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Angles of Refraction and Directions of Group Velocity Vectors of TE and TM Polarization Waves at the Interface between an Isotropic Medium and a Semi-Space with the Magnetoelectric Effect

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Abstract—The directions of phase and group velocities of TE and TM polarization waves refracted at the interface between an isotropic dielectric and uniaxial crystal with magnetoelectric properties are investigated. The angles that determine the directions of phase and group velocities of TE and TM waves are obtained in analytical form in different directions of the incident wave vector. The results are analyzed for the case of uniaxial crystals without magnetoelectric properties.

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INTRODUCTION

The magnetoelectric effect is described by the relations given in [1]. The interest in media with the magnetoelectric effect is due to the possibility of using them in creating, e.g., wireless energy sources and multiferroic structures in logical elements in constant magnetic fields [2–6]. Theoretical and experimental investigations have been conducted to create composite heterostructures with the magnetoelectric effect needed for practical applications due to a combination of piezoelectric and magnetostrictive effects. This requires comprehensive study of the wave processes in such media, particularly those with magnetoelectric effects.

In this work, wave processes involving the magnetoelectric effect are considered in uniaxial crystals with tetragonal (class 422), trigonal (class 32), and hexagonal (622) symmetries [2]. Formulated problems are solved using a matrix technique that was applied earlier in considering different wave processes in anisotropic elastic media, electromagnetic waves in crystals, and the propagation of coupled elastic and electromagnetic waves in piezoelectric and piezomagnetic media with a magnetoelectric effect [7–16].

MAXWELL EQUATIONS AND MATERIAL RELATIONSHIPS

The propagation of electromagnetic plane waves in the given medium is described by the Maxwell equations

$$\begin{aligned} &\mathrm{rot}\vec{E}(g,t) = -\frac{d\vec{B}(g,t)}{dt}; \\ &\mathrm{rot}\vec{H}(g,t) = \frac{d\vec{D}(g,t)}{dt}, \\ &\mathrm{div}\vec{D}(g,t) = 0; \quad \mathrm{div}\vec{B}(g,t) = 0 \end{aligned} \tag{1}$$

and material relations

$$\begin{aligned} \vec{D}(g,t) &= \vec{\varepsilon}\vec{E}(g,t) - \vec{\alpha}\vec{H}(g,t), \\ \vec{B}(g,t) &= \vec{\mu}\vec{H}(g,t) - \vec{\alpha}\vec{E}(g,t). \end{aligned} \tag{2}$$

Here $g \in (x,o,z) \in R^2$. Wave vector $\vec{\kappa}$ lies in plane xz. $\overline{\varepsilon}, \overline{\mu}$ are dielectric and magnetic permeability tensors, and $\overline{\alpha}$ is the magnetoelectric tensor, defined as

$$\overline{\varepsilon} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_x & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}; \quad \overline{\mu} = \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_x & 0 \\ 0 & 0 & \mu_z \end{pmatrix};$$

$$\overline{\alpha} = \begin{pmatrix} \alpha_x & 0 & 0 \\ 0 & \alpha_x & 0 \\ 0 & 0 & \alpha_z \end{pmatrix}.$$

Using the matrix technique, the system of Eqs. (1) and (2) is reduced to a set of ordinary first-order differential equations (ODEs) relative to vector components $\vec{E}$ and $\vec{H}$ of the electric and magnetic fields, based on the solutions

$$f(g,t) = \overline{f}(z)\exp[i\omega t - ik_x x]$$

and has a form

$$\frac{d\vec{W}}{dz} = B\vec{W}(z) \qquad (3)$$

relative to vector $\vec{W}(z) = (E_y, H_x, H_y, E_x)^t$. Here, t is the sign of the transponation of a line vector into a column vector. Matrix $B = (b_{i,}j)^4$ has the structure

$$B = \begin{pmatrix} 0 & b_{12} & 0 & b_{14} \\ b_{21} & 0 & b_{23} & 0 \\ 0 & -b_{14} & 0 & b_{34} \\ -b_{23} & 0 & b_{43} & 0 \end{pmatrix}. \qquad (4)$$

Elements b_{ij} are $b_{(12)} = i\omega\mu_x$; $b_{(14)} = -i\omega\alpha_x$; $b_{(21)} = i\omega\left(\varepsilon_x - \frac{k_x^2}{\omega^2\beta}\varepsilon_z\right)$;

$$b_{(23)} = -i\omega\left(\alpha_x - \frac{k_x^2}{\omega^2\beta}\alpha_z\right);\quad b_{(34)} = -i\omega\varepsilon_x;$$
$$b_{(43)} = -i\omega\left(\mu_x - \frac{k_x^2\mu_z}{\omega^2\beta}\right);\quad \beta = \varepsilon_z\mu_z - \alpha_z^2. \qquad (5)$$

Since the system of ODEs is in our case considered relative to variable z, assuming transverse electromagnetic waves, Eqs. (1) and (2) allow us to exclude field components E_z and H_z, oriented along axis z.

ANGLES OF REFRACTION OF TE AND TM POLARIZATION WAVES

The indicatrices of wave vectors of TE and TM polarization waves in uniaxial crystals with a magnetoelectric effect are described by the equations [16]

$$k_{\mathrm{TE}}^2 = \frac{\omega^2\beta_x}{\cos^2\theta + (A+B)\sin^2\theta}, \qquad (6)$$

$$k_{\mathrm{TM}}^2 = \frac{\omega^2\beta_x}{\cos^2\theta + (A-B)\sin^2\theta}. \qquad (7)$$

Here,

$$\beta_x = \varepsilon_x\mu_x - \alpha_x^2;\quad \beta_z = \varepsilon_z\mu_z - \alpha_z^2,$$
$$A = \frac{1}{2\beta_z}(\mu_x\varepsilon_z + \varepsilon_x\mu_z - 2\alpha_x\alpha_z);$$
$$B = \frac{1}{2\beta_z} \times \sqrt{(\varepsilon_z\mu_x - \varepsilon_x\mu_z)^2 - (\alpha_x\mu_z - \mu_x\alpha_z)(\varepsilon_x\alpha_z - \alpha_x\varepsilon_z)}. \qquad (8)$$

The refraction of plane waves at the interface requires that the following conditions be met:

$$k_{x_0} = k_0\sin\theta_0 = k_{\mathrm{TE}}\sin\theta_{\mathrm{TE}}, \qquad (9)$$

$$k_{x_0} = k_0\sin\theta_0 = k_{\mathrm{TM}}\sin\theta_{\mathrm{TM}}. \qquad (10)$$

From condition (10) based on Eq. (6), we obtains

$$k_0^2\sin^2\theta_0 = \frac{\omega^2\beta_x\sin^2\theta_{\mathrm{TE}}}{\cos^2\theta_{\mathrm{TE}} + (A+B)\sin^2\theta_{\mathrm{TE}}} = \frac{\omega^2\beta_x\tan^2\theta_{\mathrm{TE}}}{1+(A+B)\tan^2\theta_{\mathrm{TE}}}. \qquad (11)$$

Dividing expression (11) by $\tan^2\theta_{\mathrm{TE}}$ produces

$$\tan^2\theta_{\mathrm{TE}} = \frac{k_0^2\sin^2\theta_0}{\omega^2\beta_x - (A+B)k_0^2\sin^2\theta_0}. \qquad (12)$$

Formula (12) defines the angle of refraction of a TE polarization wave at the interface between the media and the direction of the phase velocity vector. As with a TM polarization wave,

$$\tan^2\theta_{\mathrm{TM}} = \frac{k_0^2\sin^2\theta_0}{\omega^2\beta_x - (A-B)k_0^2\sin^2\theta_0} \qquad (13)$$

and given Eqs. (12) and (13), we can determine the angle between the phase velocity vectors of TE and TM waves.

Since

$$A+B \succ A-B,\quad A \succ 0,\quad B \succ 0\quad A \succ B, \qquad (14)$$

we have

$$\theta_{\mathrm{TE}} \succ \theta_{\mathrm{TM}}. \qquad (15)$$

For angular difference $\Delta\theta = \theta_{\mathrm{TE}} - \theta_{\mathrm{TM}}$ we have

$$\tan(\theta_{\mathrm{TE}} - \theta_{\mathrm{TM}}) = \frac{\tan\theta_{\mathrm{TE}} - \tan\theta_{\mathrm{TM}}}{1 + \tan\theta_{\mathrm{TE}}\tan\theta_{\mathrm{TM}}}. \qquad (16)$$

Substituting formula (16) into expressions (12) and (13), we obtain

$$\tan\Delta\theta = \frac{k_0\sin\theta_0\left[\sqrt{\omega^2\beta_x - (A-B)k_0^2\sin^2\theta_0} - \sqrt{\omega^2\beta_x - (A+B)k_0^2\sin^2\theta_0}\right]}{\sqrt{\left(\omega^2\beta_x - Ak_0\sin\theta_0\right)^2 - B^2k_0^2\sin^2\theta_0 + k_0^2\sin^2\theta_0}}. \qquad (17)$$

The angle of total internal reflection can be determined using formulas (12) and (13) at $\sin\theta_0 = 1$, $\theta_0 = \frac{\pi}{2}$:

$$\tan^2\theta_{\mathrm{TE}} = \frac{k_0^2}{\omega^2\beta_x - (A+B)k_0^2},\quad \tan^2\theta_{\mathrm{TM}} = \frac{k_0^2}{\omega^2\beta_x - (A-B)k_0^2}. \tag{18}$$

For $\Delta\theta = \theta_{\mathrm{TE}} - \theta_{\mathrm{TM}}$,

$$\tan\Delta\theta = \frac{k_0\left[\sqrt{\omega^2\beta_x - (A-B)k_0^2} - \sqrt{\omega^2\beta_x - (A+B)k_0^2}\right]}{\sqrt{\left(\omega^2\beta_x - Ak_0\right)^2 - B^2k_0^2 + k_0^2}}, \tag{19}$$

allowing us to determine the angle between phase velocity vectors of TE and TM waves at the angle of total internal reflection.

DIRECTIONS OF THE GROUP VELOCITY VECTORS OF TE AND TM POLARIZATION WAVES

An analysis of phase velocity indicatrices, based on the Raleigh relation for determining group velocity in [16], allowed us to establish that

$$\tan\beta_{\mathrm{TE}} = (A+B)\tan\beta_{\mathrm{TE}},\quad \tan\beta_{\mathrm{TM}} = (A-B)\tan\beta_{\mathrm{TM}}. \tag{20}$$

Here, angle β_{TE} determines the group velocity direction of a TE wave in uniaxial crystals with a magnetoelectric effect. The direction of the group velocity of a TM polarization wave is determined by a similar wave. Substituting expressions (20) into formulas (12) and (13) results in the relations

$$\tan\beta_{\mathrm{TE}} = (A+B)\frac{k_0\sin\theta_0}{\sqrt{\omega^2\beta_x - (A+B)k_0^2\sin^2\theta_0}},\quad \tan\beta_{\mathrm{TM}} = (A-B)\frac{k_0\sin\theta_0}{\sqrt{\omega^2\beta_x - (A-B)k_0^2\sin^2\theta_0}}. \tag{21.1}$$

Formulas (21.1) determine the group velocities of TE and TM polarization waves as a function of wave incidence angle θ_0. At $\theta_0 = \frac{\pi}{2}$ from expressions (20) and (21), angles β_{TE} and β_{TM} can be evaluated at total internal reflection. The dependence of $\beta_{\mathrm{TE}} - \beta_{\mathrm{TM}} = \Delta\beta$ on angle θ_0 of wave incidence can also be established using a relation analogous to formula (16):

$$\left[\tan(\beta_{\mathrm{TE}} - \beta_{\mathrm{TM}}) = \frac{\tan\beta_{\mathrm{TE}} - \tan\beta_{\mathrm{TM}}}{1 + \tan\beta_{\mathrm{TE}}\mathrm{tg}\beta_{\mathrm{TM}}}\right],$$
$$\tan\Delta\beta = \frac{k_0\sin\theta_0\left[(A+B)\sqrt{\omega^2\beta_x - (A-B)k_0^2\sin^2\theta_0} - (A-B)\sqrt{\omega^2\beta_x - (A+B)k_0^2\sin^2\theta_0}\right]}{\sqrt{\left(\omega^2\beta_x - Ak_0\sin\theta_0\right)^2 - B^2k_0^2\sin^2\theta_0 + k_0^2\sin^2\theta_0}}. \tag{21.2}$$

Expression (21.2) allows values $\Delta\beta$ to be obtained at total internal reflection $\left(\sin\theta_0 = 1,\ \theta_0 = \frac{\pi}{2}\right)$.

CONSEQUENCES OF THE RESULTS FOR UNIAXIAL CRYSTALS WHEN THERE IS NO MAGNETOELECTRIC EFFECT

If magnitudes β_x, β_z, A, B are free of the magnetoelectric effect, we must assume that $\alpha_x = 0, \alpha_z = 0$. Then

$$\beta_x = \varepsilon_x\mu_x,\quad \beta_z = \varepsilon_z\mu_z,\quad A = \frac{1}{2}\left(\frac{\mu_x}{\mu_z} + \frac{\varepsilon_x}{\varepsilon_z}\right),\quad B = \frac{1}{2}\left(\frac{\mu_x}{\mu_z} - \frac{\varepsilon_x}{\varepsilon_z}\right) \tag{22}$$

and formulas (6) and (7) can be rewritten as

$$k_{\mathrm{TE}}^2 = \frac{\omega^2\varepsilon_x\mu_x\mu_z}{\mu_z\cos^2\theta + \mu_x\sin^2\theta},\quad k_{\mathrm{TM}}^2 = \frac{\omega^2\mu_x\varepsilon_x\varepsilon_z}{\varepsilon_z\cos^2\theta + \varepsilon_x\sin^2\theta}. \tag{23}$$

Given formulas (22) and (23), we obtain

$$\tan^2\theta_{\mathrm{TE}} = \frac{\mu_z k_0^2\sin^2\theta_0}{\mu_x(\omega^2\varepsilon_x\mu_z - k_0^2\sin^2\theta_0)},\quad \tan^2\theta_{\mathrm{TM}} = \frac{\varepsilon_z k_0^2\sin^2\theta_0}{\varepsilon_x(\omega^2\varepsilon_z\mu_x - k_0^2\sin^2\theta_0)}, \tag{24}$$

$$\tan\beta_{\mathrm{TE}} = \sqrt{\frac{\mu_x}{\mu_z}}\frac{k_0\sin\theta_0}{\sqrt{\omega^2\varepsilon_x\mu_z - k_0^2\sin^2\theta_0}},\quad \tan\beta_{\mathrm{TM}} = \sqrt{\frac{\varepsilon_x}{\varepsilon_z}}\frac{k_0\sin\theta_0}{\sqrt{\omega^2\varepsilon_z\mu_x - k_0^2\sin^2\theta_0}}, \tag{25}$$

$$\tan\Delta\theta = \frac{k_0\sin\theta_0\left[\sqrt{\frac{\varepsilon_x}{\varepsilon_z}}\sqrt{\omega^2\varepsilon_z\mu_x - k_0^2\sin^2\theta_0} - \sqrt{\frac{\mu_x}{\mu_z}}\sqrt{\omega^2\varepsilon_x\mu_z - k_0^2\sin^2\theta_0}\right]}{\sqrt{\frac{\varepsilon_x\mu_x}{\varepsilon_z\mu_z}}\sqrt{\omega^2\varepsilon_z\mu_x - k_0^2\sin^2\theta_0}\sqrt{\omega^2\varepsilon_x\mu_z - k_0^2\sin^2\theta_0} + k_0^2\sin^2\theta_0}, \quad (26)$$

$$\tan\Delta\beta = \frac{k_0\sin\theta_0\left[\sqrt{\frac{\mu_x}{\mu_z}}\sqrt{\omega^2\varepsilon_z\mu_x - k_0^2\sin^2\theta_0} - \sqrt{\frac{\varepsilon_x}{\varepsilon_z}}\sqrt{\omega^2\varepsilon_x\mu_z - k_0^2\sin^2\theta_0}\right]}{\sqrt{\left(\omega^2\varepsilon_z\mu_x - k_0^2\sin^2\theta_0\right)\left(\omega^2\varepsilon_x\mu_z - k_0^2\sin^2\theta_0\right)} + \sqrt{\frac{\varepsilon_z\mu_z}{\varepsilon_x\mu_x}}k_0^2\sin^2\theta_0}. \quad (27)$$

Formulas (24)–(27) at $\theta_0 = \frac{\pi}{2}$, $\sin\theta_0 = 1$ determine the relevant magnitudes at total internal reflection. Formula (24) thus defines the angle of total internal reflection for uniaxial crystals.

It also follows from expressions (24) that for the uniform isotropic media

$$\tan^2\theta_{TE} = \frac{k_0^2\sin^2\theta_0}{k^2\cos^2\theta} \Rightarrow \frac{\sin^2\theta}{\sin^2\theta_0} = \frac{k_0^2}{k^2} = \frac{\upsilon^2}{c^2} = \frac{1}{n^2},$$

$$\tan^2\theta_{TM} = \frac{k_0^2\sin^2\theta_0}{k^2\cos^2\theta} \Rightarrow \frac{\sin^2\theta}{\sin^2\theta_0} = \frac{1}{n^2},$$

$$\tan^2\theta_{TE} = \tan^2\theta_{TM} = \tan^2\beta_{TE} = \tan^2\beta_{TM} = \frac{k_0^2\sin^2\theta_0}{k^2\cos^2\theta}, \quad \tan\Delta\theta = 0; \ \tan\Delta\beta = 0.$$

CONCLUSIONS

The angles of refraction of TE and TM polarization waves were studied at the interface between a uniform isotropic dielectric and the uniaxial crystal with a magnetoelectric effect at different angles of wave incidence. These dependences determine the directions of phase velocities of TE and TM waves. The group velocities of these waves were also found as functions of incident wave angles θ_0. Relations that determine the angular differences between the phase and group velocities for TE and TM polarization waves were obtained as well. The consequences of the results for uniaxial crystals in a lack of the magnetoelectric effect and those for isotropic media were analyzed comprehensively. The angles of total internal reflection for TE and TM waves were considered along with the relevant directions of group velocities.

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