Necessary Conditions for Infinite Horizon Optimal Control Problems Revisited

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Optimal control problems with infinite horizon play an important role in economic theory. For instance, in the theory of economic growth, Pontragin's maximum principle is the workhorse for many researchers. The proof of the maximum principle for infinite time horizon (see e.g., Halkin, 1974) does not include transversality conditions. Moreover it is known (Halkin, 1974; Kamihigashi, 2001) that the following usually used forms of transversality conditions can be not necessary:

$$\lim_{t\to\infty} \psi(t) = 0, \quad \text{or} \quad \lim_{t\to\infty} \langle \hat{x}(t), \psi(t) \rangle = 0, \tag{1}$$

where $\hat{x}$ is the optimal state trajectory, ψ is the corresponding adjoint variable, and brackets $\langle \cdot, \cdot \rangle$ denote scalar product of two vectors. Transversality condition obtained in Michel (1982) under assumptions including that the objective functional takes only finite values, has the form of Hamiltonian $\mathcal{H}$ converging to zero

$$\lim_{t\to\infty} \mathcal{H}(\hat{x}(t), \hat{u}(t), t, \psi(t)) = 0, \tag{2}$$

where $\hat{x}$ and $\hat{u}$ are the optimal state trajectory and control. In works by Aseev, Besov, Kryazhimskii, and Veliov (2007-2014) the authors determine the adjoint variable uniquely by a Cauchy-tipe formula, which is equivalent to the adjoint equation with the following transversality condition:

$$\lim_{t\to\infty} Y(t)\, \psi(t) = 0, \tag{3}$$

where $Y(t)$ is the fundamental matrix of the state equation linearized about the optimal solution, see also Khlopin (2015).

All aforementioned transversality conditions fail to select the optimal solution of Ramsey problem without discounting, where we consider diverging objective functional. It can be done with the necessary conditions obtained in the present paper (arxiv.org/abs/1512.01206). The proposed conditions do not contain explicitly the adjoint variable and include condition (3) as a special case, extending its domain of applicability.