References for the Course
Algebra, Geometry and Analysis of Commuting Ordinary Differential Operators

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Abstract

This list of references are compiled for the students of the *I International Undergraduate Mathematics Summer School, 2018*. It is by no means an extensive list of reference on the subject covered by the class, but just a suggestion for a starting point (or for further reading) with most relevance for this class.

History: First articles about commuting differential operators

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Classification of commutative subalgebras of ODOs

J. Burchnall, T. Chaundy, *Commutative ordinary differential operators*, Proc. London Math. Soc. **21** (1923) 420–440.

J. Burchnall, T. Chaundy, *Commutative ordinary differential operators*, Proc. Royal Soc. London (A) **118**, 557–583 (1928).

J. Burchnall, T. Chaundy, *Commutative ordinary differential operators. II: The identity* $P^n = Q^m$, Proc. Royal Soc. London (A) **134**, 471–485 (1931).

I. Krichever, *Methods of algebraic geometry in the theory of nonlinear equations*, Uspehi Mat. Nauk **32** (1977), no. 6 (198), 183–208, 287.

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V. Drinfeld, *Commutative subrings of certain noncommutative rings*, Funct. Anal. Appl. **11** (1977), no. 1, 11–14, 96.

D. Mumford, *An algebro–geometric construction of commuting operators and of solutions to the Toda lattice equation, Korteweg deVries equation and related nonlinear equation*, Proceedings of the International Symposium on Algebraic Geometry, 115–153, Kinokuniya Book Store, Tokyo (1978).

J.-L. Verdier, *Équations différentielles algébriques*, Séminaire Bourbaki, 30e année (1977/78), Exp. no. 512, 101-122, Lecture Notes in Math. **71**, Springer (1979).

M. Mulase, *Category of vector bundles on algebraic curves and infinite-dimensional Grassmannians*, Internat. J. Math. **1** (1990), no. 3, 293–342.

Explicit examples of commuting ODOs and explicit determination of the spectral data

J. Dixmier, *Sur les algèbres de Weyl*, Bull. Soc. Math. France **96** (1968) 209–242.

I. Krichever, S. Novikov, *Holomorphic bundles over algebraic curves and nonlinear equations*, Russian Math. Surveys, **35**:6 (1980), 47–68.

P. Grinevich, *Rational solutions of equations of commutation of differential operators*, Func. Anal. Appl. **16** (1982), no. 1, 19–24, 96.

O. Mokhov, *Commuting differential operators of rank 3 and nonlinear equations*, Math. USSR–Izv. **35** (1990), no. **3**, 629–655.

E. Previato, G. Wilson, *Differential operators and rank 2 bundles over elliptic curves*, Compositio Math. **81** (1992), 107–119.

A. Mironov, *Self–adjoint commuting ordinary differential operators*, Invent. Math. **197** (2014), no. 2, 417–431.

O. Mokhov, *Commuting ordinary differential operators of arbitrary genus and arbitrary rank with polynomial coefficients*, Topology, geometry, integrable systems, and mathematical physics, 323–336, Amer. Math. Soc. Transl. Ser. 2, **234**, Amer. Math. Soc. (2014).

A. E. Mironov, A. B. Zheglov, *Commuting ordinary differential operators with polynomial coefficients and automorphisms of the first Weyl algebra*, Int. Math. Res. Not. IMRN, **10**, 2974–2993 (2016).

I. Burban, A. Zheglov, *Fourier-Mukai transform on Weierstrass cubics and commuting differential operators*, Oberwolfach Preprints (OWP), **3**, Mathematisches Forschungsinstitut Oberwolfach Oberwolfach, Germany, 1–32 (2016); https://arxiv.org/abs/1602.08694

Survey articles and books (contain not only the theory of commuting ODOs, but are mainly devoted to various aspects of nonlinear differential equations

I. M. Krichever, *Integration of nonlinear equations by the methods of algebraic geometry*, Russian Math. Surveys, **32** (6), 183—208 (1977).

Y. Manin, *Algebraic aspects of nonlinear differential equations*, Itogi Nauki Tekh., Ser. Sovrem. Probl. Mat. **11**, 5—152 (1978).

G. Segal, G. Wilson, *Loop groups and equations of KdV type*, Inst. Hautes Études Sci. Publ. Math. no. **61** (1985), 5–65.

M. Mulase, *Algebraic theory of the KP equations*, Perspectives in Mathematical Physics, R.Penner and S.Yau, Editors, 151—218 (1994).

E. Previato, *Seventy years of spectral curves: 1923–1993*, Integrable systems and quantum groups 419–481, Lecture Notes in Math. **1620**, Springer (1996).

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D. Mumford, *Tata lectures on Theta II*, Birkhäuser, Boston, 1984

Special lectures for students

E. E. Demidov, *The KadomtsevPetviashvili hierarchy and the Schottky problem*, Fundam. Prikl. Mat., 4:1 (1998), 367460; available at: http://www.mathnet.ru