Neural Network Algorithm for Forecasting and Parameter Estimation of the Coriolis Vibratory Gyroscope

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STATEMENT OF THE PROBLEM

The problem is to identify the dynamic parameters of the Coriolis Vibratory Gyroscope (CVG) operation (angular rate, angular acceleration) and the parameters of the standing wave drift caused by inhomogeneities in the physical characteristics of the resonator material.

PRINCIPLE OF OPERATION

George Bryan's effect - precession of elastic standing waves in axisymmetrical shells.

Fig. 1. The "wine-glass" or hemispherical resonator gyroscope (HRG)

Bryan, G. H., On the beats in the vibrations of a revolving cylinder or bell // Proc. Camb. Phil. Soc. Math. Phys Sci, Vol. 7, 101–111, 1890.

A standing wave is excited in the resonator along the 2nd form of plane vibrations, characterized by 4 nodes and 4 antinodes alternating through 45°.

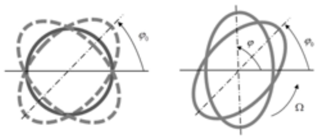

Fig. 2. The second vibrational mode of the CVG and the wave precession

When an angular velocity Ω occurs, the initial wave rotates over time t through an angle, with a coefficient of 0.4 (the "Bryan factor") proportional to the angle of rotation of the basement.

$$\varphi(t) = \varphi_0 - \frac{2}{5}\int_0^t \Omega(s)ds$$

MATHEMATICAL MODEL

The PDE of free vibrations of an imperfect resonator:

$$\ddot{w}'' - \ddot{w} + 4\Omega\dot{w}' + \left[\kappa^2(w''+w)\right]^{IV} + \left[\kappa^2(w''+w)\right]''$$
$$+\xi\left[\kappa^2(\dot{w}''+\dot{w})\right]^{IV} + \xi\left[\kappa^2(\dot{w}''+\dot{w})\right]'' = 0,$$

where $w(\varphi,t)$ is the radial movement of a point on the edge; Ω is the angular rate; $\kappa^2 = EJ\,(\rho SR^4)^{-1}$; ρ is the density; S is the cross-section area; R is the radius of the middle neutral line; E is Young's modulus of the material; $J=h^4/12$ is the moment of inertia of the cross section relative to the neutral axis; h is the thickness; ξ is the decay time characterizing the Q-factor.

Physical parameters are inhomogeneous in angle:

$$\kappa^2 = \kappa^2(\varphi), \quad \rho = \rho(\varphi).$$

The presence of 4th harmonics of the damping defect and elastic mass anisotropy

$$\xi = \xi_0 + \Delta_c^{(1)}\cos 4\varphi + \Delta_s^{(1)}\sin 4\varphi,$$
$$\kappa^2 = \kappa_0^2 + \Delta_c^{(2)}\cos 4\varphi + \Delta_s^{(2)}\sin 4\varphi$$

yields the frequency split:

$\omega_1 < \omega_0 < \omega_2$ ($\omega_0 = 6\kappa/\sqrt{5}$)

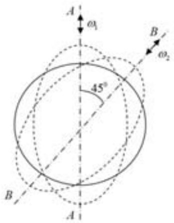

Fig. 3. Frequency split

Solution is found in the form

$$w(\varphi,t) = p(t)\sin(2\varphi) + q(t)\cos(2\varphi)$$

To identify the parameters of the 4th harmonics of defects, one should consider the resonator on a fixed basement (Ω=0). This leads to the system of ODEs:

$$\ddot{p} + \omega_0^2\left(\xi_0 + \Delta_c^{(1)}\right)\dot{p} + \Delta_s^{(1)}\dot{q} + \left(\omega_0^2 + \frac{36}{5}\Delta_c^{(2)}\right)p + \frac{36}{5}\Delta_s^{(2)}q = 0$$
$$\ddot{q} + \omega_0^2\left(\xi_0 - \Delta_c^{(1)}\right)\dot{q} + \Delta_s^{(1)}\dot{p} + \left(\omega_0^2 - \frac{36}{5}\Delta_c^{(2)}\right)q + \frac{36}{5}\Delta_s^{(2)}p = 0 \quad (*)$$

The problem of identification:
given the values of time series $p_n=p(t_n)$, $q_n=q(t_n)$ for a time interval $[0,T]$, evaluate all the model parameters

$$\omega_0^2,\ \xi_0,\ \Delta_c^{(1)},\ \Delta_s^{(1)},\ \Delta_c^{(2)},\ \Delta_s^{(2)} \quad (**)$$

SOLUTION OF THE PROBLEM: NNAR

Single-layer linear neural autoregression network (NNAR) for time series forecast:

$$p_j^* = \sum_{i=1}^m w_{1,2i-1}p_{j-i} + \sum_{i=1}^m w_{1,2i}q_{j-i}$$
$$q_j^* = \sum_{i=1}^m w_{2,2i-1}p_{j-i} + \sum_{i=1}^m w_{2,2i}q_{j-i} \quad (***)$$

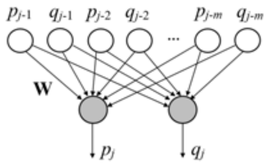

Fig. 4. NNAR

Finding the weights of NNAR: Widrow-Hoff (delta) rule.

The shortened forecast model:

$$p_j^* = \sum_{i=1}^m w_i p_{j-i} \quad (****)$$

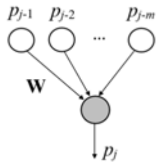

Fig. 5. Simplified NNAR

Identification algorithm

1.Replace the time derivatives in (*) by finite differences of the m–th order, obtaining an FD-approximation of the system of ODEs (*).

2.Represent the extreme right values of the grid functions (p_j, q_j) through the previous values (p_{j-k}, q_{j-k}) in a form (***).

3.Comparing both representations, express the desired parameters (**) in terms of weight coefficients w_{1i}, w_{2i}.

NUMERICAL EXPERIMENT (IMPERFECT CVG)

Let in (*) be $\Delta_c^{(1)} = 0.0114,\ \Delta_s^{(1)} = 0.1462;\ \Delta_c^{(2)} = 0,\ \Delta_s^{(2)} = -0.0154.$

The length of the forecast window m=4, no. of training eras: 50,000.

Weighting coefficients: $w_1 = -0.5231,\quad w_2 = -0.063,\quad w_3 = 0.4042,\quad w_4 = 0.8109.$

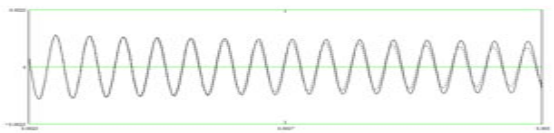

Fig. 6. Forecasting the time series for the imperfect CVG on interval (T,2T)

NUMERICAL EXPERIMENT. PERFECT CVG

Consider the case: $\kappa^2 = \kappa_0^2 = \text{const},\ \xi = \xi_0 = \text{const}$
(perfect resonator)

The system of ODEs (*):

$$\ddot{p} + \omega_0^2\xi_0\dot{p} + \omega_0^2 p = 0,$$
$$\ddot{q} + \omega_0^2\xi_0\dot{q} + \omega_0^2 q = 0.$$

Due to the fact that the equations are independent, the forecast is carried out according to scheme (****). For example, for the length of the forecast window m=2, we have for p

$$p_j^* = w_1 p_{j-1} + w_2 p_{j-2}$$

Using the s

$$\ddot{p}(t_{j-1}) \approx \frac{p_j - 2p_{j-1} + p_{j-2}}{\tau}, \quad \dot{p}(t_{j-1}) \approx \frac{p_j - p_{j-2}}{2\tau}.$$

we obtain

$$\boxed{w_1 = \frac{2-\omega_0^2\tau^2}{0,5\omega_0^2\xi_0\tau+1}, \quad w_2 = \frac{0,5\omega_0^2\xi_0\tau-1}{0,5\omega_0^2\xi_0\tau+1}}$$

Physical parameters:
- oscillation frequency ω_0=38598.5 rad / s;
- attenuation factor $\xi_0=2\cdot10^{-7}$ s.

Parameters of computational experiment:
- sampling step τ=1·10-5 s;
- training interval length T=0.0025 s (M = 250 samples);
- learning rate η=1.

In order to achieve a forecast quadratic error of less than 0.1 % on the interval (T, 4T), it was necessary to implement 250,000 training eras. The obtained values of the weights:

w_1=-1, w_2=0.851.

The relative error in calculating ω_0 and ξ_0 is less than $1\cdot10^{-7}$.
Increase in the window length m leads to decrease in the number of epochs providing the same forecast accuracy.

CONCLUSION

1. Parameters of an imperfect CVG are successfully estimated based on the linear neural network autoregression models.
2. The method allows one to get an explicit interpretation of the weight coefficients of the forecast model of the two-dimensional time series of SVG dynamics.
3. NNAR model also allows predicting the dynamics of oscillations, which is very important in optimizing inertial navigation algorithms.