, d) xz of beams of the (a, c) first and (b, d) second harmonics inside a waveguide with a radius of saturation $a_{1,2} = 0.75$.

case impossible when there is positive group dispersion [5]. The saturation of the transverse focusing profile of the refractive index thus prevents the formation of light bullets upon generation of the second harmonic in the region of positive group dispersion.

CONCLUSIONS

We considered the formation of optical bullets in waveguides with (a) a parabolic profile and (b) a profile with saturation of the refraction index at positive dispersion. The formation of these bullets was shown to be possible inside a parabolic waveguide. In a waveguide with a saturating profile, bullets form when the parameter of saturation is greater than their width; otherwise, the initial beam is scattered.

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