

Referee Report of Mitrofanov Ivan,  
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On a master Thesis of student Mohammadmehdi Golafshatan «Unipotent dynamics on a torus and complexity functions of subwords of powers of two»

The research area of this work is between combinatorics of words, number theory and dynamical systems.

For an infinite sequence  $x_0x_1\dots$  of letters its *combinatorial complexity* is the function  $c(n)$  = number of different subwords of length  $n$ . The following statement is proved in the paper:

Given a positive integer  $k$ . The sequence  $(W_n)$  is considered,  $n$ -th term of which is the first digit of the number  $2^{n^k}$  in decimal representation. Then there is a polynomial  $P(k)$  of degree  $d(d+1)/2$  such that the combinatorial complexity of  $(W_n)$  coincides with  $P(k)$  for sufficiently large  $k$ .

The first part of the paper describes the relationship of the result with various fields of mathematics, describes the classical results on combinatorial complexity (i.e. Sturmian words).

In the second (main) part in Ch.4 the author analyzes in detail the case  $k = 1$ .

The Ch.5 is devoted to the proof of the general case.

The proof uses the methods of dynamical systems (dynamics on torus), linear algebra and combinatorics.

The results are interesting and should be published.

The work is written well, fully meets the requirements for a master's thesis, and deserves a 10 point mark. M.Golafshatan deserves a recommendation for graduate school at the Moscow Institute of Physics and Technology.

Ivan Mitrofanov.