COLLOQUIUM EUROMECH 531
"VORTICES AND WAVES:
IDENTIFICATIONS AND MUTUAL INFLUENCES"

Numerical simulation of the flow-structure interaction by the VVD method

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Good afternoon. My name is Yaroslav Dynnikov, and I present the paper «Numerical simulation of the flow-structure interaction by the VVD method»

Features of viscous vortex domains (VVD) method

- It's purely Lagrangian method for solving 2D Navier-Stokes equations
- It doesn't have any empirical parameters, all formulas are well based.
- The method can be applied for the flows with arbitrary moving boundaries. In particular it's useful for investigating flow-structure interaction.
- Body motion equations can be included in general system with fluid dynamics equations, therefore there are no limitations on body inertial property (in particular they can be negligibly small)
- The method allows to resolve boundary layer correctly with high resolution
- It allows to calculate friction forces on bodies surfaces.

G. Ya. Dynnikova “The Lagrangian approach to solving the time-dependent Navier-Stokes equations” Doklady Physics. 2004, V. 49, No. 11, p. 648-652

P.R. Andronov, S.V. Guvernyuk, G.Ya. Dynnikova, “Vortex methods for calculating unsteady hydrodynamic loads” Moscow : MSU, 2006. 184p. (in Russian)

VVD is an abbreviation for Viscous Vortex Domains. This method has been proposed by GD in this work. More detailed it's described in this book.

Among the well-known numerical schemes for the fluid flow simulation, the closest to VVD method is Diffusion Velocity method of Ogami and Akamatsu.

Далее по слайду

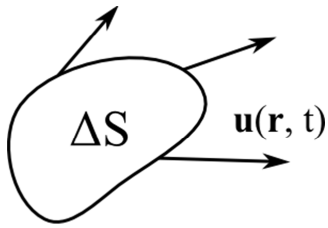
Basis of the VVD method

$$\frac{\partial \mathbf{V}}{\partial t} = \mathbf{V} \times \mathbf{\Omega} + \nu \nabla^2 \mathbf{V} - \frac{1}{\rho} \nabla \left(p + \frac{V^2}{2} \right), \ \nabla \mathbf{V} = 0$$

$$\frac{\partial \mathbf{\Omega}}{\partial t} = \nabla \times \left[\mathbf{V} \times \mathbf{\Omega} \right] + \nu \nabla^2 \mathbf{\Omega}, \quad \mathbf{\Omega} = \nabla \times \mathbf{V}$$

$$\boxed{\mathbf{u} = \mathbf{V} + \mathbf{V}_d}, V_d = -\frac{\nu}{\Omega} \nabla \Omega, \ \gamma = \int_{\Delta S} \Omega dS$$

$$\frac{\partial \Omega}{\partial t} = -\nabla \left(\mathbf{u} \Omega \right) \Rightarrow \boxed{\gamma = \mathrm{const}}$$



$\mathbf{V}$ - convective velocity

$\mathbf{V}_d$ - diffusion velocity

Y. Ogami, T. Akamatsu "Viscous flow simulation using the discrete vortex model. The Diffusion Velocity Method" Computers and Fluids. 1991, V. 19, No. 3/4, p. 433-441

Similar to the Diffusion Velocity method of Ogami and Akamatsu, the VVD method is based on the fact that circulation in a viscous fluid is conserved on contours moving with velocity u=V+Vd (u is equal to the sum of fluid velocity V and so called diffusion velocity Vd)

This follows from the Navier-Stokes equation

Difference between the VVD and Diffusion Velocity methods (DVM)

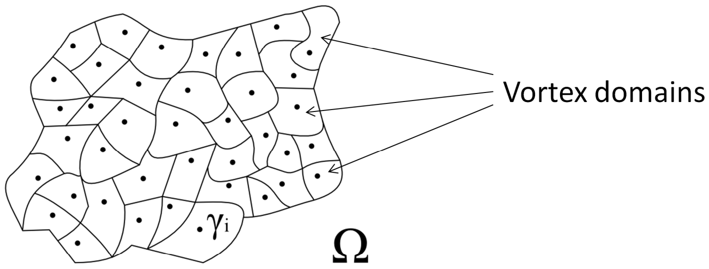
DVM	**VVD**
$\Omega(\mathbf{r}_i) = \sum_j \frac{\gamma_i}{\pi\sigma_j^2} \exp\left(-\left(\frac{\mathbf{r}_i - \mathbf{r}_j}{\sigma_j}\right)^2\right)$ $\mathbf{V}_{di} = \frac{2\nu}{\sigma_i^2 \Omega_i} \sum_j \mathbf{r}_{ij} \gamma_j \exp\left(-\frac{r_{ij}^2}{\sigma_j^2}\right)$	$\Omega(\mathbf{R}) = \lim_{\varepsilon\to 0} \frac{\int_S \Omega(\mathbf{r}) \mathrm{e}^{-\xi/\varepsilon}\, ds}{\int_S \mathrm{e}^{-\xi/\varepsilon}\, ds}, \quad \boldsymbol{\xi} = \mathbf{R} - \mathbf{r}$ $\mathbf{V}_{di} \approx \frac{1}{\mathrm{Re}} \frac{\sum_{j\neq i} \frac{\gamma_j \mathbf{r}_{ij}}{\varepsilon_i r_{ij}} \exp\left(-\frac{r_{ij}}{\varepsilon_i}\right)}{\sum_j \gamma_j \exp\left(-\frac{r_{ij}}{\varepsilon_i}\right)} +$ $+ \frac{1}{\mathrm{Re}} \sum_k \frac{\mathbf{n}_k d_k}{I_i} \exp\left(-\frac{r_{ik}}{\varepsilon_i}\right)$ $I_i = 2\pi\varepsilon_i^2 - \sum_k \varepsilon_i (\mathbf{n}_k \mathbf{r}_{ik}) \frac{(r_{ik} + \varepsilon_i)}{r_{ik}^2} \exp\left(-\frac{r_{ik}}{\varepsilon_i}\right)$
Fails near surface	Valid near surface

In VVD method we don't do any assumptions about the vorticity distribution and shape of vortex elements.

The formula for calculating the diffusion velocity is based on this expression, which is valid for any smooth function. These discrete formulas are more accurate than the formulas of Ogami, especially near the surfaces.

In Diffusion Velocity method the vorticity is expressed as superposition of Gaussian-shaped vortices with fixed core radius σ. Their method has a difficulty, connected with choosing of value of σ, because vortices should overlap everywhere in the flow for accurate computing. The calculation of the vorticity distribution and diffusive velocity near the surface isn't accurate.

Basis of the VVD method

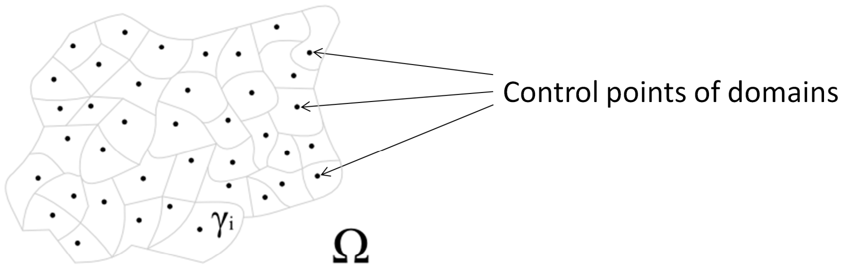


$$\mathbf{u} = \mathbf{V} + \mathbf{V}_d,\ \nabla \mathbf{u} \neq 0 \Rightarrow \text{ domains can expand and contract}$$

In VVD method we consider flow region to be covered with an invisible grid, which moves according to the previous slide, and therefore circulation of every cell remains constant. We call such cells as vortex domains.

Actually we don't have to remember the shape of contour. For each domain it's enough to control coordinates of the only point.

Basis of the VVD method



We move control points with speed $\mathbf{u} = \mathbf{V} + \mathbf{V}_d$
and remember their circulations $\gamma_i = \text{const}\ \forall i$
thus we can calculate all flow characteristics.

We move these points with velocity u and remember domains circulation gamma sub i, which is constant. Thus we can calculate all flow characteristics.

Generation of new domains takes place on the streamlined surfaces at every time step at each node of the contour. Points of the flow separation are obtained automatically.

Force and moment expressions

$$\mathbf{F}_h = m_*\dot{\mathbf{u}}_* - 2m_*\mathbf{r}_* \times \dot{\boldsymbol{\omega}} - \frac{\rho}{\Delta t}\sum_k (\mathbf{r}_k - \mathbf{r}_0)\times \boldsymbol{\gamma}_k - \nu\rho\sum_k \frac{d_k}{\pi\varepsilon_k^2}\mathbf{n}_k \times \sum_j \boldsymbol{\gamma}_j \exp\left(-\frac{r_{jk}}{\varepsilon_k}\right)$$

$$\mathbf{M}_h = \frac{1}{\Delta t}\frac{\rho}{2}\sum_k (\mathbf{r}_k - \mathbf{r}_0)^2 \boldsymbol{\gamma}_k + 2I_*\dot{\boldsymbol{\omega}} - m_*\dot{\mathbf{u}}_0 \times \mathbf{r}_* +$$

$$+\nu\rho\sum_k \frac{d_k}{\pi\varepsilon_k^2}\mathbf{n}_k \mathbf{r}_k \sum_j \boldsymbol{\gamma}_j \exp\left(-\frac{r_{jk}}{\varepsilon_k}\right) - 4\nu m_*\boldsymbol{\omega}$$

$$\mathbf{r}_* = \mathbf{R}_* - \mathbf{r}_0, \quad \mathbf{u}_* = \dot{\mathbf{R}}_* = \mathbf{u}_0 + \boldsymbol{\omega}\times\mathbf{r}_*, \quad I_* = \rho\int_{S_b} (\mathbf{r} - \mathbf{r}_0)^2\, ds, \quad \mathbf{J} = -\nu\frac{\partial\Omega}{\partial\mathbf{n}}$$

m_* – mass of displaced liquid

$\mathbf{R}_*$ – center of mass

$\mathbf{r}_0$ – axis

P.R. Andronov, S.V. Guvernyuk, G.Ya. Dynnikova, “Vortex methods for calculating unsteady hydrodynamic loads” Moscow : MSU, 2006. 184p. (in Russian)

The expressions for hydrodynamic force, force moment and pressure have been derived in this work. They all depend on circulations of newly generated domains. It's remarkable, that they depend on circulations linearly.

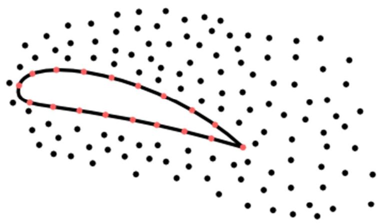
Here're written formulas for solid body, But we also have ones for deformable and semipermeable surfaces.

Domains generation

$$\mathbf{n}\mathbf{V}\left(\mathbf{R}_S\right) = \mathbf{n}\mathbf{V}_S\left(\mathbf{R}_S\right) \text{ - no-leaking condition}$$

$$\mathbf{V}(\mathbf{R}) = \int_S \mathbf{K} \times \mathbf{\Omega} dS - \oint_C \mathbf{K}(\mathbf{V}_S \mathbf{n}) + \oint_C \mathbf{K} \times \left[\mathbf{V}_S \times \mathbf{n}\right] + \mathbf{V}_\infty$$

$$\mathbf{K}\left(\mathbf{R}, \mathbf{r}\right) = \frac{\mathbf{R} - \mathbf{r}}{2\pi \left|\mathbf{R} - \mathbf{r}\right|}$$



$$\boxed{\begin{array}{l} \sum_{j=1}^{N_S} a_{ij}\gamma_j + \sum_k \mathbf{b}_{ik}\mathbf{V}_{S_k} = c_i,\ i = \overline{1, N_S - 1} \\ \sum_{j=1}^{N_S} \gamma_j = -\frac{\partial}{\partial t}\Gamma_S \end{array}}$$

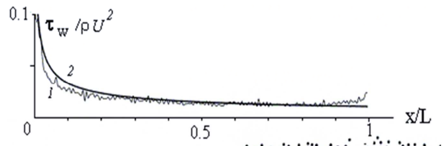
If velocity of the surface is unknown the system should be supplemented by the body dynamic equations and by the expressions of hydrodynamic forces.

As it has already been mentioned, formation of new domains takes place on the streamlined surfaces at every time step at each node of the contour. To find circulations we use linear system, which includes:

boundary no-leaking conditions, and body motion equations.

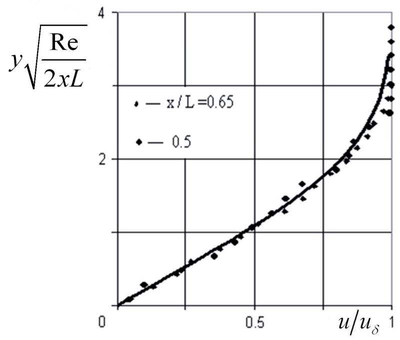
The no-slip condition is ensured by these 2 terms. All new domains represent free vorticity.

Testing the method: flow around the flat plate



Shear stress (smooth line corresponds to Blasius velocity distribution)

Vortices pattern



Velocity profile (full line – Blasius velocity distribution)

$$Re = \frac{Uh}{\nu} = 20, \quad \frac{h}{L} = 0.02$$

h - plate thickness

Testing of the method is shown here. This is classical problem of longitudinal flow around the flat plate at low Re number. This problem has an analytic solution.

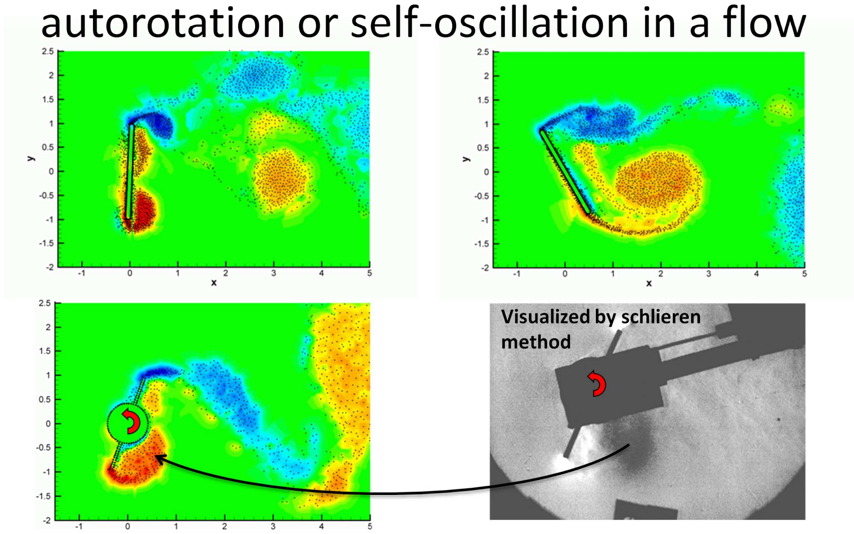
Figure 1 represents shear stress over the plate (smooth line represents shear stress for Blasius velocity distribution, and sharp line corresponds to the numerical results),

figure 2 shows position of domains control points. Flow moves from the left to the right.

On this figure there're depicted Blasius velocity distribution (solid line) and the numerical result obtained by the VVD method (dots)

Thus we can see good coincidence between VVD method results and theoretical data.

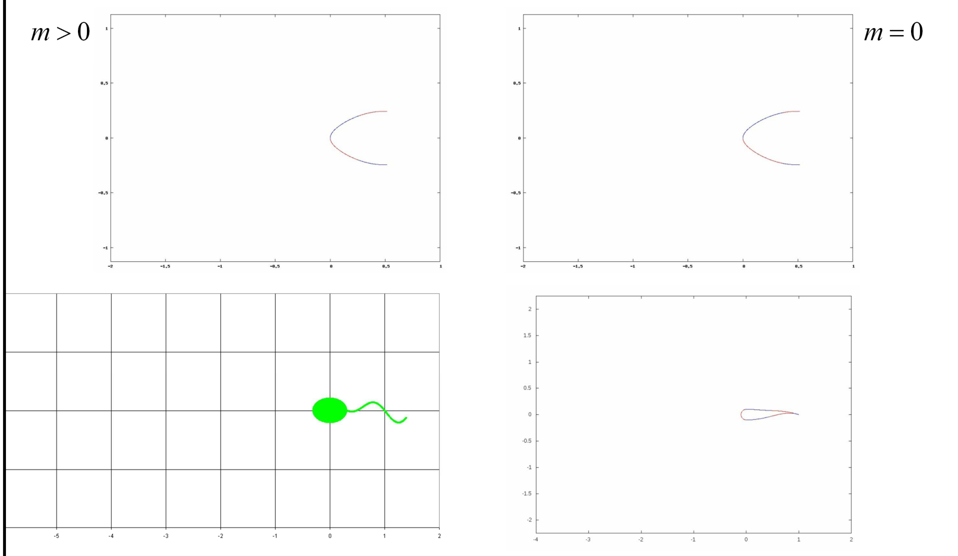
Coupled problem of a body autorotation or self-oscillation in a flow



P. R. Andronov, D. A. Grigorenko, S. V. Guvernyuk, G. Ya. Dynnikova // Fluid Dynamics 2007, Vol. 42, No.5, p. 719-731

The VVD method has been used for solving coupled task of autorotation of plates and impellers around their axes. It was obtained numerically and confirmed experimentally, that, depending on initial conditions, two different regimes with autorotation and self-oscillations of the considered bodies exist. During the autorotation regime a «vortex satellite» is formed ahead of the plate moving downstream. This vortex generates lower pressure that supports autorotation.

Method application: coupled problem of body locomotion



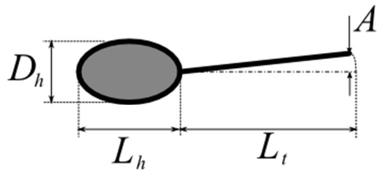
The flow-body interaction have been investigated using this method. It's a coupled problem of body locomotion due to proper shape deformation.

1) Jellyfish with finite mass
2) Masses jellyfish
3) Tadpole
4) Fish

Statement of the problem

$$\frac{D_h}{L_h} = \frac{\sqrt{5}-1}{2} \text{ - golden ratio}$$

$$\theta(t) = \begin{cases} A_\theta, t < 0 \\ A_\theta \cos(2\pi f t), t \geq 0 \end{cases}$$



$A_\theta = \arcsin(A / L_t)$

A - amplitude

f - frequency

The only parameter of problem - dimensionless frequency

Problem of tadpole swimming was considered more detailed. The task was to investigate the energy efficiency of its motion. Here's depicted tadpole model, and here it's shown on the background of real photo.

To simplify the task we consider only one degree of freedom – the movement along straight line without rotation.

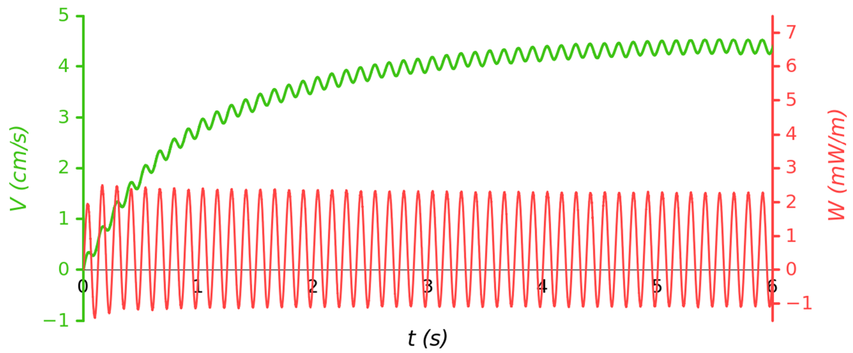
Our tadpole has ellipsoid head with proportions of the golden ratio, and straight unbendable tail. Density of tadpole and liquid are equal. Tadpole swings its tail in this law.

Example of obtained results

$L_h = 1cm, \quad L_t = 2.25cm, \quad \nu = 10^{-2}\, cm^2/s$ (water), $\quad A = 0.12cm, \quad \overline{f} = 4Hz$

$W = \omega \cdot \int_0^{L_t} \Delta p \cdot x dx$ - power per unit

Example of obtained results



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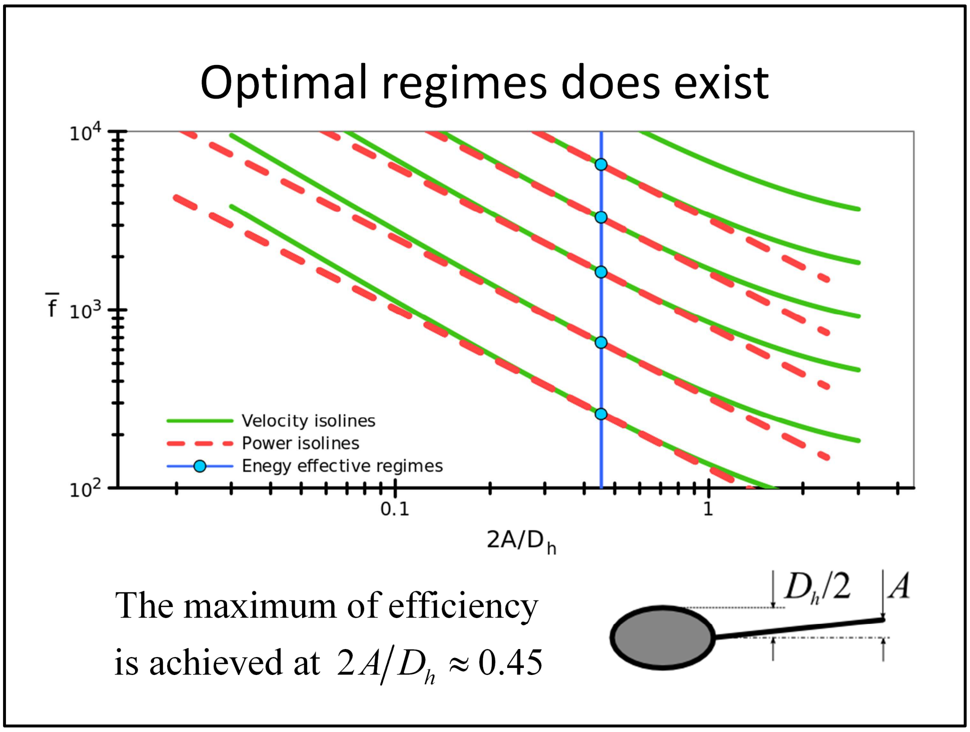
$W = \omega \cdot \int_0^{L_t} \Delta p \cdot x dx$ - power per unit

Here's shown an example of obtained results. For illustrativeness they're plotted in dimensional variables in case of 1cm tadpole, swimming in water.

Instant power is being computed using this formula. From this chart we can see its value serially becomes negative. It means that tail usually does negative work. In nature recuperation usually doesn't take place, thus cases with setting power zero were also considered.



We've modeled a lot of regimes, and analyzed dependency of tadpole's velocity and power consumption on frequency and amplitude of tail oscillations.

Here are shown isolines of velocity (green) and isolines of power (red). It's visible that if tadpole wants to move at a concrete speed, he should deviate his tail at about half of his width.

Conclusion

VVD method is effective for 2D nonstationary viscous flow simulation and for solving coupled problems of flow-structure interaction.

Thanks for your attention