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Pattern Analysis of Approximant Characteristics in Fractal-Like Multilayered Systems with Metalayers

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Abstract—The effect structural changes in fractal-like multilayered structures with metamaterial inserts have on the stability of the shape of fractal patterns in their optical spectra is investigated. Calculations using the model of approximants allows scaling parameters to be obtained that establish the relationship between structural features and their optical spectra.

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INTRODUCTION

The general theoretical aspects of multilayered structures with metalayers comprise part of the important fundamental problem of establishing relationships between the structural features of fractal-like systems and the scaling properties of probing radiation [1, 2]. A comprehensive solution to it would enable us to improve fractal means of the optical diagnostics of multilayered systems with different geometries.

Numerous studies are now being devoted to the properties of different multilayered structures with fractal signs, and to the possibilities of their practical application [2–4]. Such multilayered structures are used particularly in designing new fractal antennas, sensors for detecting biological and chemical agents, narrow-band filters, broadband absorbers, and other technical devices.

Nevertheless, the optical characteristics of approximants of fractal-like multilayered systems with metamaterials remain poorly studied. The fractality inherent in many of these systems has a strong effect on their optical characteristics. There is also the need to estimate the effect of various factors on the self-similarity of their spectral parameters.

Applying approximants of different geometries allows is to technically simplify the manufacture of structures with a given set of optical characteristics. The preliminary trials in [5, 6] showed the promise of using approximants to improve new tools of optical diagnostics. Recording patterns (individual elements in probing radiation characteristics) allows us in particular to identify multilayered systems of certain symmetries, and to estimate the degree of their structural defects.

The aim of this work was to establish the effect changes in multilayered structures with metamaterial inserts have on the stability of the shape of the fractal patterns of their optical spectra. The changes are assumed to be introduced into a multilayered system by moving to a model of approximants [6], preparing a series of layers based on widely used metamaterials with different ratios of dispersion [7, 8], or varying the structure of a system [9]. Special attention is given to pattern analysis [10, 11] in regard to the spectral characteristics of multilayered structures, allowing for absorption in metalayers.

MODEL OF MULTILAYERED STRUCTURES WITH METALAYERS

Let us consider a model of multilayered structures and their approximants composed of two types of layers: A and B. Layers A are made of a metamaterial that is characterized by a negative refractive index within a certain spectral range. Layers B are based on porous quartz with refractive index n_B. The layer thicknesses are assumed to be $d_A = 0.6$ cm and $d_B = 1.2$ cm, respectively [12].

The phase incursions in the layers of multilayered systems are determined as

$$\Phi_{j,k} = \frac{2\pi f_k \mathrm{N}_{j,k} d_j}{c}, \qquad (1)$$

where j is the number of the layer; c is the speed of light; d_j is the thickness of the j-th layer; $\mathrm{N}_{j,k}$ is the refractive index of the j-th layer, given frequency f of radiation and the geometric alternating of layers A and B. Parameter f is discretized as

$$f_k = 1.5(1 + 0.0033k), \qquad (2)$$

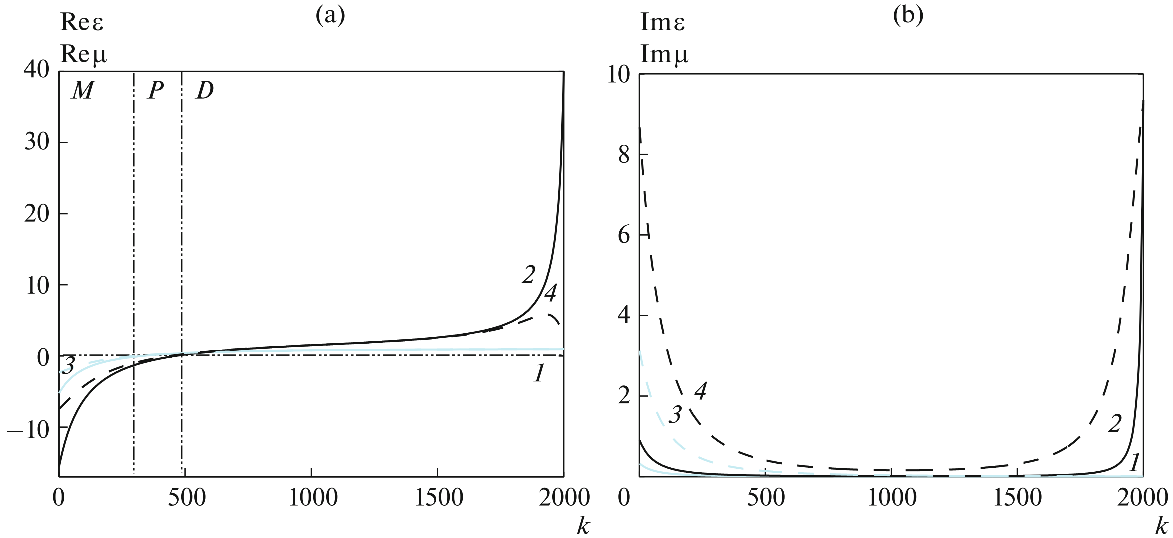

Fig. 1. Dispersion functions $\varepsilon(k, \gamma)$, $\mu(k, \gamma)$ at different losses γ: (a) real part of (b) imaginary part of $\varepsilon(k, \gamma)$, $\mu(k, \gamma)$. Functions *1*, *2*: $\gamma = 0.05$ GHz; *3*, *4*: $\gamma = 0.9$ GHz. Dependences $\varepsilon(k, \gamma)$ are shown in black. *M*, *P*, *D* are the relevant regions.

where $k = 0\ldots\tilde{N}_{\max}$ are the coefficients of discretization and $\tilde{N}_{\max}$ is the discrete value limiting the range of frequencies.

To construct approximants of multilayered structures, the parent structure is set on the basis of fractal-like numerical sequences $S_l = \{A, B\}$, where l is the level of generation and A and B are constituent elements of a sequence. Sequences S_l determine the law of alternating elements in a primary structure. In multilayered structures, this law determines the distribution of layers with low and high refractive indices. For example, approximants A_l for an m-bonacci system ($m = 2$) are

$$\mathrm{A}_0 = \Big\{\underbrace{B}_{S_0}\Big\}^p, \quad \mathrm{A}_1 = \Big\{\underbrace{A}_{S_1}\Big\}^p, \quad \mathrm{A}_2 = \Big\{\underbrace{AB}_{S_2}\Big\}^p,$$
$$\mathrm{A}_3 = \Big\{\underbrace{ABA}_{S_3}\Big\}^p, \quad \mathrm{A}_4 = \Big\{\underbrace{ABAAB}_{S_4}\Big\}^p, \tag{3}$$
$$\mathrm{A}_5 = \Big\{\underbrace{ABAABABA}_{S_5}\Big\}^p, \quad \ldots, \quad \mathrm{A}_{l+1} = \{S_l S_{l-1}\}^p,$$

where p is the approximant period (i.e., the number of unit cells S_l in the considered multilayered system).

Self-similar structural properties of m-bonacci systems can be described by scaling parameter ζ, which is defined as the limit of the number of elements in the sequence S_{l+1} to the number of elements in S_l at $l \to \infty$. At $m = 2$, we thus obtain $\zeta \approx 1.618$. For $m = 3$ and $m = 4$, we have $\zeta \approx 1.839$ and $\zeta \approx 1.927$, respectively. The values of structural parameters correspond to those reported in the literature [5, 9]. It is worth noting that the m-bonacci and Fibonacci systems match at $m = 2$.

Dielectric constant ε_A and magnetic susceptibility μ_A of the metamaterial are determined as [7]

$$\varepsilon_A(f) = 1 + \frac{5^2}{0.9^2 - f^2 - if\gamma} + \frac{10^2}{11.5^2 - f^2 - if\gamma};$$
$$\mu_A(f) = 1 + \frac{3^2}{0.902^2 - f^2 - if\gamma}, \tag{4}$$

where f is the frequency of the electromagnetic radiation; i is an imaginary unit; and γ is the parameter characterizing loss (f and γ are expressed in GHz). Figure 1 presents dispersion dependences (4) for metamaterial layers, given frequency discretization (2).

Analysis of relations (4) reveals that patterns of dispersion can be divided into three areas (Fig. 1). In first area M, layers A have the properties of metamaterials with $\varepsilon_A < 0$, $\mu_A < 0$; second area P is transitional, with $\varepsilon_A < 0$, $\mu_A > 0$; and third area D has $\varepsilon_A > 0$, $\mu_A > 0$, and the material of layers A becomes dielectric. Varying $\gamma \in [0, 0.9]$ slightly alters the boundaries of these areas. The parameters of the environment were assumed to be $\varepsilon = 1$, $\mu = 1$.

SPECTRAL CHARACTERISTICS OF STRUCTURES WITH METAMATERIALS

The spectral characteristics of fractal-like multilayered structures are calculated using the matrix approach in [13] with a logarithmic representation [1] and the Fresnel law for metamaterials [14]

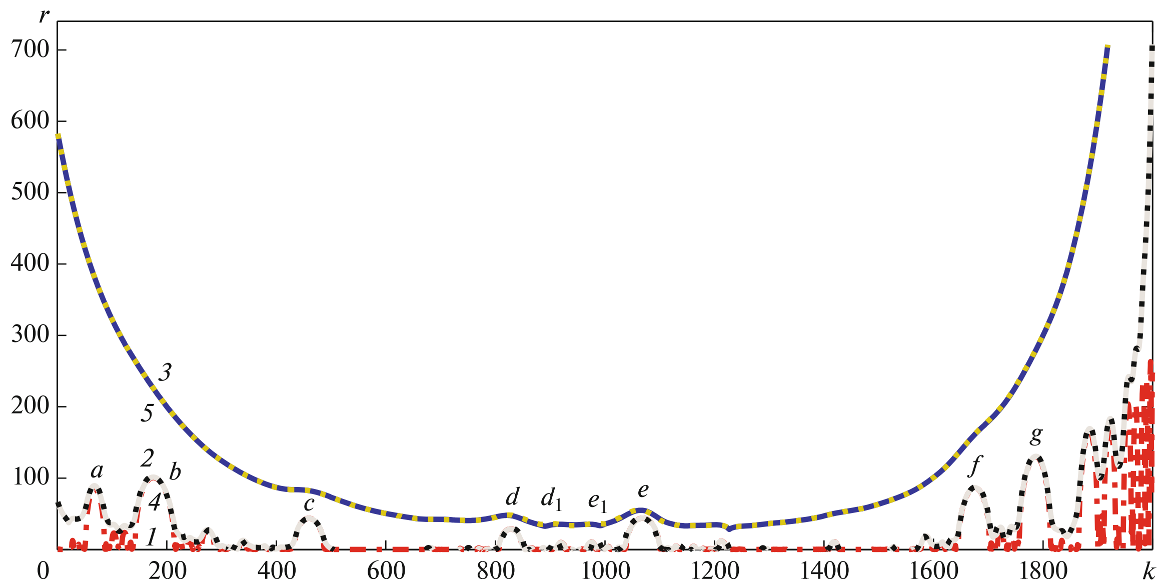

Fig. 2. Spectral characteristics of a multilayered *m*-bonacci system for $m = 2$, $n_B = 1.5$ at coefficient of frequency discretization $k = 0$–2000. Bottom line *1* corresponds to the primary structure with number of layers $J = 234$ at $\gamma = 0$ GHz; lines *2* and *3* refer to the primary structure with absorption $\gamma = 0.05$ and $\gamma = 0.9$ GHz, respectively. Dotted lines *4* and *5* correspond to approximant $A_6 = \{S_6\}^{18}$ for systems 2 and 3, respectively. Areas *ab*, *bc*, *de*, *fg* are the relevant patterns.

$r = -\ln(T_k)$, where T_k is the coefficient of power transmission.

Modeling shows that given their absorption, including metamaterials in a multilayered system notably alters the spectral characteristics of multilayered fractal-like systems in all of the above areas (Fig. 1). These alterations were investigated via pattern analysis [11] based on fixing and determining the features of individual fragments in the spectral characteristics of multilayered structures.

Figure 2 presents parameter r as a function of absorption γ of a multilayered *m*-bonacci system for $m = 2$, allowing for dispersion effects (4). Areas *ab*, *bc*, *de*, *fg* are the patterns. As is seen from Fig. 2, absorption has a strong effect on the fractal-like structure of the initial spectral characteristics corresponding to $\gamma = 0$. At the same time, it becomes smoother for all k, and the fractal properties of the spectrum vanish. Upon a change in γ, the most stable area from the viewpoint of scaling is the central part of the spectrum (pattern *de*) up to $\gamma = 0.55$ GHz. Pattern *de* is associated with area *D* (Fig. 1). For other spectral ranges, pattern features can be detected with some distortions at $\gamma \leq 0.1$ GHz. Note that the spectral characteristics of the given multilayered structure are close to those of its approximants. The coefficients of mutual correlation between the spectra of analyzed structures 2 and 4, and 3 and 5 (Fig. 2), assumed values from 0.91 to 0.98 for different spectral fragments.

According to our study, the absorption of metalayers has a pronounced smoothing effect on the self-similar properties of the spectral characteristics of multilayered structures and their approximants.

Our modeling results describe a case where phase incursions (1) in the layers are equal in absolute value. When the relationships between the geometric sizes of the layers change, there is an imbalance in phase incursions, and strong distortions of shape patterns make it difficult to identify them.

Our modeling data show that the fractal properties of the optical characteristics of *m*-bonacci systems manifest at $m = 2$ and $m = 3$. An increase in m transforms the shape of the pattern. At the same time, phase compensation eliminates some resonance peaks, bringing the spectrum closer to those of periodic systems.

A series of identifying parameters (local scaling coefficients) characteristic of the observed patterns of the optical characteristics of the above systems was determined in this work. The local scaling parameters for pattern *de* with the best absorption resistance were thus found to be $\zeta_1 = de/de_1 \approx 1.62$, $\zeta_2 = de_1/e_1e \approx 1.6$. The scaling parameters for the *de* pattern tend toward Golden section coefficient $\zeta \approx 1.618$. Estimating them allows us to establish a quantitative correlation between the structural features of the studied objects and the fractal properties of the light radiation interacting with them.

Although the pattern profiles vary greatly from one spectral range to another, their geometries determined by the mutual arrangement of the internal peaks are close to one another. We may therefore state that under certain conditions, the patterns recorded in the spectral characteristics can be taken as references in assessing the scaling properties of multilayered systems and their approximants.

The results obtained in this work enable us to supplement considerably the diagnostic approach in [1] based on pattern analysis in regard to approximants of fractal-like multilayered structures with metalayers.

CONCLUSIONS

Our modeling data testify to the notable effect of absorption and the dispersion of metalayers, and of phase compensation and phase incursion imbalance inside layers, when analyzing scaling in the optical characteristics of multilayered structures with metalayers. Dispersion distorts the shapes of visible patterns, and the absorption of metalayers smooths the spectral characteristics in all considered ranges of frequency. In some cases, phase compensation effects can suppress the formation of pattern elements (systems with equal amounts of primary elements A and B), depending on the correlation between the number of different types of layers.

Analysis of characteristics of multilayered structures with metalayers reveals the possibility of obtaining one-to-one relationships between their structural features and transmittance spectra, based on pattern formation and determining the coefficients of scaling within narrow range of absorption γ.

The results obtained in this work largely supplement the pattern approach to optical diagnostics based on determining the scaling parameters of fractal-like multilayered structures of approximants with metalayers [6, 15].

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