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Damage evolution in the process of plane strain bending under tension at large strains

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Abstract

A semi-analytic solution for plane strain bending under tension of a sheet is proposed for elastic-plastic, isotropic, incompressible, strain hardening material with damage evolution at large strains using a Lagrangian coordinate system. Numerical treatment is only necessary to evaluate ordinary integrals and solve transcendental equations. No restriction is imposed on the hardening law. Quite a general uncoupled continuum damage evolution model independent of the third invariant of the stress tensor is used. It is shown that the solution for the model adopted is facilitated by choosing the equivalent plastic strain as one of the independent variables. An illustrative example is provided for Swift's hardening law and two widely used damage evolution equations.

Keywords: Bending; damage; large strain; semi-analytic solution

1. Introduction

Ductile fracture is one of the most important limiting factors in metal forming. Several ductile fracture theories are available in the literature. The present paper deals with uncoupled continuum damage mechanics models that are independent of the third invariant of the stress tensor. Overviews of such models have been provided in [1, 2].

All sheet metal forming processes incorporate some bending [3]. Several solutions for plane strain pure bending are available in the literature. A comprehensive overview of such solutions has been given in [4]. A semi-analytic solution for plane strain bending under tension for elastic/plastic, strain hardening material has been found in [5] assuming that the material is incompressible. A numerical method has been developed in [6] to determine the through-thickness distribution of damage in the process of plane strain pure bending of rigid/plastic, strain hardening material. The damage evolution model proposed in [7] has been adopted. The present paper provides a semi-analytic solution for the process of plane strain bending under tension. Quite a general uncoupled damage evolution model is adopted. It is shown that the solution is facilitated if the equivalent plastic strain is used as one of the independent variables.

2. Material model

The Cauchy stress and Hencky strain are adopted in the present paper. The classical Eulerian theory of finite elastoplasticity is used. A description of the theory can be

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found in [8]. It is assumed that the material is incompressible (i.e., Poisson's ratio is equal to 1/2). The plane strain yield criterion is taken in the form

$$|\sigma_1 - \sigma_2| = \frac{2}{\sqrt{3}}\sigma_0 \Phi\left(\varepsilon_{eq}^p\right) \tag{1}$$

where σ_1 and σ_2 are the principal stresses, σ_0 is the initial yield stress in tension, ε_{eq}^p is the equivalent plastic strain, $\Phi\left(\varepsilon_{eq}^p\right)$ is an arbitrary function of its argument satisfying the conditions $\Phi(0) = 1$ and $d\Phi\left(\varepsilon_{eq}^p\right)/d\varepsilon_{eq}^p \geq 0$ for all ε_{eq}^p. The equivalent plastic strain is defined by the equation

$$\varepsilon_{eq}^p = \int_0^t \sqrt{\frac{2}{3}\xi_{ij}^p \xi_{ij}^p}\, d\tau \tag{2}$$

where ξ_{ij}^p are the plastic components of the strain rate tensor and t is the time. Integration in (2) should be performed over the strain path. The elastic portions of the principal strain rates, ξ_1^e and ξ_2^e, are connected to the stress components by the following rate constitutive equations:

$$(\dot{\sigma}_1 - \dot{\sigma}) = 2G\xi_1^e \quad \text{and} \quad (\dot{\sigma}_2 - \dot{\sigma}) = 2G\xi_2^e. \tag{3}$$

Here G is the shear modulus of elasticity, σ is the hydrostatic stress and the superimposed dot denotes the material derivative. In general, the left-hand sides of the equations in (3) should involve an objective derivative. However, it will be seen later that in the case under consideration all objective derivatives reduce to the material derivative. The hydrostatic stress is given by

$$\sigma = \frac{\sigma_1 + \sigma_2}{2}. \tag{4}$$

The total principal strain rates are

$$\xi_1 = \xi_1^e + \xi_1^p \qquad \text{and} \qquad \xi_2 = \xi_2^e + \xi_2^p. \tag{5}$$

The plastic flow rule associated with the yield criterion (1) results in conditions that plastic deformation is incompressible and that the principal axes of the stress and strain rate tensors coincide.

Uncoupled damage evolution equations are adopted in the present paper. It is assumed that the damage parameter takes the form [1]

$$D = \int_{t_0}^t g\left(\frac{\sigma}{\sigma_{eq}}\right)\xi_{eq}^p\, d\tau. \tag{6}$$

Here $g(\sigma/\sigma_{eq})$ some function of the ratio of the hydrostatic stress to the equivalent stress and t_0 is the instant of time at which the damage starts to develop. The equivalent stress is defined as

$$\sigma_{eq} = \frac{\sqrt{3}}{2}|\sigma_1 - \sigma_2| = \sigma_0 \Phi\left(\varepsilon_{eq}^p\right). \tag{7}$$

3. Elastic/plastic solution

A general elastic/plastic solution for the process of bending under tension of a wide sheet has been proposed in [5]. The solution satisfies the material model described in Section 2 and the equilibrium equations. It is outlined below for subsequent use in Section 4.

The solution is based on the following mapping between Eulerian Cartesian coordinates (x, y) and Lagrangian coordinates (ζ, η):

$$\frac{x}{H} = \sqrt{\frac{\zeta}{a} + \frac{s}{a^2}}\cos(2a\eta) - \frac{\sqrt{s}}{a}, \quad \frac{y}{H} = \sqrt{\frac{\zeta}{a} + \frac{s}{a^2}}\sin(2a\eta). \tag{8}$$

The Lagrangian coordinates are non-dimensional. The mapping (8) transforms an initial rectangular into a sector of a hollow cylinder (Fig. 1). In Eq. (8), H is the initial thickness of the sheet, a is a dimensionless time-like variable, $a = 0$ at the initial instant, and s is a function of a. This function should be found from the solution and should satisfy the condition $s(0) = 1/4$. This condition ensures that $x = \zeta H$ and $y = \eta H$ at the initial instant. Then, it is seen from Fig.1 that $\zeta = 0$ on the surface AB and $\zeta = -1$ on the surface CD throughout the process of deformation. The trajectories of the principal stresses and principal strain rates coincide with the coordinate curves of the Lagrangian coordinate system.

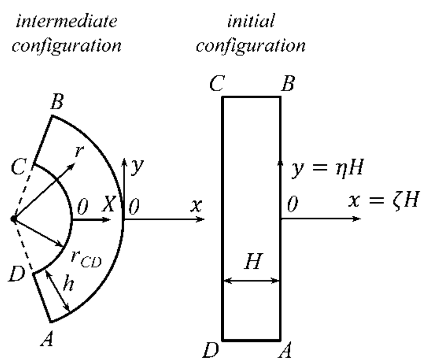

Fig. 1. Initial and intermediate/final configurations.

In general, there are three stages of the process. The entire sheet is elastic at the beginning of the process. The second stage starts when plastic yielding appears at the surface AB (Fig. 1). During this stage, there is one elastic region, $-1 \leq \zeta \leq \zeta_1$, and one plastic region $\zeta_1 \leq \zeta \leq 0$. Here $\zeta = \zeta_1(a)$ is the elastic/plastic boundary. The third stage starts when plastic yielding appears at the surface CD (Fig. 1). During this stage, there is one elastic region, $\zeta_2 \leq \zeta \leq \zeta_1$, and two plastic regions $-1 \leq \zeta \leq \zeta_2$ and $\zeta_1 \leq \zeta \leq 0$. Here $\zeta = \zeta_1(a)$ and $\zeta = \zeta_2(a)$ are the elastic/plastic boundaries. This stage ends when

$d\zeta_2(a)/da = 0$. This condition implies that the thickness of one of the plastic regions has attained its maximum. The solution given in [5] is not valid beyond this point.

The solution in the elastic region is not of interest for the present study since the damage parameter does not change without plastic deformation. The distribution of the equivalent strain in the plastic region during the second stage is given by

$$\varepsilon_{eq}^{p} = \frac{1}{\sqrt{3}}\ln\left[4(\zeta a + s)\right] - k\Phi\left(\varepsilon_{eq}^{pl}\right) \tag{9}$$

where $k = \sigma_0/(3G)$. The dependence of the hydrostatic stress involved in (6) on the equivalent plastic strain is

$$\frac{\sigma}{\sigma_0} = \Psi\left(\varepsilon_{eq}^{p}\right) - \Psi(\varepsilon_1) + \frac{k}{2}\left[\Phi^2\left(\varepsilon_{eq}^{p}\right) - \Phi^2(\varepsilon_1)\right] + \frac{\Phi\left(\varepsilon_{eq}^{p}\right)}{\sqrt{3}}. \tag{10}$$

Here $\Psi\left(\varepsilon_{eq}^{p}\right)$ is an anti-derivative of $\Phi\left(\varepsilon_{eq}^{p}\right)$ and ε_1 is the value of ε_{eq}^{p} at $\zeta = 0$. The solution to the following system of equations determines the dependence of s, ε_1 and ζ_1 on a:

$$\begin{gathered}\ln\left[4(\zeta_1 a + s)\right] = \sqrt{3}k, \quad \sqrt{3}\varepsilon_1 = \ln(4s) - \sqrt{3}k\Phi(\varepsilon_1), \\ \frac{fa}{\sqrt{s-a}} + \frac{1}{6k}\ln^2\left[4(s-a)\right] = \Psi(\varepsilon_1) - \Psi(0) + \frac{k}{2}\Phi^2(\varepsilon_1).\end{gathered} \tag{11}$$

Here f is the tensile force that is supposed to be constant. Once the system (11) has been solved, (9) and (10) provide the dependence of σ and ε_{eq}^{p} on a and ζ (or the dependence of σ and ζ on a and ε_{eq}^{p}).

Equations (9) and (10) are valid in the plastic region $\zeta_1 \le \zeta \le 0$ during the third stage of the process. However, equation (11) is replaced with

$$\begin{gathered}\ln\left[4(\zeta_1 a + s)\right] = \sqrt{3}k, \quad \sqrt{3}\varepsilon_1 = \ln(4s) - \sqrt{3}k\Phi(\varepsilon_1), \\ \ln\left[4(\zeta_2 a + s)\right] = -\sqrt{3}k, \\ \sqrt{3}\varepsilon_2 = -\ln\left[4(s-a)\right] - \sqrt{3}k\Phi(\varepsilon_2), \\ 2\left[\Psi(\varepsilon_2) - \Psi(\varepsilon_1)\right] + k\left[\Phi^2(\varepsilon_2) - \Phi^2(\varepsilon_1)\right] + \frac{2fa}{\sqrt{s-a}} = 0.\end{gathered} \tag{12}$$

The solution of this system supplies the dependence of s, ε_1, ε_2, ζ_1, and ζ_2 on a. Here ε_2 is the value of ε_{eq}^{p} at $\zeta = -1$. The distribution of the equivalent strain in the plastic region $-1 \le \zeta \le \zeta_2$ during the third stage is given by

$$\varepsilon_{eq}^{p} = -\frac{1}{\sqrt{3}}\ln\left[4(\zeta a + s)\right] - k\Phi\left(\varepsilon_{eq}^{pl}\right). \tag{13}$$

The dependence of the hydrostatic stress involved in (6) on the equivalent plastic strain is

$$\begin{gathered}\frac{\sigma}{\sigma_0} = \Psi\left(\varepsilon_{eq}^{p}\right) - \Psi(\varepsilon_2) + \\ \frac{k}{2}\left[\Phi^2\left(\varepsilon_{eq}^{p}\right) - \Phi^2(\varepsilon_2)\right] - \frac{fa}{\sqrt{s-a}} - \frac{\Phi\left(\varepsilon_{eq}^{p}\right)}{\sqrt{3}}.\end{gathered} \tag{14}$$

Once the system (12) has been solved, equations (9), (10), (13), and (14) provide the dependence of σ and ε_{eq}^{p} on a and ζ (or the dependence of σ and ζ on a and ε_{eq}^{p}) in both plastic regions.

4. Damage distribution

Equation (6) can be rewritten as

$$\dot{D} = g\left(\frac{\sigma}{\sigma_{eq}}\right)\xi_{eq}^{p}. \tag{15}$$

Since the $(\zeta, \eta)-$coordinate system is Lagrangian, equation (15) in this coordinate system becomes

$$\frac{\partial D}{\partial t} = g\left(\frac{\sigma}{\sigma_{eq}}\right)\xi_{eq}^{p}. \tag{16}$$

Using (9) and (13) the equivalent plastic strain rate is determined as

$$\xi_{eq}^{p} = \pm\frac{(\zeta + ds/da)}{\sqrt{3}(\zeta a + s)\left[1 + k\Phi'\left(\varepsilon_{eq}^{p}\right)\right]}\frac{da}{dt} \tag{17}$$

where $\Phi'\left(\varepsilon_{eq}^{p}\right) \equiv d\Phi\left(\varepsilon_{eq}^{p}\right)/d\varepsilon_{eq}^{p}$, the upper sign corresponds to the plastic region $0 \ge \zeta \ge \zeta_1$ and the lower sign to the plastic region $\zeta_2 \ge \zeta \ge -1$. Then, substituting (17) into (16) gives

$$\frac{\partial D}{\partial a} = \pm\frac{(\zeta + ds/da)}{\sqrt{3}(\zeta a + s)\left[1 + k\Phi'\left(\varepsilon_{eq}^{p}\right)\right]}g\left(\frac{\sigma}{\sigma_{eq}}\right). \tag{18}$$

It is seen from (7), (10) and (14) that the argument of the function g is rather a simple function of ε_{eq}^{p} and a. One can also eliminate ζ in (18) by means of (9) or (13). As a result, the right hand side of (18) is a function of ε_{eq}^{p} and a. Therefore, it is convenient to use ε_{eq}^{p} as one of the independent variables instead of ζ. In this case, equation (18) becomes

$$\begin{gathered}\frac{\partial D}{\partial a} \pm \frac{(\zeta + ds/da)}{\sqrt{3}(\zeta a + s)\left[1 + k\Phi'\left(\varepsilon_{eq}^{p}\right)\right]}\frac{\partial D}{\partial \varepsilon_{eq}^{p}} = \\ \pm\frac{(\zeta + ds/da)}{\sqrt{3}(\zeta a + s)\left[1 + k\Phi'\left(\varepsilon_{eq}^{p}\right)\right]}g\left(\frac{\sigma}{\sigma_{eq}}\right).\end{gathered} \tag{19}$$

The equation of characteristics is

$$\frac{d\varepsilon_{eq}^{p}}{da}=\pm\frac{(\zeta+ds/da)}{\sqrt{3}(\zeta a+s)\left[1+k\Phi'(\varepsilon_{eq}^{p})\right]} \tag{20}$$

and the relation along characteristics is

$$\frac{dD}{d\varepsilon_{eq}^{p}}=g\left(\frac{\sigma}{\sigma_{eq}}\right). \tag{21}$$

Equations (20) and (21) should be solved numerically using the conditions $\varepsilon_{eq}^{p}=0$ and $D=0$ at $\zeta=\zeta_1$ and $\zeta=\zeta_2$.

5. Illustrative example

Assume that the material obeys Swift's law. Then,

$$\Phi(\varepsilon_{eq}^{pl})=\left(1+\frac{\varepsilon_{eq}^{pl}}{\varepsilon_0}\right)^{n},\quad \Psi(\varepsilon_{eq}^{pl})=\frac{\varepsilon_0}{(1+n)}\left(1+\frac{\varepsilon_{eq}^{pl}}{\varepsilon_0}\right)^{n+1},$$
$$\Phi'(\varepsilon_{eq}^{pl})=\frac{n}{\varepsilon_0}\left(1+\frac{\varepsilon_{eq}^{pl}}{\varepsilon_0}\right)^{n-1}. \tag{22}$$

The numerical solution has been obtained for half-hard aluminum with $n=0.25$ and $\varepsilon_0=0.222$ [9]. In all calculations $k=0.003$. Several widely used representations of the function $g(\sigma/\sigma_{eq})$ have been provided in [1,2]. Consider two functions,

$$g\left(\frac{\sigma}{\sigma_{eq}}\right)=g_1\left(\frac{\sigma}{\sigma_{eq}}\right)=\frac{\sigma}{\sigma_{eq}} \text{ and}$$
$$g\left(\frac{\sigma}{\sigma_{eq}}\right)=g_2\left(\frac{\sigma}{\sigma_{eq}}\right)=\exp\left(\frac{3}{2}\frac{\sigma}{\sigma_{eq}}\right) \tag{23}$$

A qualitative difference between these functions is that the ductile fracture criterion based on the function $g_1(\sigma/\sigma_{eq})$ predicts no change in the damage parameter if $\sigma\le 0$, and the function $g_2(\sigma/\sigma_{eq})$ predicts the increasing change in the damage parameter independently of the value of σ. The through-thickness variation of D for several values of f is depicted in Fig.2 for $g(\sigma/\sigma_{eq})=g_1(\sigma/\sigma_{eq})$ and Fig. 3 for $g(\sigma/\sigma_{eq})=g_2(\sigma/\sigma_{eq})$. In these figures, $X=r-r_{CD}$ (Fig. 1). The solutions illustrated in Figs. 2 and 3 correspond to the instant when $d\zeta_2(a)/da=0$. The region where $D=0$ in Fig 2 combine the elastic region and the plastic region adjusted the surface CD (Fig.1). The region where $D=0$ in Fig. 3 is the elastic region.

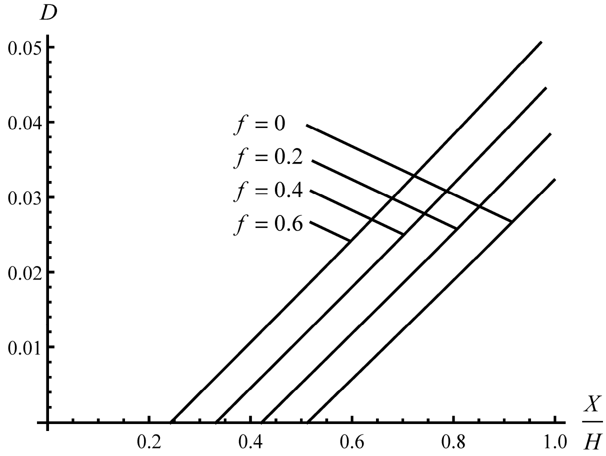

Fig. 2. Through-thickness distribution of the damage parameter for $g(\sigma/\sigma_{eq})=g_1(\sigma/\sigma_{eq})$ (see (23)).

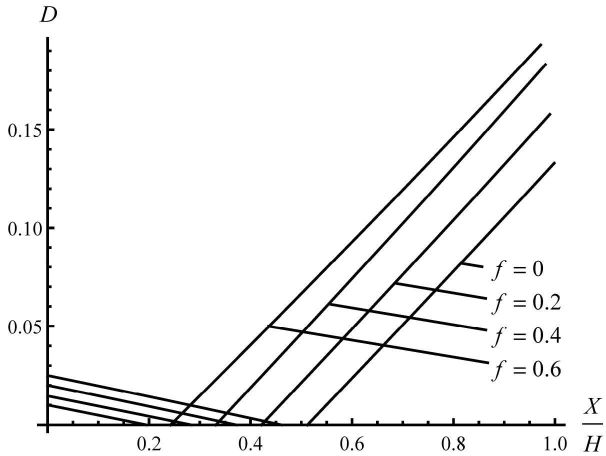

Fig. 3. Through-thickness distribution of the damage parameter for $g(\sigma/\sigma_{eq})=g_2(\sigma/\sigma_{eq})$ (see (23)).

6. Conclusions

Presented is a new semi-analytic elastic/plastic solution for the process of bending under tension at large strains. The solution accounts for an arbitrary strain-hardening law and an arbitrary uncoupled damage evolution equation that is independent of the third invariant of the stress tensor. Numerical treatment reduces to solving one partial differential equation in two variables (equation (19)). The general solution is illustrated for Swift's hardening law and two damage evolution equations. The final results shown in Figs. 2 and 3 are in qualitative agreement with physical expectations. In particular, the value of D is the highest at the surface $\zeta=0$. Moreover, the value of D increases at the surface $\zeta=0$ (Figs. 2 and 3) and decreases at the surface $\zeta=-1$ (Fig. 3) as f increases.

References

[1] Atkins A. Fracture in forming. J Mater Process Tech 1996;56:609-18.

[2] Hambli R, Reszka M. Fracture criteria identification using an inverse technique method and blanking experiment. Int J Mech Sciences 2002;44:1349–61.
[3] Hosford WF, Caddell RM. Metal forming: mechanics and metallurgy. Englewood Cliffs: Prentice Hall; 1950.
[4] Bruhns OT, Gupta NK, Meyers ATM, Xiao H. Bending of an elastoplastic strip with isotropic and kinematic hardening. Arch Appl Mech 2003;72:759-78.
[5] Alexandrov S, Manabe K, Furushima T. A general analytic solution for plane strain bending under tension for strain-hardening material at large strains. Arch Appl Mech 2011;81:1935-52.
[6] Alexandrov S, Gelin J-C. Plane strain pure bending of sheets with damage evolution at large strains. Int J Solids Struct 2011;48:1637-43.
[7] Hartley P, Hall FR, Chiou JM, Pillinger I. Elastic-plastic finite-element modelling of metal forming with damage evolution. In: Predeleanu M., Gilormini P, editors. Advanced methods in materials processing defects. Elsevier, Amsterdam; 1997. p. 135-142.
[8] Xiao H, Bruhns OT, Meyers A Elastoplasticity beyond small deformations. Acta Mech 2006;182:31-111.
[9] Rees DWA. Basic engineering plasticity. Butterworth-Heinemann: Oxford; 2006.