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Tokamak plasma models development for plasma magnetic control systems design by first principle equations and identification approach

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Abstract

To design plasma magnetic control systems in modern tokamaks one needs models of the plasma. These models are linear parameter varying (LPV) because of relatively small variations of outputs of the feedback loops around scenarios and may be obtained by first-principle equations, identification approach or by their combinations. The challenge of obtaining models of plasma in the tokamak becomes more complicated by the existence of plasma position and poloidal field control loops with diagnostics and actuators which are needed as inner cascades for plasma current and shape control in particular on the Globus-M2 tokamak (Ioffe Inst., St. Petersburg, RF). The plasma equilibrium is reconstructed on the base of magnetic measurements outside the plasma by Picard iterations, moving filaments or neural networks, and linear plasma models are developed around the equilibrium with the help of the Kirchhoff's law and force balance. In order to ensure the operability of plasma current and shape feedback control systems, the identification approach (controlled plant model design on the base of experimental input-output signals) is planned to be used. The basic methods of the identification are supposed to be applied as follows: subspaces, wavelets, linear matrix inequalities (LMIs), adaptive state observers, and dynamical neural networks which are able to automatically adjust their states to an unknown plant (elements of artificial intelligence). The solutions of the identification problem will be compared with the models obtained by the first principles with the aim to get the sufficient accuracy of coincidence. The approaches to be developed of getting tokamak plasma models may be applied to any vertically elongated (D-shaped) operating tokamak such as Globus-M2 (RF), D-IIID, NSTX (US), JET, ST40 (GB), ASDEX Upgrade (Germany), TCV (Switzerland), EAST (China), Damavand (Iran) etc.

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1. Introduction

1.1. State-of-the-art

At present, the challenge of the controlled fusion problem is supposed to be solved on the base of tokamak-reactors [1]. Modern tokamaks have a common feature: plasma in them is vertically elongated to get higher plasma pressure at the same toroidal field. In spite of that, all modern tokamaks have different poloidal systems. From all the variety of designs of modern tokamaks with D-shaped cross-sections, the following groups can be conventionally distinguished [2,3] in line with their poloidal systems:

- "warm" (non-superconducting) poloidal field (PF) coils located inside toroidal field (TF) coil, vertical position control coil is outside the vacuum vessel (VV): NSTX (National Spherical Torus Experiment), DIII-D (both are in the USA), JT-60U (Japan Torus-60U, Japan), TCV (Variable Configuration Tokamak, Switzerland);
- "warm" (non-superconducting) PF coils located outside the TF coil, vertical position control coils are outside the VV: JET (Joint European Torus, UK), ASDEX-Upgrade (Germany), Damavand (Iran) (Fig. 1(a));
- "warm" (non-superconducting) PF coils, a part of the coils is located outside the TF coil, the other part is inside the TF coil, the vertical and horizontal positions control coils are outside the VV: Globus-M2 (RF) (Fig. 1(b));
- "warm" (non-superconducting) PF coils are located inside the VV: MAST-U (Mega Ampere Spherical Tokamak, UK);
- superconducting PF coils are located outside the TF coil, vertical position control coils are located inside the VV: EAST (Experimental Advanced Superconducting Tokamak, China), KSTAR (Korean Superconducting Tokamak Reactor, South Korea), ITER (International Thermonuclear Experimental Reactor, France). In the KSTAR tokamak and in the JT-60SA tokamak project the horizontal plasma position control coil is additionally placed inside the VV, which improves the plasma control efficiency.

1.2. Motivation

Because poloidal systems of tokamaks are different, they have different plasma magnetic automatic control systems [2, 3]. These control systems, as well as the D-shaped tokamaks, have a common feature: they are based on linear plasma model-based design. In this case, linear plasma models are used because the plasma is inside feedback loops and its output variations around references are relatively small [3, 4]. So, obtaining reliable LPV plasma models is necessary for the design of reliable plasma magnetic control systems with a high level of performance and stability margins.

1.3. Novelty

In this paper the methodology of plasma control systems [4] is planned to be developed further in the sense of application of a new direction of the most advanced identification approaches to obtain plasma linear models and compare them with linear models to be obtained by first principle equations approach.

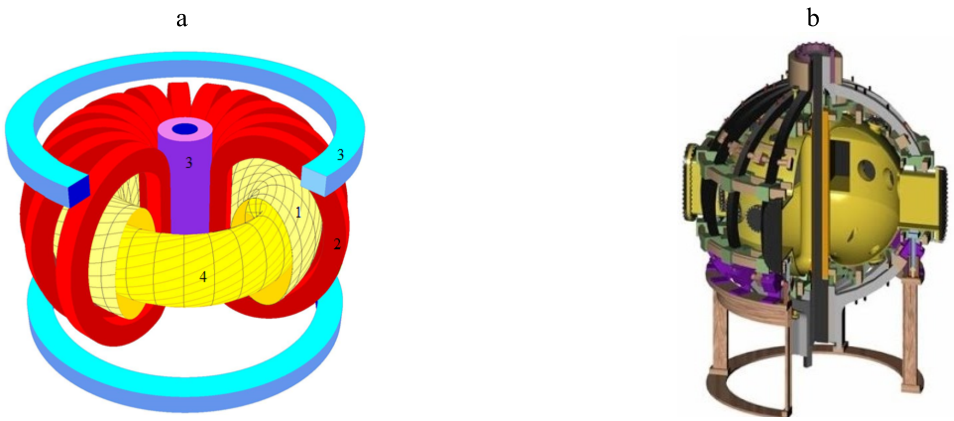

Fig. 1. Tokamaks: (a) General scheme of an elongated tokamak with the air central solenoid (CS), (1 - vacuum vessel; 2 - TF-coils; 3 - CS&PF-coils; 4 - plasma with magnetic screw lines); (b) Globus-M2

2. Plasma equilibrium reconstruction methods based on magnetic measurements outside the plasma

2.1. Plasma equilibrium in a tokamak

In order to construct a linear model of a tokamak plasma from first principle equations one should first obtain a corresponding magnetohydrodynamic equilibrium of the plasma, i.e. plasma current and magnetic flux distribution. These plasma parameters cannot be measured directly and must be obtained from magnetic measurements taken outside the plasma. The problem of identifying the plasma equilibrium from magnetic measurements of a tokamak is called plasma equilibrium reconstruction.

A useful function for describing plasma position and shape is the distribution of the poloidal magnetic flux $\psi_p(r,\, z)$, as the plasma is contained within the largest closed isoline of the poloidal flux. The poloidal magnetic field $\vec{B}_p \equiv [B_r,\, B_z]$ can be calculated from the poloidal flux as $\vec{B}_p = \nabla\psi_p \times \nabla\varphi$ and can be used to calculate a force enacted on the plasma by the magnetic field: $\vec{F} = 2\pi\int r\vec{J}\times\vec{B}dS$. Here r, φ , z are cylindrical coordinates. From the Maxwell's equation the differential equation for the poloidal flux in the tokamak is derived [1]: $\Delta^*\psi_p \equiv r\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}\psi_p) + \frac{\partial^2}{\partial z^2}\psi_p = -\mu_0 r J_\varphi$. The solution of this equation depends on density of toroidal plasma current J_φ . Two main approaches to plasma equilibrium reconstruction problems are described further: Picard iterations and moving filaments methods.

2.2. Picard iteration method

In the Picard iterations method [5,6] the plasma current density is modelled as a continuous function of coordinates r, z. The force balance equation is used to express toroidal current density in the form: $J_\varphi = r\frac{d}{d\psi_p}p(\psi_p) + \frac{1}{2\mu_0 r}\frac{d}{d\psi_p}F^2(\psi_p)$, where p is the plasma pressure and F describes the toroidal magnetic field: $F \equiv rB_\varphi$. If functions p and F are known, then poloidal flux can be found through the iterative process $\Delta^*\psi_p^{(m+1)} = -\mu_0 r J_\varphi(r, \psi_p^{(m)})$. However, in the equilibrium reconstruction problem functions p and F are usually not known beforehand and are approximated as polynomial functions of the poloidal flux with coefficients of the polynomials found by minimization of the errors between measured and calculated values of the poloidal flux outside the plasma and the total plasma current.

2.3. Moving filaments method

The method of plasma modeling using filaments [7] is the approximation of the toroidal plasma current distribution by a set of ring-filament currents: $J_\varphi(\vec{r}) = \sum_{m=1}^{M} I_m \delta(\vec{r} - \vec{r}_m)$, where I_m is the current in the *m*-th filament, $\vec{r}_m$ is the coordinate of the *m*-th filament, *M* is the number of filaments, $\delta(\vec{r} - \vec{r}_m)$ is the Dirac delta function. As a result of this approximation, the solution of the poloidal flux distribution equation for the plasma can be found using the Green's function method: $\psi_p(\vec{r}) = \sum_{m=1}^{M} I_m G(\vec{r}_m, \vec{r})$, where *G* is the Green's function. The Improved Moving Filaments method [8] was developed for the non-iterative simultaneous determination of coordinates and currents of filaments using magnetic measurements outside the plasma. The possibility of real-time application of this method in the plasma shape feedback loop for the Globus-M/M2 tokamak was demonstrated [9].

2.4. Neural networks method

To estimate plasma parameters that need to be controlled, but which are not available for direct measurements, an approach using an artificial neural network (ANN) can be applied, taking into account that there is enough data to train the ANN. The use of the ANN as the magnetic signal's estimator for plasma shape control in the Globus-M/M2 tokamak was shown [10]. Here, for training the ANN, the dataset was created based on the developed numerical Tokamak Plasma Magnetic Evolution Code (TOPMEC) [11]. The task of the ANN is a nonlinear mapping of the input vector $\vec{x}$ consisting of the plasma current, currents in the CS&PF-coils, the poloidal fluxes and the VV current to the output vector $\vec{y}$ corresponding to magnetic signals that determine the plasma shape.

3. LPV plasma models design by first principle equations

Evolution of plasma position and currents in the tokamak is described by the Faraday's law $\frac{d}{dt}\Psi(J_\varphi, \psi_p) + RI = U$ and motion equations $m\frac{d^2}{dt^2}\vec{r}_p = \vec{F}(J_\varphi, \psi_p)$. The plasma shape is described by the gaps between the plasma surface and the VV $g(J_\varphi, \psi_p)$. Here $I \equiv [I_c^T, I_v^T, I_p]^T$ is the vector of currents in tokamak coils, elements of VV, and plasma, Ψ, *U,* and *R* are vectors of the magnetic flux through these circuits, the voltage applied to them, and the diagonal matrix of circuits electrical resistance, respectively, $\vec{r}_p \equiv [r_p, z_p]$ is coordinates of the plasma centre of a mass, *m* is the plasma mass. These equations are to be linearized around the reconstructed plasma equilibrium, characterized by distributions of the toroidal current density J_φ and the poloidal flux ψ_p:

$$M(J_\varphi, \psi_p)\frac{d}{dt}\delta I + R\delta I + \frac{\partial}{\partial \vec{r}_p}\Psi(J_\varphi, \psi_p)\frac{d}{dt}\delta\vec{r}_p = \delta U, \quad m\frac{d^2}{dt^2}\delta\vec{r}_p = \frac{\partial}{\partial I}\vec{F}(J_\varphi, \psi_p)\delta I + \frac{\partial}{\partial \vec{r}_p}\vec{F}(J_\varphi, \psi_p)\delta\vec{r}_p,$$
$$\delta g = \frac{\partial}{\partial I}g(J_\varphi, \psi_p)\delta I + \frac{\partial}{\partial \vec{r}_p}g(J_\varphi, \psi_p)\delta\vec{r}_p.$$

In modern tokamaks, plasma configurations are usually unstable in regard to vertical displacements and stable in regard to radial displacements. Therefore, small plasma mass may be neglected in radial motion equation and radial displacement δr_p can be expressed through vertical displacement δz_p and current disturbances δI. Introducing state vector $x \equiv [\delta I^T, \delta z_p, \delta \dot{z}_p]^T$, input vector $u \equiv \delta U$, and output vector $y = [\delta r_p, \delta z_p, \delta I_p, \delta I_c^T, \delta g^T]^T$, the model equations take a state space form: $\dot{x} = A(t)x + B(t)u,\ y = C(t)x.$

The plasma equilibrium may significantly change during a tokamak discharge and therefore matrices A, $B,$ and C are dependent on time. One way to obtain them is to reconstruct sequence of plasma equilibria, calculate corresponding time-invariant matrices, and interpolate them, resulting in a linear time-varying model (LPV).

4. Identification approaches based on input-output experimental data

4.1. Subspaces method

The difference equations of a discrete linear time-invariant system and their solution have the following form,

$$\begin{cases} x(k+1) = Ax(k) + Bu(k), \\ y(k) = Cx(k) + Du(k), \end{cases} \quad x(k) = A^k x(0) + \sum_{i=0}^{k-1} A^{k-i-1} Bu(k).$$

To estimate the matrices of the system model and the initial value of the state vector with accuracy to the similarity transformation by a finite number of values of input and output sequences the identification procedure [12] is used. The subspace method [12,13] of the system identification is based on the representation of the relationship between the measured data and the system model matrices as follows, $Y_{0,s,N} = O_s X_{0,N} + T_s U_{0,s,N}$, where $n < s << N$, O_s is the extended observability matrix and

$$Y_{0,s,N} = \begin{bmatrix} y(0) & \cdots & y(N-1) \\ \vdots & \ddots & \vdots \\ y(s-1) & \cdots & y(N+s-2) \end{bmatrix}, \; O_s = \begin{bmatrix} C \\ \vdots \\ CA^{s-1} \end{bmatrix}, \; X_{0,N} = \begin{bmatrix} x(0) \\ \vdots \\ x(n-1) \end{bmatrix}^{\mathrm{T}},$$
$$T_s = \begin{bmatrix} D & \cdots & 0 \\ \vdots & \ddots & \vdots \\ CA^{s-2}B & \cdots & D \end{bmatrix}, \; U_{0,s,N} = \begin{bmatrix} u(0) & \cdots & u(N-1) \\ \vdots & \ddots & \vdots \\ u(s-1) & \cdots & u(N+s-2) \end{bmatrix}.$$

By knowing T_s and subtracting $T_sU_{0,s,N}$ from $Y_{0,s,N}$, one can define the O_s column space (its linear span, the intersection of all linear subspaces containing every column-vector in the set) and define the system matrices. The matrix T_s is estimated from the solution of the following least squares problem,

$$\min_{T_s} \left\| Y_{0,s,N} - \hat{T}_s U_{0,s,N} \right\|_F^2, \; where \; \|A\|_F = \sqrt{\sum_{i=1}^{s} \sum_{j=1}^{N} \left| a_{i,j} \right|^2}$$

is the Frobenius norm of matrix $A^{s \times N}$. When the input data provide the full rank of $U_{0,s,N}$,the solution has the form,

$$\hat{T}_s = Y_{0,s,N} U_{0,s,N}^{\mathrm{T}} \left(U_{0,s,N} U_{0,s,N}^{\mathrm{T}} \right)^{-1} \quad \Rightarrow \quad Y_{0,s,N} - \hat{T}_s U_{0,s,N} = Y_{0,s,N} \Pi_{U_{0,s,N}}^{\perp},$$

where the matrix of the orthogonal projection of a vector-row onto the column space of the matrix $U_{0,s,N}$ has the form,

$$\Pi_{U_{0,s,N}}^{\perp} = I_N - U_{0,s,N}^{\mathrm{T}} \left(U_{0,s,N} U_{0,s,N}^{\mathrm{T}} \right)^{-1} U_{0,s,N}, \; and \; U_{0,s,N} \Pi_{U_{0,s,N}}^{\perp} = 0.$$

The expression, $Y_{\Pi} = Y_{0,s,N} \Pi_{U_{0,s,N}}^{\perp} = O_s X_{0,N} \Pi_{U_{0,s,N}}^{\perp}$, shows that the column space of Y_Π is contained in the column space of O_s, and if the measured signal sequences provide $rank(Y_\Pi) = n$, then these spaces coincide. Thus, it is possible to estimate the column space of the extended observability matrix and calculate the system matrices from Y_Π with accuracy to some similarity transformation F:

$$O_s F = \left[CF;\ CF(F^{-1}AF);\ \ldots\ CF(F^{-1}AF)^{s-1} \right]$$

The system matrices B, D, and initial state $x(0)$ are calculated from the solution of the least squares problem.

4.2. Wavelets method

Wavelets-transformation of 1-dimentional signal is representation of this signal as generalized series or Fourier integral over a system of basic functions. The wavelet is a localized and fast damping wave and it is characterized by scale (frequency) and time localization by operations of shifting and changing time scale. Time scale is similar to an oscillation period, the shift is a displacement of the signal in time. In compassion with the Fourier transformation wavelets represent not only frequency domain but represent properties of the signal in frequency domain by time-scaling and time domain by time shifting. Mother (initial) wavelet $\psi(t)$ should have the finite norm: $\int_{-\infty}^{\infty} |\psi(t)|^2\, dt < \infty$, and a mean value equals to zero: $\int_{-\infty}^{\infty} \psi(t)dt = 0$. Time scale and time shift change mother wavelet by: $\psi_{ab}(t) = \frac{1}{\sqrt{a}}\psi(\frac{t-b}{a})$ or in the discrete case: $\psi_{mn}(t) = \frac{1}{\sqrt{a^m}}\psi(\frac{t-nb}{a^m})$. The discrete wavelet-transformation is a signal representation as series:

$$x(t) = \frac{1}{K_\psi} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} c_{mn}\psi_{mn}(t), \qquad K_\psi = 2\pi \int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega,\ c_{mn} = \int_{-\infty}^{\infty} x(t)\psi^*_{mn}(t)dt.$$

The continuous wavelet-transformation is the signal representation as an integral:

$$x(t) = \frac{1}{K_\psi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \gamma(a,b)\psi_{ab}(t)\frac{dadb}{a^2}, \quad K_\psi = 2\pi \int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega, \qquad \gamma(a,b) = \int_{-\infty}^{\infty} x(t)\psi^*_{ab}(t)dt.$$

Here $\Psi(\omega)$ is the Fourier representation of $\psi(t)$, $\psi^*(t)$ is a complex conjugate of $\psi(t)$. The discrete wavelets-transformation could be useful for system identification [14,15]. A model in the discrete state-space form could be tuned using the wavelet transform of the difference between an output signal of the model and plant or using the difference between the wavelet transformed output signal of the model and the output signal of the model under influence of the wavelet transformed input signal [15]. The model represents sum of models in the special form:

$$M_{i,j}(z) = P_{i,j}(z)Q_{i,j}(z),\ P_{i,j}(z) = \left(\frac{1}{1-z^{-1}}\right)^{s_{i,j}},\ s_{i,j} \in \mathbb{Z},\ Q_{i,j}(z) = \alpha_{i,j} + \beta_{i,j}z^{-1},\ \alpha_{i,j}, \beta_{i,j} \in \mathbb{R}.$$

In this way each model corresponds to a different frequency. Parameters α and β are tuned for each model with different values of parameter s and the best set is chosen. For choosing the best parameters for each model the mean-square error between the model output and the plant measured output could be used.

4.3. Linear Matrix Inequalities (LMIs) method

The proposed method [16] is based on the fact that the problem of system identification for ARX models (autoregressive with exogenous terms) can be formulated as a problem of LMI optimization. Assume that there are $N+1$ samples of input and output data of the plant P_{ARX} denoted by $u(k)$ and $y(k)$, $k = 0,...,N$ from a single identification experiment. In the ARX model the prediction error $e(k,\theta)$ is given by

$$e(k,\theta) = D_{ARX}(z)y(k) - N_{ARX}(z)u(k) = y(k) - D_{IO}(k)\theta,\ \ P_{ARX} = \frac{N_{ARX}(z)}{D_{ARX}(z)},\ \ \theta = [a_1\ \dots\ a_n\, b_1\ \dots\ b_n]^T,$$

$$D_{IO}(k) = \left[-D_O(k)\ \ D_I(k)\right],\quad D_O(k) = \left[y(k-n)\ \dots\ y(k-1)\right],\ \ D_I(k) = \left[u(k-n)\ \dots\ u(k-1)\right],$$
$$u(k-j) = y(k-j) = 0,\quad \forall k-j<0,\ \ j = 1,\dots,n.$$

The objective function for the system identification, the sum of the squared prediction errors, is given as

$$J_p = \sum_{k=1}^{N} e^2(k,\theta) = \sum_{k=1}^{N} (y(k) - D_{IO}(k)\theta)^2.$$

Using the Schur complement lemma, we can present this problem as an LMI problem. Optimal solution of the least squares problem can be obtained as a solution vector θ_{opt}, which is the solution of the LMI optimization problem:

$$\min\ f\left(\alpha(k)\right) = \sum_{k=1}^{N} \alpha(k),\ \text{s.t.}\ \alpha(k) > 0,\ \begin{bmatrix} \alpha(k) & y(k) - D_{IO}(k)\theta \\ y(k) - D_{IO}(k)\theta & 1 \end{bmatrix} > 0.$$

This method can be extended for use with the Windsurfer-Like Approach [17] to obtain MIMO LPV models using LMI.

4.4. Adaptive state observer method

The plasma movement in the tokamak along horizontal direction is described by the 1st order differential equation with additive disturbance $\dot{x}_p(t) = -a(t)x_p(t) + b(t)\left[I_c(t) + w(t)\right], \dot{w}(t) = \mu(t), \dot{\mu}(t) = 0$ where $x_p, I_c \in \mathbb{R}$ are plasma displacement and current in the vertical field coil respectively, a and b are parameters, w is the disturbance and μ is its speed [19]. The observer equation in the state space form is $\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}I_c + K\left(y - C\hat{x}\right)$ where $\hat{x} = \left[\hat{x}_p\ \ \hat{w}\ \ \hat{\mu}\right]^T$ is the vector of state estimation, $K = \left[K_1\ \ K_2\ \ K_3\right]^T$, matrices $\hat{A}, \hat{B}$ are the same as matrices A, B of the original plant model namely $\dot{x} = Ax + BI_c, y = Cx, A = \left[-a, b, 0; 0, 0, 1; 0, 0,\right], B = \left[b, 0, 0\right]^T$, $C = \left[1\ 0\ 0\right], x = \left[x_p\ \ w\ \ \mu\right]^T$ where x the plant model state vector, y is the output signal and y=x_p. To tune the parameters $\hat{a}, \hat{b}$ to a,b the criterion is introduced $Q = \left(x_p - \hat{x}_p\right)^2$ and the parameter estimate vector $\chi = \left[\hat{a}\ \ \hat{b}\right]^T$ should be changed along antigradient of Q:, $\dot{\chi} = -\Gamma\nabla_{\hat{a},\hat{b}}Q = -\Gamma\left[\partial Q/\partial\hat{a}\ \ \partial Q/\partial\hat{b}\right]^T$, $\Gamma = diag\left[\lambda_a\ \ \lambda_b\right]$. Then with these designations $\dot{\hat{a}} = 2\lambda_a\left(x_p - \hat{x}_p\right)\alpha_1$, $\dot{\hat{b}} = 2\lambda_b\left(x_p - \hat{x}_p\right)\alpha_2$ .The sensitivity functions $\alpha_1 = \partial\hat{x}_p/\partial\hat{a}, \beta_1 = \partial\hat{w}/\partial\hat{a}$, $\gamma_1 = \partial\hat{\mu}/\partial\hat{a}$, $\alpha_2 = \partial\hat{x}_p/\partial\hat{b}, \beta_2 = \partial\hat{w}/\partial\hat{b}, \gamma_2 = \partial\hat{\mu}/\partial\hat{b}$ give the set of differential equations:

$$\begin{aligned}\dot{\alpha}_1 &= -\hat{x}_p - \left(\hat{a} + K_1\right)\alpha_1 + \hat{b}, \dot{\beta} = \gamma_1 - K_2\alpha_1, \dot{\gamma}_1 = -K_1\alpha_1; \dot{\alpha}_2 = \\ &= -\left(\hat{a} + K_1\right)\alpha_2 + \hat{b}\beta_2 + I_c + \hat{w}, \dot{\beta}_2 = \gamma_2 - K_2\alpha_2, \dot{\gamma}_2 = -K_3\alpha_2\end{aligned}$$

This approach of on-line identification of the additive disturbance w and two plasma parameters a,b was applied in experiments on the Tuman-3 tokamak (Ioffe Inst.) in the self-oscillations adaptive control system of the horizontal plasma position [19].

4.5. Dynamical neural networks method

The ideology of neuro control is a complete antipode of classical control theory and robust control here should be interpreted more widely than the uncertainties in the mathematical model, because it is not used. One of the advances in this direction is the development of dynamic neural networks (DNN), in particular, the synthesis of such neural networks uses methods for the synthesis of dynamic control systems [20]. The DNN is a dynamic system described by the system of ordinary differential equations, in which on the right side there is a sum of linear and nonlinear components together with nonlinear activating functions (σ, φ), which depend on a DNN state vector $\hat{x}$:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + W_\sigma(t)\sigma(\hat{x}(t)) + W_\varphi(t)\varphi(\hat{x}(t))u(y(t)).$$

The two nonlinear summands on the left depend on weight matrices ($W_\sigma(t)$, $W_\varphi(t)$). There are different learning methods which are used to develop models of processes, while adopting the network to the changing environment and discovering useful knowledge, namely, supervised, unsupervised, reinforcement learning (RL). For the application of the DNN in the control systems of dynamic plants the observer DNNO is constructed by analogy with the Kalman's filter and Luenberger's observer. For the quasi-linear dynamic system:

$$\dot{x}(t) = Ax(t) + Bu(x(t),t) + \xi(x(t),t), \;\; y(t) = Cx(t) + \eta(t),$$

where ξ is an external perturbation, η is a noise in the output, A, B are known matrices, C is a given output matrix, the DNNO have the following structure:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu + W_\sigma(t)\sigma(\hat{x}(t)) + W_\varphi(t)\varphi(\hat{x}(t))u + L[y(t) - C(\hat{x}(t))].$$

Two nonlinear terms are introduced into the Luenberger observer, which depend on the weight functions, sigmoidal functions, and input vector. The RL process consists in the realization of an adequate adjustment of the weight matrixes $W_\sigma(t)$ and $W_\varphi(t)$ in such a way that the current state estimates $\hat{x}(t)$ would be as closed as possible to the current real state $x(t)$ of the modelled real-life system:

$$\dot{W}_\sigma(t) = \Phi_\sigma(W_\sigma(t), \hat{x}(t), u(y(t))), \;\; \dot{W}_\varphi(t) = \Phi_\varphi(W_\varphi(t), \hat{x}(t), u(y(t))).$$

This reflects the proximity of the current outputs of the applied DNN to a desired trajectory realizing the tracking-process. The number of artificial neurons is defined by the number of activating functions σ, φ components, which in fact determines the complexity of the used DNN.

5. Conclusion

Today, as many decades ago, tokamak plasma [1] remains a very complex controlled plant with difficulties to identify properties, subject to non-universal empirical patterns of behavior, strongly dependent on the modes of installation systems in general and discharge scenarios. Therefore, the use of "magnetic-kinetic" codes and first principle equations approach are not enough to obtain plasma models suitable for reliable plasma magnetic control systems design and their application [2, 3]. For this reason, identification methods, both from the observation data obtained and during the operation of control systems, are becoming increasingly necessary.

In the paper a short survey has been presented of off-line and on-line identification methods applied already or are supposed to be applied for the experimental plasma in tokamaks. The first priority to control plasma shape specifically the location of the plasma separatrix nearby the firs wall at the divertor phase of plasma discharges are on-line algorithms (codes) to reconstruct the plasma equilibrium based on magnetic measurements outside plasma. These are Flux and Current Distribution Identification (FCDI) code [6], moving filaments code [8], and neural network as a plasma equilibrium reconstruction algorithm (Section 2) [10]. These reconstruction algorithms give a chance by means of their applications to the experimental data firstly to use the off-line identification proceedures beetween the references for VFC&HFC&CS&PF-currents control loops and outputs of the reconstruction codes and plasma current diagnostics to get linea models for design of the plasma current and shape control system with the feedback on the Globus-M2 tokamak. The off-line identification methods may be as subspaces (Subsection 4.1) [12, 13], wavelets (Subsection 4.2) [14, 15], LMIs (Subsection 4.3) [16, 17]. After getting such experience of off-line identification and application of plasma robust control systems on the base of the real time testbed [18] one will be able to apply on-line identification methods as part of plasma control systems for instance based on adaptive state observers (Subsection 4.4) [19] or dynamical neural networks (Subsection 4.5) [20].

The direction of dynamical system identification is being developed not only for linear approaches [12-17] but for nonlinear approaches as well [21]. But the second one is much more complicated and may be applied to plasma control after getting experience of application of linear identification approaches and on the base of statements of new nonlinear plasma control challenges for instance plasma kinetic control.

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