The determination of the snowmelt rate and the meltwater outflow from a snowpack for modelling river runoff generation

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Abstract

Two procedures to estimate the area-averaged snowmelt and meltwater outflow from a snowpack were compared for a river basin in the south part of European Russia. Both methods are based on the same model of melting snow but use two different methods for computing the snowmelt rate at the surface of the snowpack, the degree-day method and the Kuzmin method. For averaging, the spatial change in the meteorological inputs and the statistical distribution of the premelting snow water equivalent before melt are taken into account. The calculated basin-averaged meltwater outflow is checked against the snowmelt input obtained from the measured runoff hydrograph by solving the inverse problem for a runoff generation model. It also gives opportunities to calibrate the basin-averaged degree-day factor, coefficients of the heat component dependences, or dependences for parameters of spatial statistical distribution of snow characteristics. The procedure based on the Kuzmin method gave better results than that based on the degree-day method, both in the case of a priori assigned parameters and in the case of parameter calibration.

1. Introduction

Most efforts in the development of models of snowmelt runoff processes have been devoted to the description of snow cover formation and of snowmelt at a point or over uniform areas (Anderson, 1976; Morris and Godfrey, 1978; Male and Gray, 1981; Kuusisto, 1984; Motovilov, 1986), perhaps reflecting the availability of experimental measurement and the relative simplicity of point processes. However, to estimate the snowmelt as an input to a runoff generation model for a real river

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basin requires knowledge of the spatial change in the heat fluxes as well as of the snow characteristics. Variations in snow depth, density, albedo, liquid water retention caused by spatial heterogeneities of meteorological processes, terrain types and land use as well as lack of the necessary observations can influence the advantages and disadvantages of models of snow cover formation and of melt rate at the river basin scale. Therefore, in estimating melt rate and the meltwater outflow from a snowpack as a input to runoff models, it is necessary to take into consideration both the accuracy of estimations of these values at a point and the possible effects of spatial averaging over various spatial scales. To improve the representation of the spatially averaged values of the snowmelt and the snowpack outflow, empirical parameters, used in the point model or for spatial averaging, need to be calibrated.

One way of estimating the accuracy of computing the area-averaged meltwater outflow from meteorological data and determining the empirical parameters is a comparison of the calculated outflow with the snowmelt input obtained from measured runoff hydrographs by solving the inverse problem for a runoff generation model.

The objective of this paper is to study the use of this approach for the choice of the most reliable snowmelt model and the comparison of the basin-averaged snowmelt rate and the melt water outflow from the snowpack based on the heat budget and degree-day methods in a case study of the River Sosna basin.

The River Sosna (the catchment area is 16 300 km^2) is one of the major tributaries of the River Don and is located in the forest–steppe zone of European Russia. The basin terrain is a smooth plain. The elevation difference between highest and lowest points of the basin is about 50 m. The dominant soils are chernozems (black soils). About 80% of the basin area is ploughed. Ravines and rills occupy 8% of the basin area and forest about 2%. The groundwater level is at a depth of 10–30 m. The climate is temperate: the mean annual air temperature is 4.5°C, the mean annual precipitation sum is 463 mm. The measured values of maximum snow water equivalent vary significantly from year to year (the maximum observed value is 180 mm and the minimum is 17 mm). The mean maximum snow water equivalent changes from the northern part of the basin to the southern part in the range of 130–80 mm. The snowmelt runoff coefficients are in the range of 0.22–0.90.

The observations used are the standard 6 h meteorological measurements and measurements of snowpack at four meteorological stations (Verchovie, Livny, Jeletz and Schigry), as well as the runoff hydrographs at the outlet of the basin during the spring seasons of 1967–1973 and 1976.

2. Computing the snowmelt and meltwater outflow at a point

If the temperature of the snowpack is 0°C, the melting rate S can be found from the heat budget equation as

$$S = (Q_{sw} + Q_{lw} - Q_{ls} + Q_T + Q_E + Q_P + Q_G)(\rho_i L)^{-1} \quad (1)$$

where Q_{sw} is the net shortwave radiation, Q_{lw} is the incoming longwave radiation, Q_{ls}

is the outgoing longwave radiation, Q_T is the sensible heat exchange, Q_E is the latent heat exchange, Q_P is the heat content of liquid precipitation, Q_G is the heat exchange at ground surface, L is the heat of ice fusion, and ρ_i is the ice density.

The use of Eq. (1) requires measurements of radiation and of heat exchange components which commonly are not available. As an alternative, the components of the energy budget can be determined from empirical relationships with the dominant meteorological characteristics (air temperature and humidity, wind velocity, cloudness) (e.g. US Army, Corps of Engineers, 1956; Kuzmin, 1972). The accuracy of such relationships is affected significantly by many physiographic and climatic factors.

The most widely applied procedure for estimation of the rate snowmelt is the degree-day method, based on the assumption that the daily mean air temperature is a integrated index of the heat budget and that the rate of snowmelt is proportional to the daily mean air temperature T_a:

$$S = m_m T_a \tag{2}$$

where m_m is the degree-day factor.

Detailed investigations of the degree-day method, carried out for different climatic and geographic zones (Komarov et al., 1969; Bergstrom, 1976; Kuusisto, 1984), have illustrated the successful application of this method in many cases. However, the degree-day factor varies strongly as a function of physiographic conditions, meteorological situations and snow properties. For the river basins of Central Russia, the degree-day factor commonly varies from 4.6×10^{-8} to 6.9×10^{-8} m °C^{-1} s^{-1} and sometimes exceeds 1.2×10^{-7} m °C^{-1} s^{-1} (Appolov et al., 1974).

Using the meteorological measurements for the springs of 1967–1973 and 1976 at each of four meteorological stations in the River Sosna basin, the snowmelt and the snowpack outflow at a point have been calculated applying the degree-day method and the Kuzmin modification of the heat balance method. The degree-day factor has been taken as equal to 6.3×10^{-8} m °C^{-1} s^{-1} (the value obtained for the Sosna River region by Deleur (1974) as a result of a detailed analysis snow measurement at a water balance station).

For estimation of the heat budget components Q_{sw}, Q_{lw}, Q_T, Q_E (in W m^{-2}), the following relationships adopted by Kuzmin (1972) were applied:

$$Q_{sw} = 17.46 h_0 (1.0 - r)(1.0 - 0.2N - 0.47N_0) \tag{3}$$

where h_0 is the angle of the shortwave radiation with the horizontal in degrees, calculated as a function of local latitude, declination and sun's hour angle, r is the snow albedo; N and N_0 are the total and the lower cloudness (in fractions of unit), respectively;

$$Q_{lw} = \epsilon\sigma(T_a + 273)^4[0.61 + 0.05(e_a)^{1/2}](1.0 + 0.12N + 0.12N_0) \tag{4}$$

where e_a is the vapour pressure (in mbar), ϵ is the effective emissivity of the atmosphere ($\epsilon = 0.99$), σ is the Stefan–Boltzmann constant;

$$Q_T = 18.85(T_a - T_s)(0.18 + 0.098u) \tag{5}$$

where T_s is the temperature of the snow surface (in °C) (T_s is assumed to be equal to

T_a if $T_a < 0°C$ and 0°C if $T_a \geqslant 0°C$), u is the wind velocity (in m s^{-1});

$$Q_E = 32.82(e_s - e_a)(0.18 + 0.098u) \tag{6}$$

where e_s is the vapour pressure over the ice (in mbar).

The outgoing longwave radiation is given by

$$Q_{lr} = \epsilon\sigma(T_s + 273)^4 \tag{7}$$

The heat content of liquid precipitation Q_P is

$$Q_P = \rho_w C_w T_a R_l \tag{8}$$

where ρ_w is the density of water, C_w is the heat capacity of water and R_l is the rainfall rate.

To calculate the snow albedo, the empirical relation derived from experimental data of Kuchment et al. (1983) is used:

$$r = 1.03 - \rho_s \tag{9}$$

where ρ_s is the density of snow (in g cm^{-3}). The heat exchange Q_G at the boundary of the melting snow cover and the ground is assumed to be equal to zero.

To calculate the change in the characteristics of the snow cover during the melting season and to estimate the meltwater outflow from the snowpack at a point, the system of vertically averaged equations of the snow cover formation described by (Motovilov, 1993) has been applied. This system is written as follows:

$$\begin{aligned} \frac{dH}{dt} &= \rho_w[R_s\rho_0^{-1} - (S + E)(\rho_i I)^{-1}] - V \\ \frac{d}{dt}(\rho_i I H) &= \rho_w(R_s - S - E) + S_i \\ \frac{d}{dt}(\rho_w \theta H) &= \rho_w(R_l + S - R_w) - S_i \end{aligned} \tag{10}$$

where H is the snow depth; I, θ are the volumetric content of ice and liquid water, respectively; R_s is the snowfall rate (it is assumed that if $T_a \geqslant 0°C$ only rainfall occurs and if $T_a < 0°C$ only snowfall occurs); ρ_0 is the density of new snow (140 kg m^{-3}); ρ_i is the density of ice; E is the rate of sublimation; S_i is the rate of refreezing melt water in snow; R_w is the melt water outflow; V is the compression rate, which is found as

$$V = 0.5K_v\rho_s \exp(0.08T_a - \beta\rho_s)H^2 \tag{11}$$

where $K_v = 2.7 \times 10^{-7}$ m^2 s^{-1} kg^{-1}, $\beta = 2.1 \times 10^{-4}$ m^3 kg^{-1}.

The rate of sublimation is calculated from

$$E = Q_E(\rho_i I L)^{-1} \tag{12}$$

The meltwater outflow is determined as

$$R_w = \begin{cases} R_0 + S, & \theta = \theta_{max} \\ 0, & \theta < \theta_{max} \end{cases} \tag{13}$$

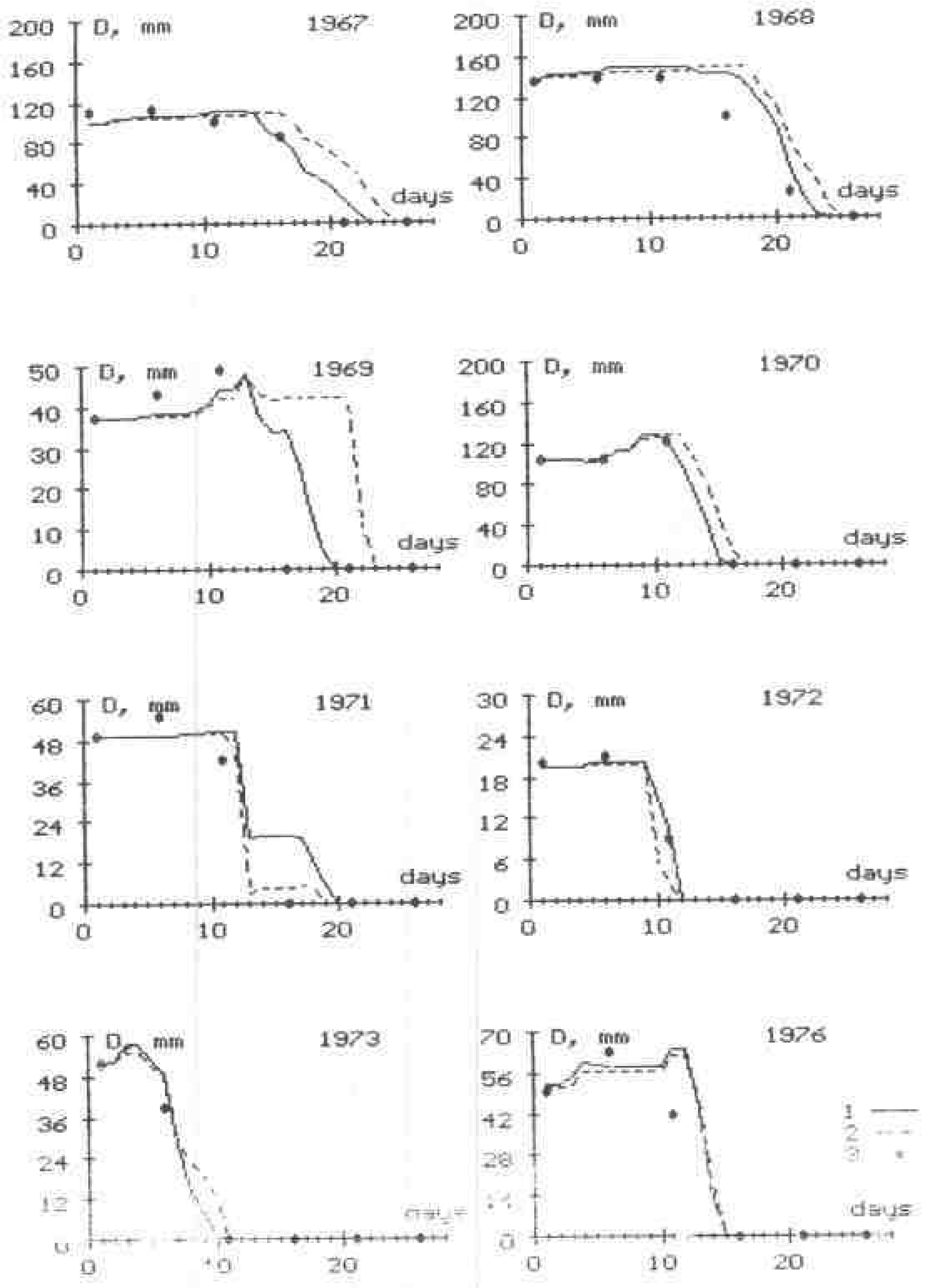

Fig. 1. Comparison of the calculated snow water equivalent (*D*, mm) for the meteorological station Livny (1, based on the Kuzmin method; 2, based on the degree-day method) with the observed one (3).

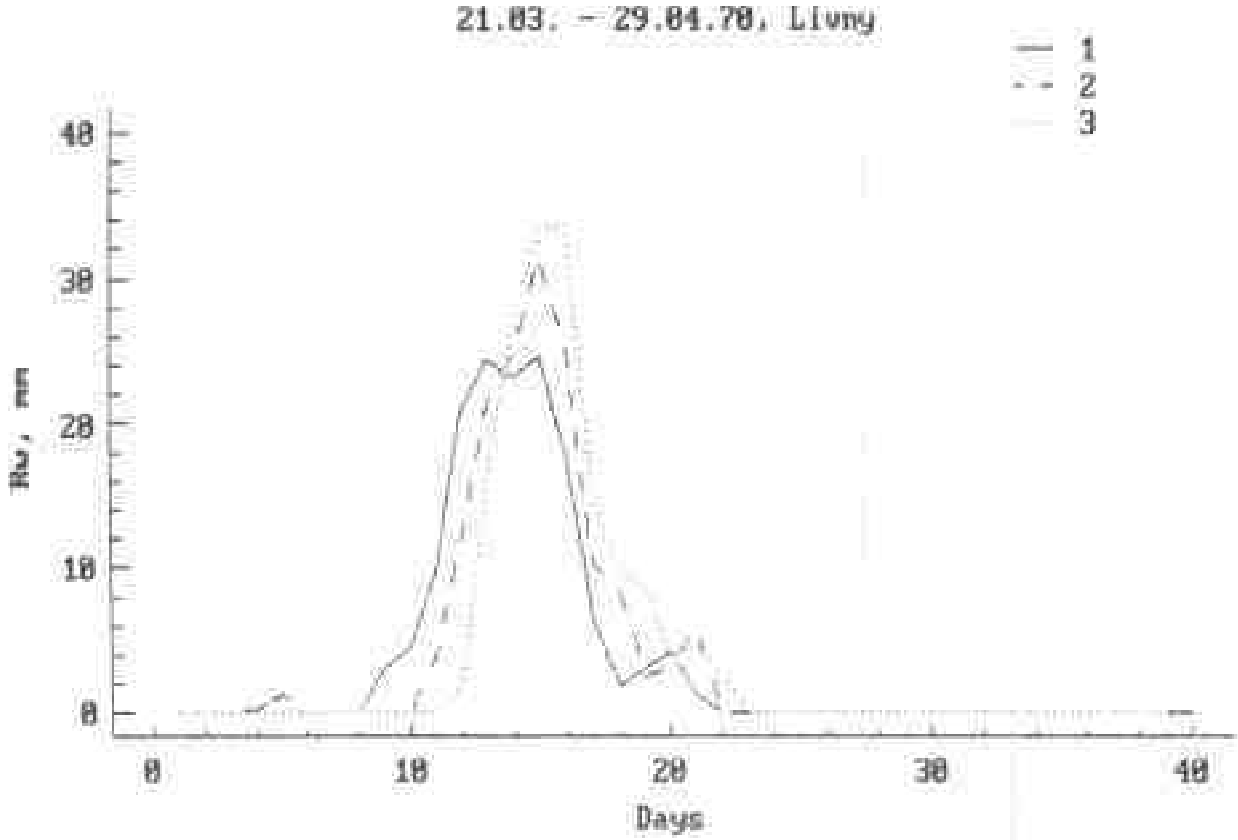

Fig. 2. Comparison of the calculated snowpack outflow based on: 1, the Kuzmin method with 6 h input data; 2, the Kuzmin method with daily input data; 3, the degree-day method.

where $R_0 = R_l + S - E - \theta_{max}(dH/dt)$, and θ_{max} is the maximum liquid water holding capacity of snow related to ρ_s as

$$\theta_{max} = 0.11 - 0.11\frac{\rho_w}{\rho_s}$$

It is assumed that the rate of refreezing $S_i = K_i|T_a|^{0.5}$ ($K_i = 5.8 \times 10^{-8}$ m s^{-1} C$^{-0.5}$). The numerical integration of the system of Eqs. (10) has been carried out using an explicit finite-difference scheme with $\Delta t = 6$ h.

The results of the computations of the snowmelt rate, the snow water equivalent and the outflow from the snowpack for all four meteorological stations are similar. In general, both the degree-day method and the Kuzmin method agreed well with observations when melting is intensive. However, under some hydrometeorological conditions, the differences between these two methods are substantial.

As an example, Fig. 1 shows the comparison of the calculated snow water equivalent with the observations for the meteorological station Livny situated in the centre of the River Sosna basin; the two methods gave similar results for the spring periods of 1972, 1973 and 1976, when melting began at high air temperatures. However, there are notable differences between the snow water equivalent calculated by these methods for the springs of 1967–1970 when the snowmelt occurred slowly and over a long time. For these four springs, the Kuzmin method results are closer to the measurements than those of the degree-day method. The largest errors of the degree-day method were for the spring period of 1969, when melting began at negative air temperatures.

One cause of the difference in the results of the degree-day and the Kuzmin method may be the different time intervals over which the meteorological data are averaged (daily average air temperature for the degree-day method and 6 h average for the

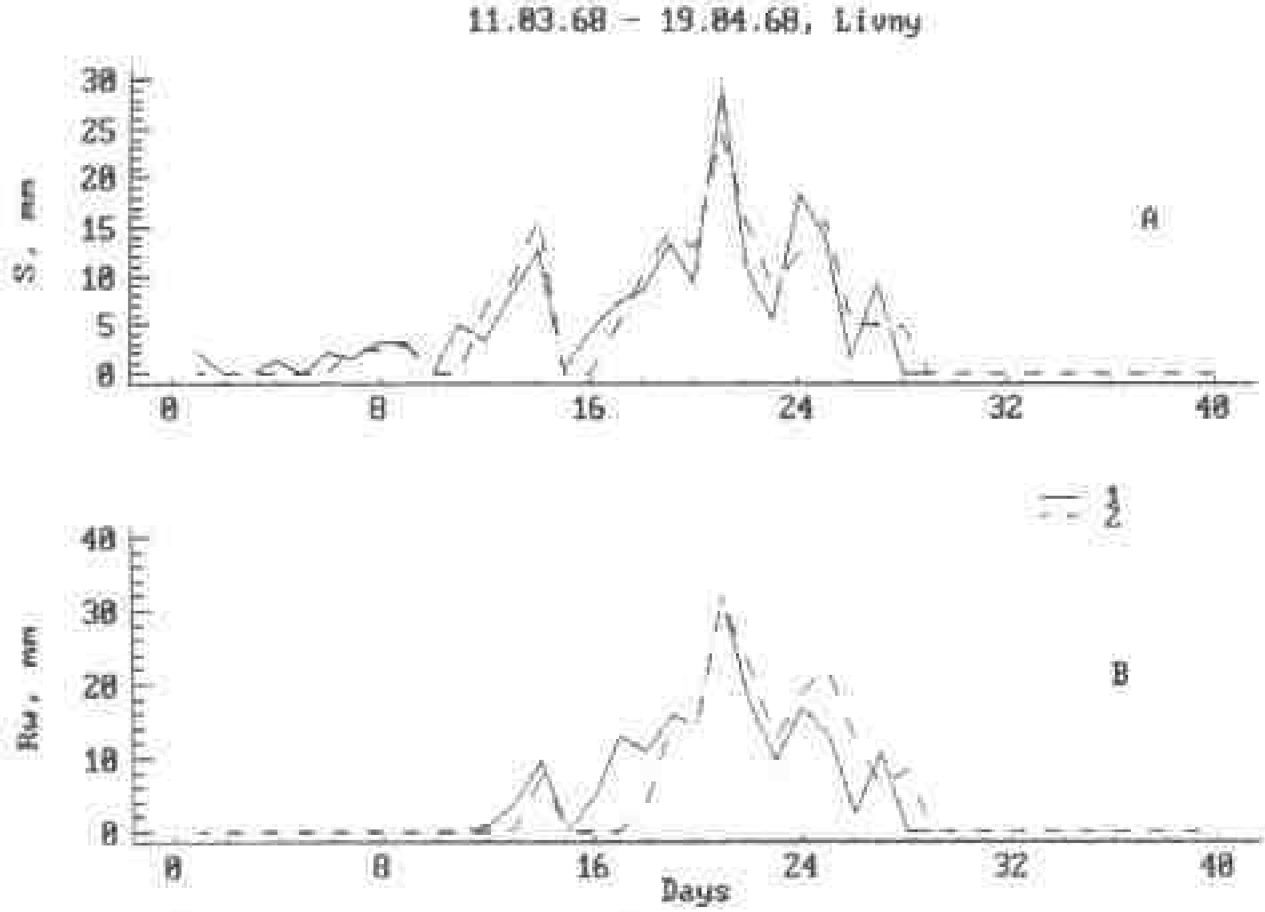

Fig. 3. Comparison of the calculated melt rate (A) and snowpack outflow (B) based on: 1, the Kuzmin method; 2, the degree-day method.

Kuzmin method). Fig. 2 presents the calculated outflow from the snowpack based on the Kuzmin method using 6 h and daily meteorological data in comparison with results of applying the degree-day method for 1970; using the same time interval gives closer agreement.

In spite of retention of the meltwater in the snowpack, the differences in the results given by the degree-day and the Kuzmin method for calculating the snowpack outflow are approximately the same as for the melting rate. For example, Fig. 3 presents the temporal changes of outflow and melting rates computed by both methods for the spring of 1973.

3. Computing the basin-averaged meltwater outflow by the meteorological data

To take into account the large-scale spatial variability of the meteorological inputs and the snowpack characteristics, the River Sosna basin was divided into approximately equal subareas corresponding to each of the four meteorological stations. To account for the stochastic variations of the snow water equivalent within each subarea, it has been assumed that these variations can be described by the same statistical distribution as for a whole basin. To choose this type, the detailed snow water equivalent measurements in the River Sosna basin during the spring of 1970 (102 point measurements) were used. The histogram of the areal distribution of snow water equivalent obtained from these data and fitted by gamma- and log-normal distributions is shown in Fig. 4. The χ^2-test gives preference to the log-normal distribution. The mean value of this distribution for each subarea has been equated to

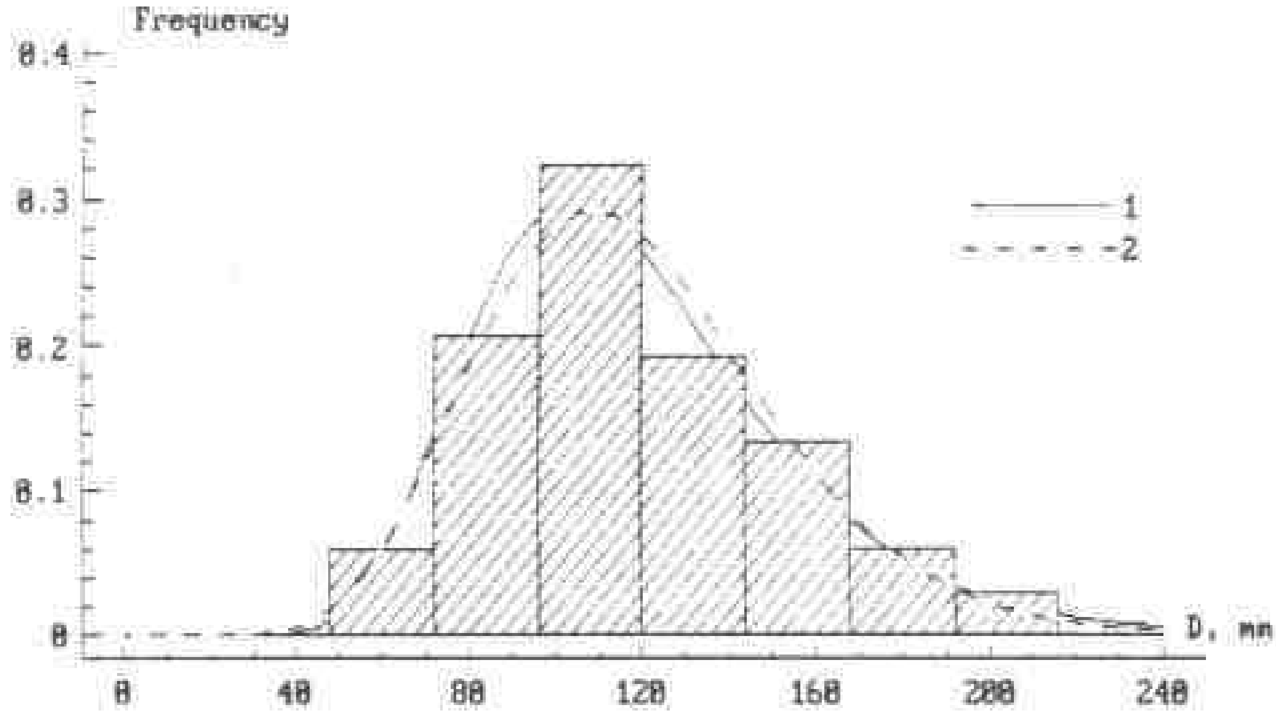

Fig. 4. Histogram of measured snow water equivalents fitted by a log-normal (1) and a gamma (2) distribution.

the snow water equivalent measured at the corresponding meteorological station. The coefficient of variation C_v calculated by the empirical formula (Kuchment et. al, 1986)

$$C_v = \begin{cases} 0.32 - 0.38D & \text{for } D > 0.08 \text{ m} \\ [(0.025 + 0.23D)(D)^{-1}]^2 & \text{for } D \leqslant 0.08 \text{ m} \end{cases} \tag{14}$$

where D is the mean value of the snow water equivalent for a given subarea.

Calculations of the outflow from the snowpack have been carried out for each subarea of the basin for snow water equivalents exceeding 5%, 22.5%, 50%, 77.5% and 95% of the area. It has been assumed that the initial value of the snow density for each subarea is equal to the area-averaged measured value before melting. Then, the weighted average values of the outflow from the snowpack have been computed for each subarea and finally for the entire basin.

4. The inverse method for determining the basin-averaged snowmelt excess

The snowmelt in the River Sosna basin usually occurs over the entire catchment simultaneously. The runoff losses depend mainly on the soil moisture content and the depth of frozen soil before the snowmelt. The runoff coefficient commonly is constant during the melting season (except for special cases when the soil is dry and very cold before the snowelt). It allows the use of a simple linear runoff model for determining the snowmelt input.

The runoff hydrograph $Q(t)$ can be calculated with the help of the Duhamel integral as

$$Q(t) = K\int_0^t P(t-\tau)R(\tau)\mathrm{d}\tau \tag{15}$$

where $P(t)$ is the response function, R is the basin-averaged outflow from the snowpack and K is the coefficient of snowmelt runoff.

The solution of (15) for the snowmelt excess $q(t) = KR$ (if the $Q(t)$ and $P(t)$ are given) or for the response function $P(t)$ (if the $q(t)$ and $Q(t)$ are given) leads to a improperly posed problem (the small errors in the initial data may result in an infinite increase in computation errors). The method of solution of (15) based on the theory of improperly posed problems and on use of a priori information on the properties of the unknown solution was suggested by Kuchment (1967). The essence of this method is as follows.

The integral in (15) in the discrete form at $\Delta t = 1$ can be expressed as

$$Q(i) = \sum_{j=1}^{i} P(i-j+1)q(j) \tag{16}$$

or, in matrix form,

$$\boldsymbol{Q} = A\boldsymbol{q} \tag{17}$$

(the symbols of (17) are clear from (16)).

To obtain the best smoothed approximation of $q(i)$, instead of (16) the following system is solved:

$$A^T\boldsymbol{Q} = A^T A\boldsymbol{q} + \lambda\boldsymbol{q} \tag{18}$$

where A^T is the transpose of A and the value λ is selected from the additional condition

$$\sum_{i=1}^{\infty} Q(i) = \sum_{i=1}^{\infty} q(i) \tag{19}$$

The same procedure with the additional condition $\sum_{i=1}^{\infty} P(i) = 1$ instead of (19) can be applied for determining $P(t)$ from (15).

To estimate the response function $P(t)$ with the help of this approach, the rainfall and runoff measurements for three large rainfall floods have been used: 20–30 July 1978, 26–31 July 1980 and 16–23 August 1980. Values of rainfall excess have been obtained by multiplication of the rainfall rate by the ratio of the measured volume of runoff to the measured amount of corresponding rainfall.

Using the response function $P(t)$ obtained from summer floods and the observed snowmelt runoff hydrographs of the Sosna River for the spring of 1967–1973 and 1976, values of the basin-averaged snowmelt excess were estimated. These values were then divided by the runoff coefficients of the corresponding snowmelt floods to obtain the basin-averaged outflow meltwater from the snowpack.

5. Comparison of the basin-averaged snowpack outflow determined by direct and inverse methods

The example of the comparison of the basin-averaged snowpack outflow calculated

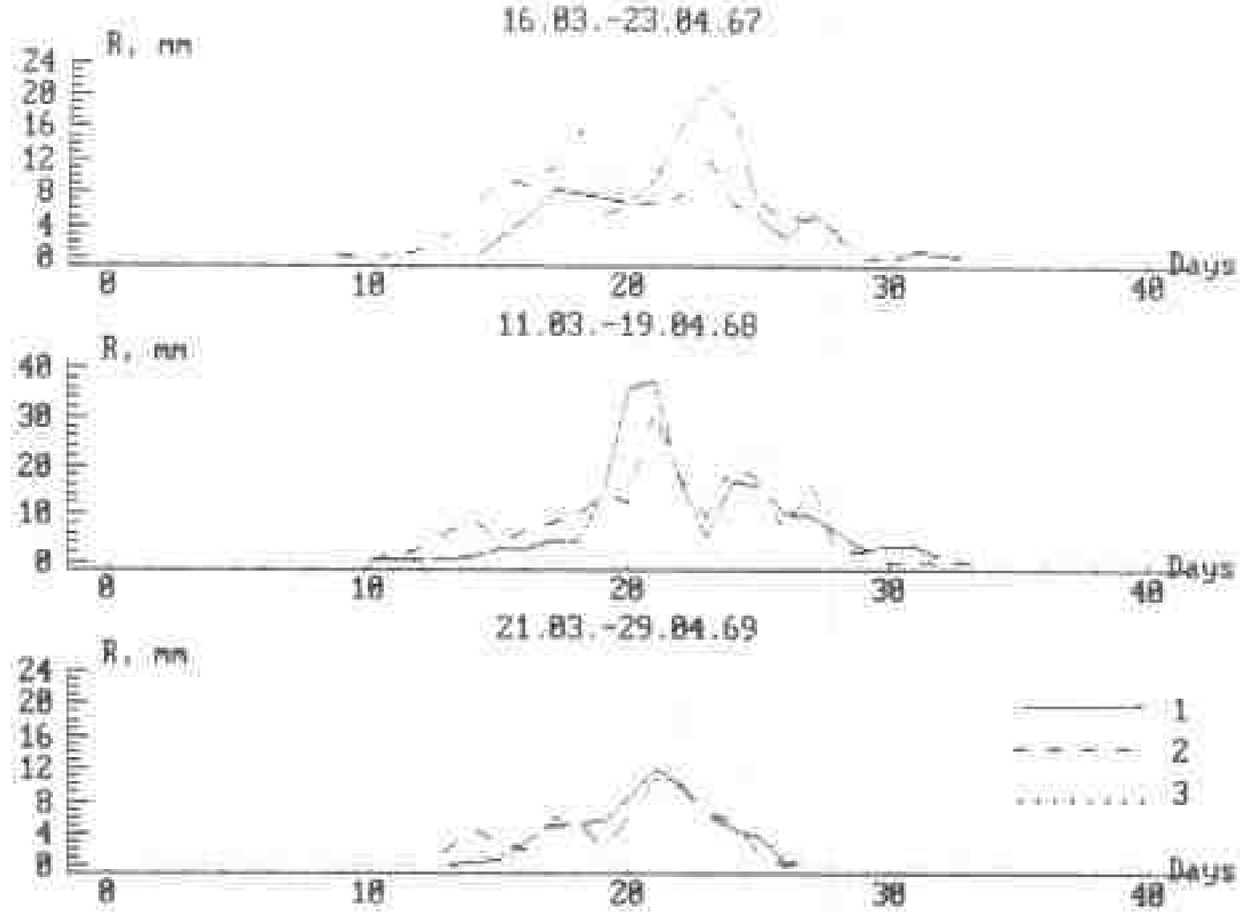

Fig. 5. Comparison of the calculated basin-averaged snowpack outflow based on: 1, the inverse problem solution; 2, the Kuzmin method; 3, the degree-day method.

from meteorological data and that obtained by the solution of the inverse problem on the basis of runoff hydrographs for 1967–1969 is presented in Fig. 5. The comparison of calculated peaks of meltwater outflow from the snowpack for all springs is presented in Table 1.

It is seen from Table 1 that the basin-averaged outflow from snowpack calculated on the basis of the Kuzmin method and the degree-day method, as well as the outflow calculated by the inverse method, have rather similar timing for all springs, except for the spring of 1969 (see Fig. 5), but the differences between the values of calculated

Table 1
Comparison of the calculated basin-averaged peaks of meltwater outflow from the snowpack

Year	Value of outflow peak (mm day^{-1})			Date of outflow peak		
	Inverse problem solution	Kuzmin method	Degree-day method	Inverse problem solution	Kuzmin method	Degree-day method
1967	21.1	10.2	10.7	08.04	08.04	08.04
1968	36.4	30.8	27.9	01.04	01.04	01.04
1969	12.0	11.8	21.1	10.04	10.04	12.04
1970	34.6	22.5	24.6	04.04	04.04	05.04
1971	19.6	21.3	23.9	24.03	24.03	24.03
1972	11.5	8.0	7.2	18.03	18.03	18.03
1973	9.0	12.2	10.3	30.03	31.03	31.03
1976	24.8	22.3	25.4	05.04	05.04	06.04

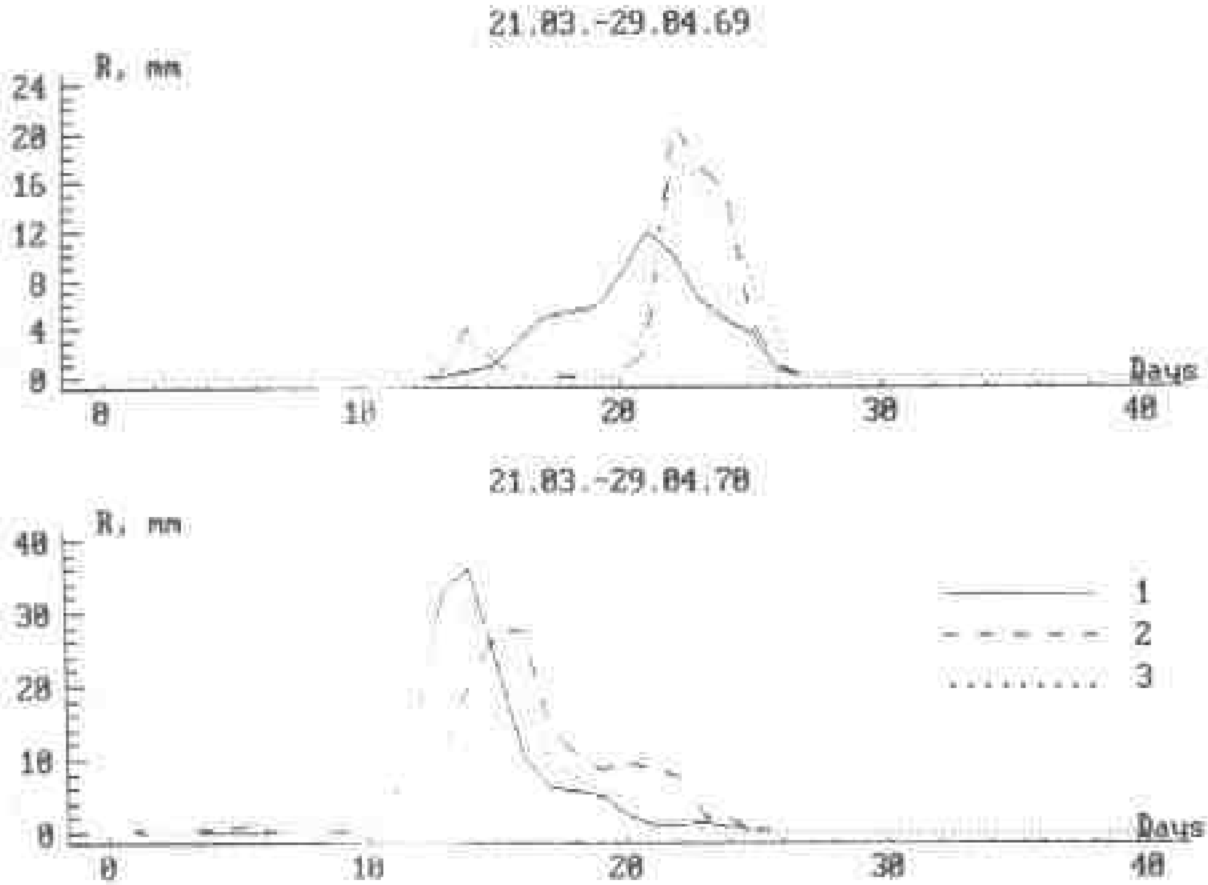

Fig. 6. Comparison of the calculated basin-averaged snowpack outflow based on: 1, the inverse problem solution; 2, the degree-day method without calibration; 3, the degree-day method with calibration.

peaks of outflow are large. In most cases, the outflow calculated by the Kuzmin method is closer to the outflow obtained by the inverse method than that calculated by the degree-day method. The largest deviations of the results of the Kuzmin method from the inverse problem solutions occurred in the springs of 1967 (see Fig. 5) and 1972. Such deviations can be explained, at least partially, by the use of too crude a model of runoff generation. For example, it is possible to assume that the runoff coefficient was increasing during the melt period of 1967 and 1972: after a cold winter, the first portion of meltwater can infiltrate into the frozen soil with overcooled soil moisture, and an impermeable layer can form at the soil surface or at some depth (such an event usually occurs in the river basins of the forested–steppe zone of Central Russia at a 4–5 year period).

The snowpack outflow data obtained by the inverse method have been used to calibrate the degree-day model and to determine some coefficients in the heat component relationships of the Kuzmin method. To account for the influence of the snow density on the degree-day factor, the latter was expressed as

$$m_m = m_d \rho_s \tag{20}$$

where m_d is a calibrating coefficient.

Analysis of the sensitivity of the Kuzmin method to separate components of the heat budget equation for conditions in the River Sosna basin has shown that shortwave radiation and sensible heat exchange play a major role in the process of snow melting in this basin. Hence, the calibrating coefficients have been included in (3) and (5). The calibration has been carried out for the springs of 1967–1969. The remainder of the data have been used for verification of the calibrated models.

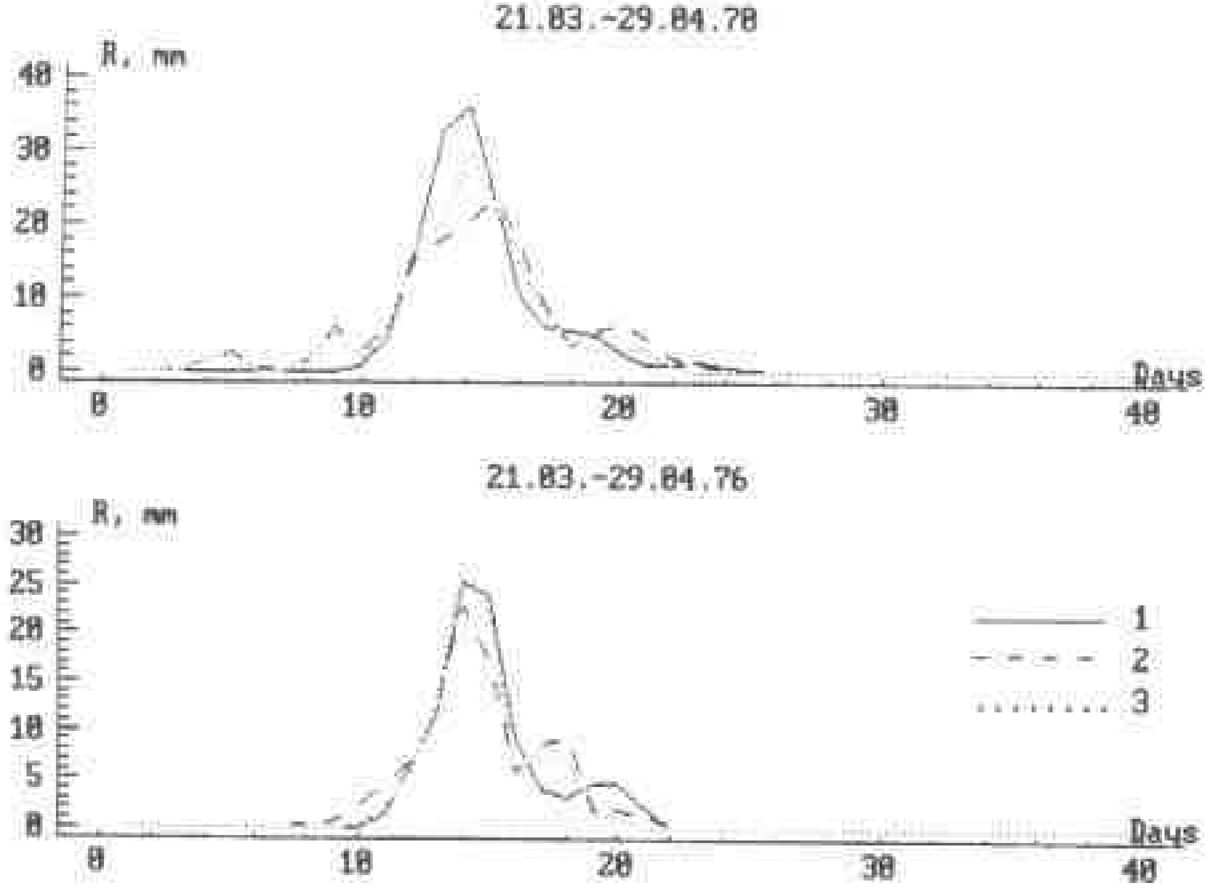

Fig. 7. Comparison of the calculated basin-averaged snowpack outflow based on: 1, the inverse problem solution; 2, the Kuzmin method without calibration; 3, the Kuzmin method with calibration.

The best results of computing the basin-averaged outflow on the basis of the degree-day method have been obtained for $m_d = 2.3 \times 10^{-10}$ m^4 kg^{-1} °C^{-1} s^{-1} (especially for 1970; see Fig. 6). Large errors were obtained only for the spring of 1969 (Fig. 6), as well as for calculations at the constant degree-day factor.

When using the Kuzmin method, the calibration gave the best results by increasing the coefficient in (3) from 17.46 to 19.56 W m^{-2} (deg)$^{-1}$ and using the unchanged coefficients in (5). Significant improvements have been achieved for the springs of 1970 and 1976 (Fig. 7).

The numerical experiments have shown that the basin-averaged outflow determined directly is sensitive to changing C_v. For example, Fig. 8 presents the outflow calculated with the different values of C_v for spring 1968. An increase in C_v from 0.26 to 0.5 leads to an increase in the outflow peak from 31 mm day^{-1} to 40 mm day^{-1}, and in the duration of melting season, by 3 days. An attempt has been made to apply the calibration of the coefficients in (14) against the basin-averaged outflow from the snowpack determined by solving the inverse problem, but with no meaningful improvement in the results has been obtained; the series of observations may be too short.

6. Discussion

The relationships and coefficients used in point computations can be reckoned as a priori assigned (obtained from laboratory and experimental data from the basin under consideration). The results of these computations can be improved by applying the calibration procedure for each point; however, it is impossible to do this for all the

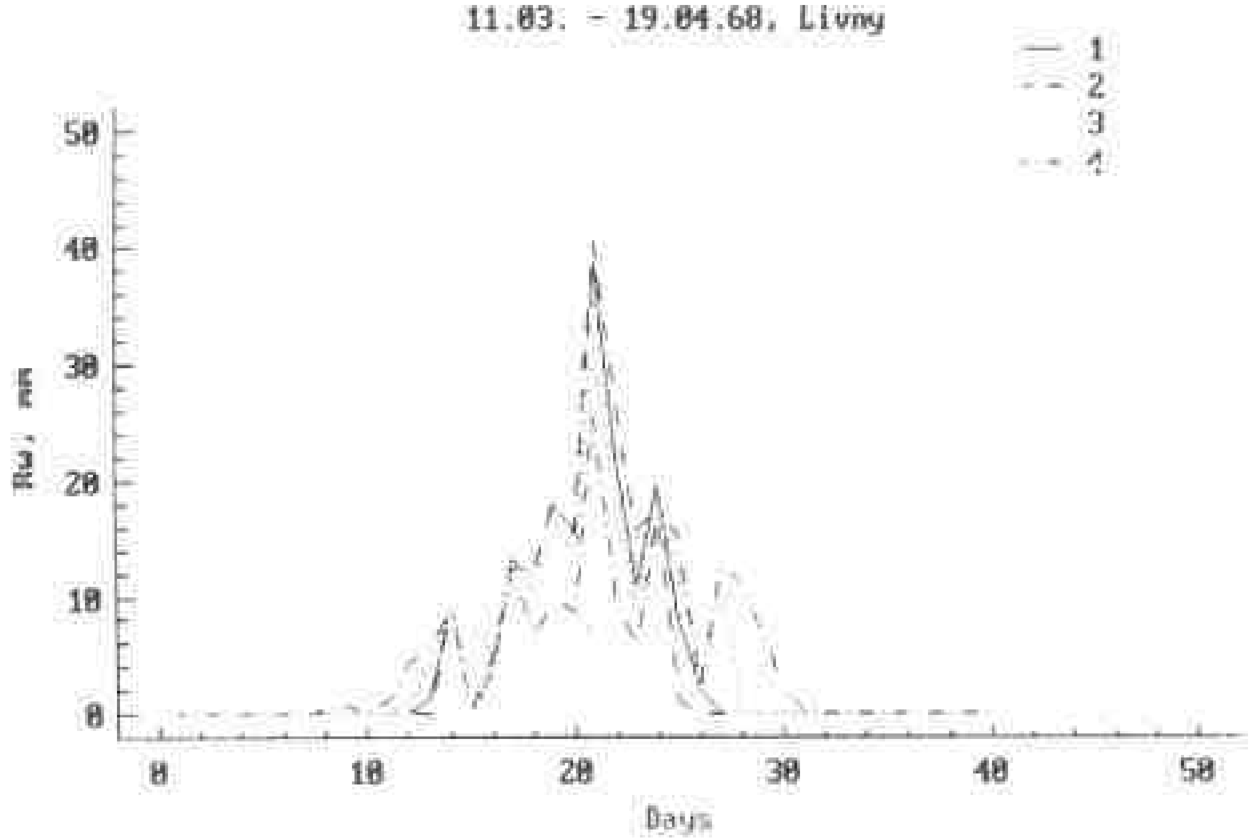

Fig. 8. Comparison of the area-averaged snowpack outflow calculated by the different variation of the maximum snow water equivalent: 1, $C_v = 0.26$; 2, $C_v = 0.1$; 3, $C_v = 0.5$; 4, $C_v = 1.0$.

types of heterogeneities in the basin. At the same time, in such a large area as the Sosna River basin this point calibration can increase the errors in determining the area-averaged snowmelt characteristics. The differences between the basin-averaged snowmelt characteristics calculated directly and the basin-averaged ones obtained by the inverse method are, in general, of the same order as the errors in computing point snowmelt characteristics, despite the lack of data for detailed representation of the spatial distribution of the meteorological inputs and the snow characteristics before melt. The improved results obtained by applying the Kuzmin method can be attributed to the use of more detailed meteorological information, better descriptions of the physical processes and improved opportunities to account for spatial changes in meteorological conditions. The procedure based on the Kuzmin method also offers opportunities for calibrating parameters taking into account the effects of spatial averaging. For calibration of the snowmelt model parameters, the entire runoff generation model including the snowmelt model as a input block can be used; however, the sensitivity of runoff hydrographs to calibrating parameters may in this case be insufficient for reliable fitting. The solution of the inverse problem for runoff of the model provides a well-expressed temporal simulation of the basin-averaged outflow and analysis of consistency even if the runoff model is crude.

7. Conclusions

The solution of the inverse problem for a snowmelt runoff generation model is an effective tool for determining the area-averaged meltwater outflow from the snowpack and for estimating the accuracy of computation of the area-averaged snowmelt characteristics by meteorological data. The calibration of the snowmelt model against the area-averaged meltwater outflow offers a significant improvement in the procedures of computing area-averaged snowmelt characteristics. The procedure based on the Kuzmin method provided better results than that based on the degree-day method, both for point calculations and for calculations of the basin-averaged snowmelt characteristics. Hence, when standard meteorological measurements are available, it is recommended that, for calculating snowmelt, the Kuzmin or other relevant heat budget method with calibration of the most important parameters is applied, instead of the degree-day method.

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