with matter ejected by supernova explosions [11].

At the same time, models of coronas without heat conduction are in complete agreement with the observational data. Such coronas could easily be relics of "hot" protogalaxies, which may exist practically unchanged during billions of years after the initial phase of rapid evolution of a hot cloud of the protogalaxy.

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ROTATION OF GAS ABOVE THE GALACTIC DISK

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The galactic disk is modeled by an oblate spheroid with confocal spheroidal isodensity surfaces. An explicit analytic expression is found for the angular velocity of the gas outside the disk. The parameters of a three-component model of a spiral galaxy (oblate spheroid with central hole, bulge, and massive corona) are chosen in such a way as to obtain in the disk a two-hump rotation curve (as in the Galaxy, M 31, and M 81). It is shown that at heights $|z| \leq 2$ kpc the gas rotates in the same manner as the disk. However, at greater heights the rotation curve ceases to have two humps. Allowance for the pressure gradient of the gas slightly changes the rotation curve directly above the disk ($r < r_{disk}$) and leads to a falling rotation curve beyond the edge of the disk ($r > r_{disk}$).

1. Introduction

The gas in the disk of the Galaxy rotates differentially and has a two-hump rotation curve (see, for example, [1]). The gas outside the disk also participates in the rotation of the Galaxy. Observations of the H I emission lines at 21 cm at intermediate latitudes ($6° \leq b \leq 20°$) [2] show that up to heights $|z| \simeq 1-2$ kpc the halo gas rotates in the same way as in the disk. However, the observed motion of the high-latitude ($b > 20°$) molecular clouds can be explained by assuming that the rotation velocity of the halo gas decreases with increasing distance from the plane of the disk [3]. There are also indications of more complicated motions than simple rotation, for example, flows of a "galactic fountain" type, meridional circulation, galactic wind, and accretion from intergalactic space. Nevertheless, differential rotation can be regarded as the predominant mode of motion of the halo gas.

Information about the distribution of the rotation are important for understanding the hydrodynamics of the galactic gas. For example, the presence of a vertical gradient of the angular velocity means that the gas pressure cannot be a function of just the density alone. An inclination between the surfaces of constant pressure and constant

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density leads to a baroclinic instability. A gradient of the angular velocity can also lead to other hydrodynamic instabilities, for example, the shear instability and the Goldreich–Schubert instability [4, 5]. Inhomogeneity of the angular velocity also leads to an enhancement of the magnetic field. Knowing the angular velocity and the gradient of the gas density in the disk and halo, one can find the mean spiral turbulence, which is a characteristic of turbulent gas motion important for generation of a magnetic field [6].

In this paper, we study the distribution of the rotation of the gas above the galactic disk. The motion of the gas is determined by the total gravitational field of the components that form the Galaxy — the bulge, disk with hole, and the massive corona (see, for example, [1]). The self-gravity of the gas in the halo can be ignored. The degree of influence of each component on the motion of the gas depends on the distance of the considered element of gas from the center and from the central plane of the Galaxy. The massive corona has a strong influence on the rotation only at large distances from the galactic center, while the bulge has a strong influence at short distances. At short distances from the central plane of the Galaxy and far from the inner and outer radii of the disk the motion of the gas above the disk is basically determined by the gravitational field of the disk.

For simplicity, we shall assume that the disk has the shape of an ellipsoid of revolution, and that its isodensity surfaces are confocal ellipsoids. We shall assume that the corona is spherically symmetric. In Sec. 2, we find the gravitational potential of the disk. In Sec. 3, we give the distribution of the angular velocity above the disk. For the transition to the real rotation curve, we introduce in the fourth section a central mass and massive corona and take into account the density deficit in the central region of the disk (the hole). The possible part played by a pressure gradient is discussed in the fifth section. Brief conclusions are given at the end.

2. The Gravitational Potential of the Disk

In this section, we obtain the gravitational potential of an ellipsoid of revolution having as isodensity surfaces confocal ellipsoids. The gravitational potential of a body of arbitrary shape is

$$\Phi(\vec{r}) = -G\int\frac{dM}{R} = -G\int\frac{\rho(\vec{r}\,')\,d^3r'}{|\vec{r}-\vec{r}\,'|}, \tag{1}$$

where the integration is over the complete volume of the body. If the body possesses some symmetry (exact or approximate), then the most effective method for finding the potential is an expansion with respect to orthogonal functions. The particular choice of the orthogonal system of functions depends on the symmetry.

We shall assume that the stellar disk of the Galaxy has the form of an oblate ellipsoid of revolution (spheroid). The section of the disk perpendicular to the plane of rotation has the form of an ellipse. It is then natural to make all the calculations in a system of oblate spheroidal coordinates:

$$\begin{cases} x = c\left[(\xi^2+1)(1-\eta^2)\right]^{1/2}\cos\varphi, \\ y = c\left[(\xi^2+1)(1-\eta^2)\right]^{1/2}\sin\varphi, \quad 0\leqslant\xi<\infty, \quad -1\leqslant\eta\leqslant 1, \quad 0\leqslant\varphi\leqslant 2\pi. \\ z = c\xi\eta, \end{cases} \tag{2}$$

The Lamé parameters in this system of coordinates have the form

$$h_\xi = c\left[\frac{\xi^2+\eta^2}{\xi^2+1}\right]^{1/2}, \quad h_\eta = c\left[\frac{\xi^2+\eta^2}{1-\eta^2}\right]^{1/2}, \quad h_\varphi = c\left[(\xi^2+1)(1-\eta^2)\right]^{1/2},$$

and the element of volume can be expressed in terms of the introduced coordinates as follows:

$$d^3r = c^3(\xi^2+\eta^2)\,d\xi\,d\eta\,d\varphi, \tag{3}$$

where $c = \sqrt{a^2 - b^2}$ is the half-distance between the focuses of the spheroid, and a and b are, respectively, the semimajor and semiminor axes of the spheroid. The boundary of the spheroid is determined by the relation $\xi = \xi_0 = b/c$.

In the oblate spheroidal coordinates we can expand R^{-1} in a series in associated Legendre functions [7, 8]:

$$\frac{1}{R} = \frac{1}{c}\sum_{n=0}^{\infty}(2n+1)\sum_{m=0}^{n}\varepsilon_m i^{m+1}\left[\frac{(n-m)!}{(n+m)!}\right]^2 \cos[m(\varphi-\varphi')]\times$$

$$P_n^m(\eta')P_n^m(\eta)\begin{cases}P_n^m(i\xi')Q_n^m(i\xi), & \xi>\xi',\\ P_n^m(i\xi)Q_n^m(i\xi'), & \xi'>\xi,\end{cases} \tag{4}$$

where

$$\begin{cases}\varepsilon_m = 1, \text{ for } m=0,\\ \varepsilon_m = 2, \text{ for } m>0,\end{cases}$$

P_n^m and Q_n^m are associated Legendre functions of the first and second kinds, respectively.

Substituting the expansion (4) for the case $\xi > \xi_0 \geq \xi'$, i.e., outside the disk, in the general formula (1) and taking into account (3), we obtain a representation of the gravitational potential of the ellipsoid in the oblate spheroidal coordinates:

$$\Phi(\xi,\eta,\varphi) = -Gc^2\int\rho(\xi',\eta',\varphi')(\xi'^2+\eta'^2)\sum_{n=0}^{\infty}(2n+1)\sum_{m=0}^{n}\varepsilon_m i^{m+1}\times$$

$$\left[\frac{(n-m)!}{(n+m)!}\right]^2\cos[m(\varphi-\varphi')]\,P_n^m(\eta')P_n^m(i\xi')P_n^m(\eta)Q_n^m(i\xi)\}\,d\xi'd\eta'd\varphi'.$$

The integration over ξ' is from 0 to ξ_0, over η' from -1 to $+1$, and over φ' from 0 to 2π.

We assume that the density does not depend on φ (axial symmetry), and then all integrals with $m > 0$ are zero. The expression for the gravitational potential simplifies,

$$\Phi(\xi,\eta) = -Gc^2\int\rho(\xi',\eta')(\xi'^2+\eta'^2)\left[i\sum_{n=0}^{m}(2n+1)P_n(\eta')P_n(\eta)P_n(i\xi')Q_n(i\xi)\right]d\xi'd\eta'd\varphi'.$$

We consider the terms of this series

$$\Phi(\xi,\eta) = \Phi_0(\xi,\eta)+\Phi_1(\xi,\eta)+\Phi_2(\xi,\eta)+\ldots, \tag{5}$$

where Φ_0, Φ_1, Φ_2, etc., are determined by

$$\begin{cases}\Phi_0(\xi,\eta) = -c^2GI_0\operatorname{arctg}(1/\xi),\\ I_0 = 2\pi\int\limits_0^{\xi_0}\int\limits_{-1}^{1}\rho(\xi',\eta')(\xi'^2+\eta'^2)P_0(\eta')P_0(i\xi')\,d\xi'd\eta';\end{cases} \tag{6}$$

$$\begin{cases}\Phi_1(\xi,\eta) = -3c^2GI_1\eta[\xi\operatorname{arctg}(1/\xi)-1],\\ I_1 = 2\pi i\int\limits_0^{\xi_0}\int\limits_{-1}^{1}\rho(\xi',\eta')(\xi'^2+\eta'^2)P_1(\eta')P_1(i\xi')\,d\xi'd\eta';\end{cases}$$

$$\begin{cases}\Phi_2(\xi,\eta) = \dfrac{5}{4}c^2GI_2(3\eta^2-1)[(1+3\xi^2)\operatorname{arctg}(1/\xi)-3\xi],\\ I_2 = 2\pi\int\limits_0^{\xi_0}\int\limits_{-1}^{1}\rho(\xi',\eta')(\xi'^2+\eta'^2)P_2(\eta')P_2(i\xi')\,d\xi'd\eta'.\end{cases} \tag{7}$$

The constants I_0, I_1, I_2, etc., can be readily determined by specifying the particular form of the density function. For $\rho = \rho(\xi)$, i.e., when the density is distributed over confocal spheroids (and also in the special case ρ = const), all the constants except

I_0 and I_2 are zero. This can be readily seen by representing $\xi^2 + \eta^2$ in the form

$$\xi^2 + \eta^2 \equiv P_0(\eta)\left(\xi^2 + \frac{1}{3}\right) + \frac{2}{3}P_2(\eta)$$

and by using in the integration over η the orthogonality property of the Legendre functions:

$$\int_{-1}^{1} P_n(\eta)\, P_m(\eta)\, d\eta = \delta_{nm}\,\frac{2}{2n+1}.$$

In the well-known (see, for example, [9]) special case ρ = const, the constants I_0 and I_2 have the form

$$I_0 = \frac{4\pi}{3}\rho\xi_0(1+\xi_0^2) = \frac{1}{c^3}\rho V = \frac{M}{c^3}, \tag{8}$$

$$I_2 = -\frac{1}{5}I_0. \tag{9}$$

In (8) $V = (4\pi/3)a^2b$ is the volume of the spheroid, and M is its mass.

After substitution of (6), (7), and (9) in (5) we obtain

$$\Phi(\xi, \eta) = -\frac{GM}{c}\left\{\operatorname{arctg}(1/\xi) + \frac{1}{4}(3\eta^2-1)[(1+3\xi^2)\operatorname{arctg}(1/\xi) - 3\xi]\right\}. \tag{10}$$

The oblate spheroidal coordinates ξ and η are related to the cylindrical coordinates r and z by

$$\xi = \frac{1}{\sqrt{2}\,c}[\varkappa + \rho]^{1/2}, \quad \eta = \frac{1}{\sqrt{2}\,c}[\varkappa - \rho]^{1/2}, \quad \varkappa = [\rho^2 + 4z^2c^2]^{1/2}, \quad \rho = z^2 + r^2 - c^2.$$

After some manipulations the expression (10) can be shown to be identical to the well-known expression for the gravitational potential of a homogeneous spheroid [9]. However, it is found that this expression is unchanged in the more general case $\rho = \rho(\xi)$, since the relation (9) between the constants I_0 and I_2 remains true. Thus, the gravitational potential outside an inhomogeneous ellipsoid of revolution whose isodensity surfaces are confocal ellipsoids is identical to the gravitational potential of a homogeneous ellipsoid of revolution of the same mass. Similarly, the gravitational potential outside a spherically symmetric mass distribution does not depend on the particular distribution of the density.

We note that the gravitational potential of an ellipsoid of revolution for the special case of isodensity surfaces in the form of confocal ellipsoids was obtained in [10], but the identity of the obtained potential and the potential of a homogeneous ellipsoid was not pointed out.

Using the connection between ξ and η (see (2)), we can rewrite (10) in the form

$$\Phi(r, z) = -\frac{3}{4}\frac{GM}{c^3}\left\{[2(c^2+z^2) - r^2]\operatorname{arctg}(1/\xi) - \frac{3z^2}{\xi} + c^2\xi\right\}.$$

From this expression we can readily find the force function $K_r = -\partial\Phi/\partial r$.

3. Distribution of the Angular Velocity

Ignoring the self-gravity of the gas outside the disk, we can assume that the motion of the gas is determined by the gravitational field of the disk. In this and the following section we shall assume that the radial component of the gravitational force of the disk is balanced by the centrifugal force.

$$\Omega^2 r = K_r, \tag{11}$$

where Ω is the angular velocity of rotation of the gas (the possible role of a radial pressure gradient will be discussed in Sec. 5). From (11) we find the angular velocity and the linear velocity corresponding to it:

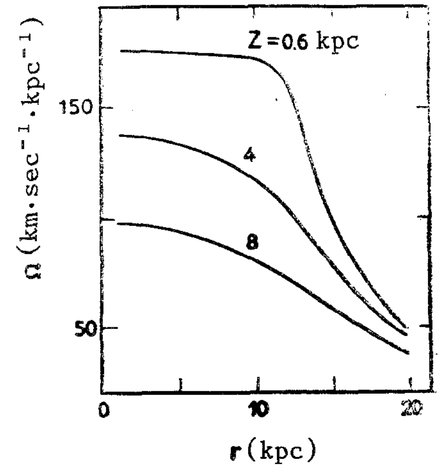

Fig. 1. Radial dependence of the angular velocity for different z.

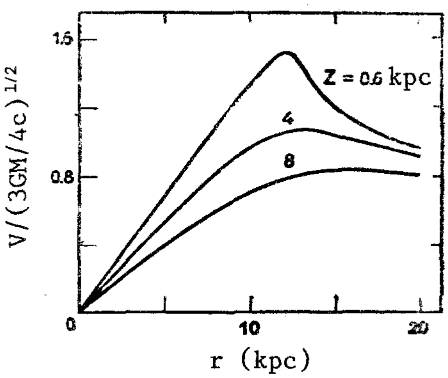

Fig. 2. Radial dependence of the linear velocity for different z.

$$\Omega = \left[\frac{1}{r} K_r\right]^{1/2}, \quad v = [rK_r]^{1/2}.$$

Figures 1 and 2 give the angular and linear velocities as functions of the radius for different z. To be specific, we have taken a disk with major and minor axes a = 12.5 kpc and b = 0.5 kpc. It can be seen from Fig. 2 that the maximum of the linear velocity near the plane of the disk is at the edge of the disk, and is shifted to larger r with increasing distance from the disk. The fact that the maximum of the linear velocity is at the edge of the disk is due to the fact that the density in the disk is distributed with respect to the confocal spheroids, i.e., is a function of only ξ. If a dependence of the density on the coordinate η is taken into account, the maximum can be shifted closer to the rotation axis. In the Galaxy it is at about r ≈ 9 kpc. In this paper we do not aim to reproduce individual properties of particular systems, for example, the Galaxy. Our main aim is a qualitative investigation of the rotation of the halo gas.

Figure 3 shows the angular velocity as a function of the distance to the plane of the disk at different r.

4. A More Realistic Model

Real spiral galaxies have other components besides a disk. We limit ourselves to a three-component model of spirals and assume that they consist of a bulge, a disk, and a massive corona (see, for example, [1]).

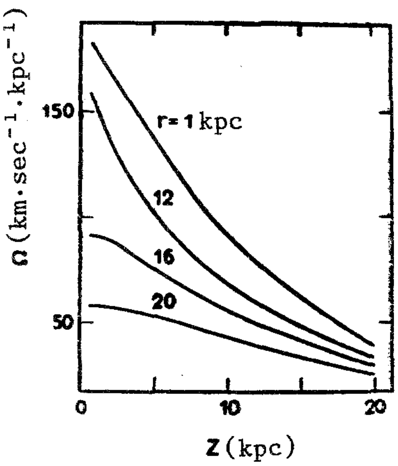

Fig. 3. Dependence of angular velocity on the distance to the plane of the disk at different r.

TABLE 1. Parameters of Bulge, Disk, and Hole.

	Major axis, a (kpc)	Minor axis, b (kpc)	Mass, M ($10^{10} \cdot \odot$)
Bulge	1	0.6	1.2
Disk	12.5	0.5	7.0
Hole	3	0.5	−0.4

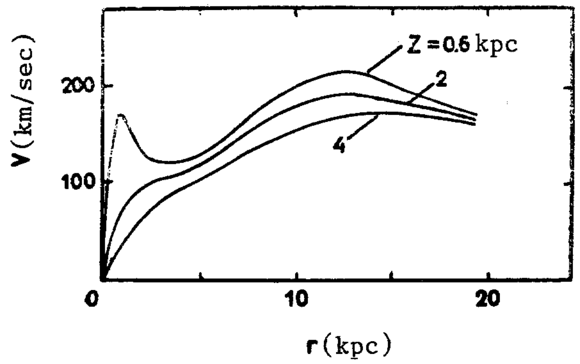

Fig. 4. Rotation curve at different distances from the plane of the Galaxy.

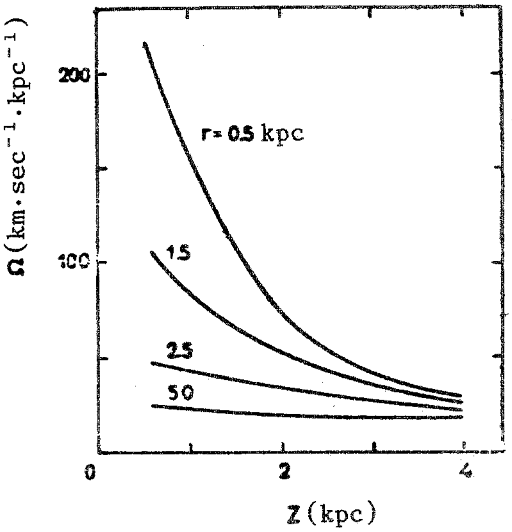

Fig. 5. Dependence of the angular velocity on the distance to the plane of the Galaxy at different r.

It is known from observations that the rotation curves of the Galaxy M 31, and M 81 have deep minima at distances $r \simeq 1-4$ kpc from the center. The observed minimum can be explained if the stellar disk has a density deficit (a hole) in the central region [11]. To be specific, we consider the Galaxy. The hole in the disk of the Galaxy has a radius $r \sim 3$ kpc. For simplicity, we shall assume that the hole and the bulge, like the disk, have a spheroidal shape. The parameters of the bulge, disk, and hole are given in Table 1.

We shall assume that the massive corona is spherically symmetric. The density in it is distributed in accordance with the law [12]

$$\rho(R) = \frac{\rho_c}{1 + (R/R_c)^2}, \tag{12}$$

where $R^2 = r^2 + z^2$, $\rho_c = 2.15 \cdot 10^{-2}\ M_\odot\ \mathrm{pc}^{-3}$, $R_c = 15.4$ kpc. The gravitational potential within the spherically symmetric mass distribution (12) is determined by the expression

$$\Phi_{\text{corona}}(R) = -4\pi G \rho_c R_c^2 \left(1 - \frac{R_c}{R} \operatorname{arctg} \frac{R}{R_c} - \frac{1}{2} \ln \frac{R^2 + R_c^2}{R_0^2 + R_c^2}\right),$$

where R_0 is the radius of the region occupied by the mass.

From the linearity of Poisson's equation in Φ and ρ it follows that the gravitational potentials of the components that form the Galaxy are added. Allowance for the hole is equivalent to introducing a component with negative mass. The total linear rotation velocity of the halo gas is

$$v^2 = v^2_{\text{bulge}} + v^2_{\text{disk}} - v^2_{\text{hole}} + v^2_{\text{corona}}, \tag{13}$$

where v_{bulge}, v_{disk}, v_{hole}, and v_{corona} are the linear rotation velocities of the corresponding components. Figure 4 gives the rotation curves for different z. We have chosen the masses of the components to make the rotation curve above the disk at height $|z| = 0.6$ kpc similar to the rotation curve in the disk (see, however, Sec. 3). The rotation curve retains a two-hump shape to heights $|z| < 2$ kpc. At greater heights, the rotation curve has only one hump. This property is a general property of rotation above disk systems with holes. Figure 5 gives the angular velocity as a function of the distance to the galactic plane for different r.

5. Allowance for Pressure

When the rotation curves of spiral galaxies in the plane of their rotation ($z = 0$) are being constructed, the influence of a pressure gradient is ignored, and it is assumed that the radial component of the gravitational force is equalized by the centrifugal

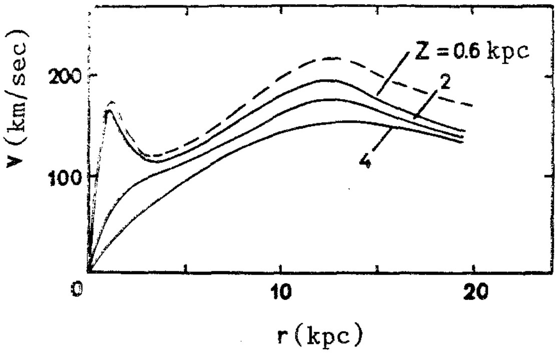

Fig. 6. Modified rotation curve at different distances from the plane of the galaxy. For comparison, the broken curve is the rotation curve ($|z|$ = 0.6 kpc) obtained without allowance for the pressure gradient.

force (see, for example, [1]). This can be done because the rms velocity of the gas (v_t ~ 10 km/sec) is small compared with the rotation velocity (v ~ 200 km/sec). We compare the two terms $\Omega^2 r$ and $(1/\rho)\partial P/\partial r$. Assuming that $P \sim \rho v_t^2$ and replacing $\partial P/\partial r$ by P/r, we obtain the estimate

$$\frac{1}{\rho}\frac{\partial P}{\partial r} \sim \frac{v_t^2}{r} \ll \frac{v^2}{r} = \Omega^2 r.$$

In the halo the situation is different. The temperature of the halo gas is about two orders of magnitude higher than in the disk, $T_{halo} \simeq 5 \cdot 10^5$°K. Therefore, the rms velocity of the gas in the halo is also higher ($v_t \simeq$ 100 km/sec). In this case, the terms $\Omega^2 r$ and $(1/\rho)\partial P/\partial r$ are comparable.

Thus, in the halo the equilibrium of the gas in the radial direction is described by the equation

$$\Omega^2 r = K_r + \frac{1}{\rho}\frac{\partial P}{\partial r}, \tag{14}$$

which differs from (11) by the edition of the pressure gradient. We shall assume that the pressure of the gas in the halo has a thermal nature, and that the rms turbulent velocity of the gas is equal to the thermal velocity:

$$P \simeq \frac{1}{3}\rho v_t^2; \quad v_t = \sqrt{\frac{3kT}{m}}.$$

For an isothermal gas

$$\frac{\partial P}{\partial r} = \frac{1}{3} v_t^2 \frac{\partial \rho}{\partial r}. \tag{15}$$

Note that in Eq. (15) the pressure is a function of the density alone, but this, strictly speaking, is possible only if the angular velocity does not depend on z. However, in a first approximation, until we consider the baroclinic instability in the obtained velocity field, the gas can be assumed to be barotropic.

We consider two model distributions of the gas density in the halo:

a) The density depends only on z. Then the radial pressure gradient is zero and the pressure does not change the rotation curve.

b) The density of the gas in the halo decreases exponentially with increasing r, $\rho(r) \sim \exp(-r/r_0)$. From (15) we obtain

$$\frac{\partial P}{\partial r} = -\frac{\rho}{3r_0} v_t^2.$$

Then from Eq. (14) we obtain for the linear velocity the expression

$$v = \left(\tilde{v}^2 - \frac{r}{3r_0} v_t^2 \right)^{1/2},$$

where $v = \Omega r$, and $\tilde{v}^2$ is determined by Eq. (13). The modified rotation curves for different z are given in Fig. 6, where we have set $r_0 = 10$ kpc.

Thus, the pressure gradient has the consequence that the rotation curve of the halo may decrease at large radii even in the presence of a massive corona, when the rotation curve of the disk is flat.

6. Conclusions

The gravitational potential outside an inhomogeneous ellipsoid of revolution whose isodensity surfaces are confocal ellipsoids is identical to the gravitational potential of a homogeneous ellipsoid of revolution of the same mass.

The halo gas rotates differentially. Near the disk ($|z| \leq 2$ kpc) it preserves a two-hump rotation curve like the rotation curve in the plane of the Galaxy (z = 0). With increasing distance from the plane of the disk the rotation curve becomes a single-hump curve, i.e., the influence of the hole becomes unimportant, and the maximum of the rotation curve is shifted to larger r.

Allowance for the pressure gradient slightly changes the rotation curve above the disk ($r < r_{disk}$) and leads to a falling rotation curve beyond the edge of the disk ($r > r_{disk}$), even in the presence of a massive corona.

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