Neural Network Algorithm for Forecasting and Parameter Estimation of the Coriolis Vibratory Gyroscope

Basarab M., Ivanov I., and Lunin B.

Bauman Moscow State Technical University, Moscow 105005, Russia
bmic@mail.ru

Abstract. The accuracy of the Coriolis vibratory gyroscope is determined by the technological factors of manufacturing its resonator and the heterogeneity of its physical characteristics. The elimination of heterogeneities primarily requires the accurate determination of their parameters (amplitudes, locations). In this work, the problem of identifying the parameters of the Coriolis vibratory gyroscope, using time series data for the main and quadrature signals obtained from information pickup sensors is considered. As a dynamic model of the gyroscope, a system of two second-order ordinary differential equations with coefficients associated with the parameters of the device is used. The algorithm for identifying unknown parameters is based on the use of an autoregressive model of the time series and a single-layer neural network that implements forecasting. The method gives us the possibility to get an explicit interpretation of the weight coefficients of the forecast model of the two-dimensional time series.

Keywords: Coriolis Vibratory Gyroscope, Time Series, Forecasting, Autoregression.

1 Introduction

The solid-state wave gyroscope (SWG) belongs to the class of the so-called Coriolis vibration gyroscopes [1-3] measuring an angular rate on the base of the precession of elastic waves excited in axisymmetric solid bodies (resonators), the effect discovered experimentally by G. Bryan [4]. In practice, the shells of revolution of cylindrical and hemispherical types (respectively, the cylindrical resonator gyroscope - CRG, and hemispherical resonator gyroscope - HRG), as well as elastic rings on torsion suspensions are most often used as such resonators.

The accuracy of the device strongly depends on technological factors of manufacturing the resonator and inhomogeneities of its geometry and physical characteristics (density, Young's modulus, etc.). The elimination of defects (called the trimming procedure) is one of the most important operations in the manufacture of the SWG and requires accurate determination of the parameters of such inhomogeneities (amplitudes and locations) [5]. The three-dimensional equations of SWG dynamics are derived based on the models of the theory of elasticity of thin-walled shells and, as a rule, are rather cumbersome. However, given the fact that the main energy of elastic vibrations falls on the narrow edge of the shell, it is convenient to study the dynamics of the resonator based on its ring model. In the resonance mode, for a specific waveform, from the equation of dynamics of the elastic ring, a system of second-order ordinary differential equations (ODEs) for non-stationary amplitudes of the main and quadrature waves can be obtained, which are used for decomposition of the initial ideal standing wave in the presence of the angular rate of the basement or in the presence of inhomogeneities of the resonator material. These amplitudes can be evaluated, with a certain sampling rate, from signals from a pair of information pickup sensors. The problem is to identify the dynamic parameters of the SWG operation (angular rate, angular acceleration) and the parameters of the standing wave drift caused by inhomogeneities in the physical characteristics of the resonator material. It is the identification of the parameters that cause drift that is the most complicated problem, for the solution of which a number of methods have been proposed [1,2,5], none of which can be considered universal. In this paper, a new approach is proposed, based on the construction of a neural network autoregressive model of time series corresponding to signals from SWG sensors. Further, by means of the autoregression coefficients, the sought parameters of the inhomogeneities of the distribution of mass and quality factor in the angular coordinate are determined.

2 Principle of SWG operation

The principle of operation of the SWG will be considered on the ring resonator model. A standing wave is excited in the resonator along one of the forms of plane vibrations, most often the second one. This wave is characterized by four nodes and four antinode points alternating through 45° (Fig. 1, a). When an angular velocity $\Omega = \Omega(t)$ occurs, the initial wave rotates over time t through an angle, with a coefficient of 0.4 (the "Bryan factor") proportional to the angle of rotation of the basement (Fig. 1, b):

$$\varphi(t) = \varphi_0 - \frac{2}{5}\int_0^t \Omega(s)ds, \tag{1}$$

where φ_0 is the initial standing wave orientation (antinodes and nodes).

Measuring the positions of antinodes oriented at 45 ° relative to each other, we can determine the angle, and from it, from Eq. (1), the desired angular velocity, in particular, at $\Omega = \text{const}$, we have

$$\Omega = \frac{5}{2} \cdot \frac{\varphi(t) - \varphi_0}{t}.$$

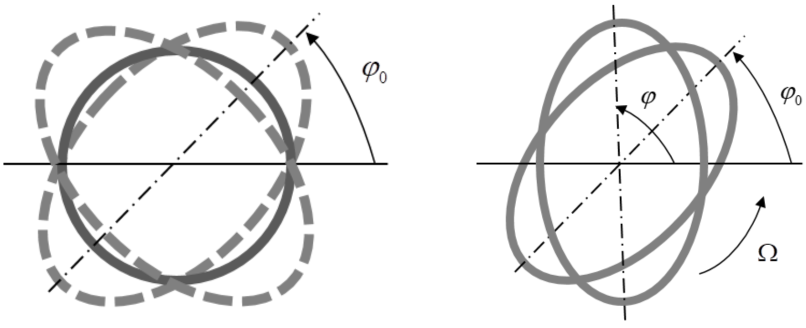

Fig. 1. The second vibrational mode of the SWG resonator (a) and the wave precession (b)

3 Mathematical model

Consider the ring model of free vibrations of an imperfect ring resonator with dissipation [2]:

$$\begin{gathered}\ddot{w}'' - \ddot{w} + 4\Omega\dot{w}' + 2\dot{\Omega}w' - \Omega^2(w^{\mathrm{IV}} + 3w'') + \left[\kappa^2(w'' + w)\right]^{\mathrm{IV}} + \\ + \left[\kappa^2(w'' + w)\right]'' + \xi\left[\kappa^2(\dot{w}'' + \dot{w})\right]^{\mathrm{IV}} + \xi\left[\kappa^2(\dot{w}'' + \dot{w})\right]'' = 0,\end{gathered} \tag{2}$$

Here, $w = w(\varphi, t)$ is the radial movement of a point on the edge of the ring; $\Omega, \dot{\Omega}$ are angular rate and acceleration respectively; $\kappa^2 = EJ/(\rho SR^4)$; ρ is the density; S is the cross-section area; R is the radius of the middle neutral line; E is Young's modulus of the material; $J = h^4/12$ is the moment of inertia of the cross section relative to the neutral axis; h is the thickness; ξ is the decay time characterizing the quality factor of the resonator.

We further assume that the physical parameters are inhomogeneous in angle and therefore $\kappa^2 = \kappa^2(\varphi)$, $\xi = \xi(\varphi)$. We expand each of these parameters in a Fourier series:

$$\begin{gathered}\xi = \xi_0 + \sum_{l=1}^{N}\left(\Delta_{lc}^{(1)}\cos l\varphi + \Delta_{ls}^{(1)}\sin l\varphi\right), \\ \kappa^2 = \kappa_0^2 + \sum_{l=1}^{N}\left(\Delta_{lc}^{(2)}\cos l\varphi + \Delta_{ls}^{(2)}\sin l\varphi\right).\end{gathered} \tag{3}$$

Here $\Delta_{lc}^{(1)}, \Delta_{ls}^{(1)}, \Delta_{lc}^{(2)}, \Delta_{ls}^{(2)}$ are the components of the amplitudes of the l-th harmonics of the damping defect and elastic mass anisotropy respectively.

Let the initial conditions include the displacements and accelerations of the resonator points:

$$w(\varphi,0)=\alpha_0(\varphi),\quad \dot{w}(\varphi,0)=\alpha_1(\varphi).$$

We will represent the desired solution (2) in the form of the evolution of a wave excited by the second form of oscillation (Fig. 1):

$$w(\varphi,t)=p(t)\sin(2\varphi)+q(t)\cos(2\varphi). \tag{4}$$

Substituting (3) and (4) into (2), and applying the Galerkin method with basic functions $\sin(2\varphi), \cos(2\varphi)$, we obtain a system of ODEs with respect to amplitudes (4):

$$\begin{gathered}
\ddot{p}+\omega_0^2\left(\xi_0+\Delta_{4c}^{(1)}\right)\dot{p}+\left(\Delta_{4s}^{(1)}-\frac{8}{5}\Omega\right)\dot{q}+\\
+\left(\omega_0^2+\frac{36}{5}\Delta_{4c}^{(2)}+\frac{4}{5}\Omega^2\right)p+\left(\frac{36}{5}\Delta_{4s}^{(2)}-\frac{4}{5}\dot{\Omega}\right)q=0,\\
\ddot{q}+\omega_0^2\left(\xi_0-\Delta_{4c}^{(1)}\right)\dot{q}+\left(\Delta_{4s}^{(1)}+\frac{8}{5}\Omega\right)\dot{p}+\\
+\left(\omega_0^2-\frac{36}{5}\Delta_{4c}^{(2)}+\frac{4}{5}\Omega^2\right)q+\left(\frac{36}{5}\Delta_{4s}^{(2)}+\frac{4}{5}\dot{\Omega}\right)p=0,
\end{gathered} \tag{5}$$

where $\omega_0=6\kappa_0\sqrt{5}$ is the natural frequency of the second form of oscillations of the ideal resonator.

To identify the parameters of the 4th harmonics of defects, one should consider the resonator on a fixed basement, then (5) takes the form

$$\begin{gathered}
\ddot{p}+\omega_0^2\left(\xi_0+\Delta_{4c}^{(1)}\right)\dot{p}+\Delta_{4s}^{(1)}\dot{q}+\left(\omega_0^2+\frac{36}{5}\Delta_{4c}^{(2)}\right)p+\frac{36}{5}\Delta_{4s}^{(2)}q=0,\\
\ddot{q}+\omega_0^2\left(\xi_0-\Delta_{4c}^{(1)}\right)\dot{q}+\Delta_{4s}^{(1)}\dot{p}+\left(\omega_0^2-\frac{36}{5}\Delta_{4c}^{(2)}\right)q+\frac{36}{5}\Delta_{4s}^{(2)}p=0.
\end{gathered} \tag{6}$$

Using the appropriate measuring circuit, one can obtain the amplitudes at discrete time moments:

$$p_n=p(t_n),\quad q_n=q(t_n)\quad (n=0,1,2,...). \tag{7}$$

The problem is as follows: given the values of time series (7) for a limited time interval $t\in[0,T]$, evaluate all the model parameters (6) ($\Delta_{4c}^{(1)}, \Delta_{4s}^{(1)}, \Delta_{4c}^{(2)}, \Delta_{4s}^{(2)}, \omega_0^2, \xi_0$).

4 Method of solution

A possible identification algorithm based on the least squares method was proposed in [6]. It allows an assessment even in the presence of measurement errors due to averaging over a large number of samples p_n, q_n. At the same time, this approach is rather cumbersome, requires the formation of the sixth order system of algebraic equations and, in some cases, especially in the presence of noise, can be unstable due to poor conditioning of the matrix of a system of linear algebraic equations.

In this paper, we propose a solution to the problem posed by solving the problem of forecasting time series (7) based on the autoregressive model [7-9]. Assuming that the dynamic process described by system (6) is quite smooth and, in addition, quasi-periodic, we restrict ourselves to a single-layer linear neural autoregression network (NNAR) [8] (Fig. 2, a):

$$p_j^* = \sum_{i=1}^{m} w_{1,2i-1} p_{j-i} + \sum_{i=1}^{m} w_{1,2i} q_{j-i}, \quad q_j^* = \sum_{i=1}^{m} w_{2,2i-1} p_{j-i} + \sum_{i=1}^{m} w_{2,2i} q_{j-i}, \quad (j = m, ..., M), \tag{8}$$

where p_j^*, q_j^* are the predicted values; w_{1i}, w_{2i} are the weighting coefficients; m is the length of the two-dimensional sliding window; M is the the length of the training segment of the two-dimensional time series.

Finding the weights of a neural network is carried out according to the delta rule, reccurently ($k = 0,1,2,...$), starting with some starting set $w_{1i}^{(0)}, w_{2i}^{(0)}$:

$$\begin{aligned} w_{1,2i-1}^{(k+1)} &= w_{1,2i-1}^{(k)} + \eta p_i \left(p_j^* - p_j \right), \quad w_{1,2i}^{(k+1)} = w_{1,2i}^{(k)} + \eta q_i \left(p_j^* - p_j \right); \\ w_{2,2i-1}^{(k+1)} &= w_{2,2i-1}^{(k)} + \eta p_i \left(q_j^* - q_j \right), \quad w_{2,2i}^{(k+1)} = w_{2,2i}^{(k)} + \eta q_i \left(q_j^* - q_j \right). \end{aligned} \tag{9}$$

Here $\eta \in (0,1]$ is the learning rate.

Taking into account the non-explicit dependence between parameters p_n, q_n, one can also use a shortened forecast model (Fig. 2b, the similar model can be presented for q_j^*):

$$p_j^* = \sum_{i=1}^{m} w_i \, p_{j-i} \quad (j = m, ..., M). \tag{10}$$

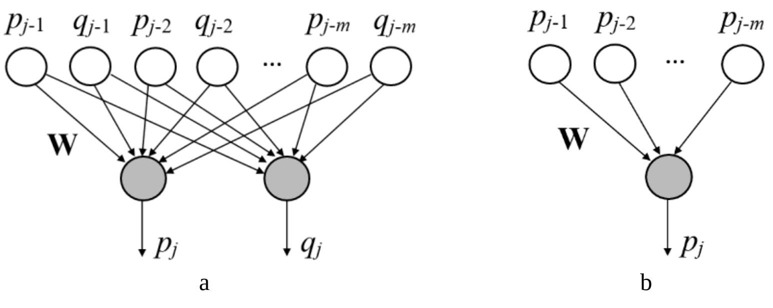

Fig. 2. Complete (a) and shortened (b) linear neural networks of autoregression

The neural network is trained by iterating through a sliding window along the training segment of the time series of length *M*, from the first sample ($j = m$) to the far right sample ($j = M$) with a single time lag.

After satisfying the convergence criterion (the smallness of the norm of the difference between the vectors of the predicted values (p_j^*), (q_j^*) and the actual measured values, (p_j), (q_j)), we will have a set of final weights of the forecast model (8).

To evaluate the parameters of model (6) by the found weight coefficients of the autoregressive model (8), the following algorithm can be used:

1. Replace the time derivatives in (6) with the help of finite differences of the *m*-th order [10], obtaining a finite-difference approximation of the system of differential equations (6).
2. Represent the extreme right values of the grid functions (p_j, q_j) through the previous values $(p_{j-1}, q_{j-1}), (p_{j-2}, q_{j-2}),...$ in a form similar to (8).
3. Comparing both representations, we express the desired parameters $\Delta_{4c}^{(1)}, \Delta_{4s}^{(1)}, \Delta_{4c}^{(2)}, \Delta_{4s}^{(2)}, \omega_0^2, \xi_0$ in terms of weight coefficients w_{1i}, w_{2i} .

4. Numerical experiment

Example 1. Let us first consider the simplest case of an ideal SWG resonator, when $\kappa^2 = \kappa_0^2 = \text{const}$, $\xi = \xi_0 = \text{const}$. Then (6) has the form:

$$\begin{aligned} &\ddot{p} + \omega_0^2 \xi_0 \dot{p} + \omega_0^2 p = 0, \\ &\ddot{q} + \omega_0^2 \xi_0 \dot{q} + \omega_0^2 q = 0. \end{aligned} \tag{11}$$

Obviously, due to the fact that both equations of system (11) are independent, the forecast must be carried out according to scheme (10). For definiteness, we choose the amplitude *p* (for the amplitude *q*, the actions will be completely analogous). Let, for example, the length of the forecast window $m = 2$, then

$$p_j^* = w_1 p_{j-1} + w_2 p_{j-2} \quad (j = m,...,M). \tag{12}$$

In accordance with the proposed algorithm, we replace the time derivatives in the first equation of (11) with finite-difference approximations of the second order of accuracy with respect to the time step τ [10]:

$$\ddot{p}(t_{j-1}) \approx \frac{p_j - 2p_{j-1} + p_{j-2}}{\tau}, \quad \dot{p}(t_{j-1}) \approx \frac{p_j - p_{j-2}}{2\tau}. \tag{13}$$

We substitute (13) into (11) and express p_j through the previous values:

$$p_j = \frac{2-\omega_0^2\tau^2}{0,5\omega_0^2\xi_0\tau+1} p_{j-1} + \frac{0,5\omega_0^2\xi_0\tau-1}{0,5\omega_0^2\xi_0\tau+1} p_{j-2} \quad (14)$$

Comparison of expressions (12) and (14) gives us a simple expression for the weights of the neural forecast network:

$$w_1 = \frac{2-\omega_0^2\tau^2}{0,5\omega_0^2\xi_0\tau+1}, \quad w_2 = \frac{0,5\omega_0^2\xi_0\tau-1}{0,5\omega_0^2\xi_0\tau+1}. \quad (15)$$

We accept the following values of physical parameters:
- oscillation frequency $\omega_0 = 38598,5$ rad / s;
- attenuation factor $\xi_0 = 2,0 \cdot 10^{-7}$ s.

In a computational experiment, we use the parameters:
- sampling step $\tau = 1 \cdot 10^{-5}$ s;
- the length of the training interval $T = 0,0025$ s (M = 250 samples);
- learning rate $\eta = 1$.

During the experiment, in order to achieve a forecast quadratic error of less than 0.1 % on the interval $(T, 4T)$, it was necessary to implement 250,000 training eras. The obtained values of the weights are as follows

$$w_1 = -1, \quad w_2 = 0,851.$$

The relative error in calculating the eigenfrequency and attenuation coefficient by formulas (15) turned out to be less than $1 \cdot 10^{-7}$.

It should be noted that even a slight increase in the window length *m* leads to a significant decrease in the number of epochs required to ensure the same forecast accuracy, however, this complicates the task of interpreting the weight coefficients.

Example 2. Next we study the dynamics of imperfect SWG. Take the data from the previous example. Let now in (6): $\Delta_{4c}^{(1)} = 0,0114$, $\Delta_{4s}^{(1)} = 0,1462$; $\Delta_{4s}^{(2)} = -0,0154$.

Let training be carried out at the same interval $(0, T)$ and with the same step τ, but the length of the forecast window $m = 4$. The forecast results for the interval $(T, 2T)$ for 50,000 training eras are shown in Fig. 3. Values of weight coefficients are the following: $w_1 = -0,5231$, $w_2 = -0,063$, $w_3 = 0,4042$, $w_4 = 0,8109$.

It can be seen that using the reduced model (10), it is possible to achieve a satisfactory forecast quality. The relationship between weights and heterogeneity parameters is more complicated than in Ex. 1, however, a continuous relationship between them is still observed, which allows us to conclude that it is possible to build empirical dependencies.

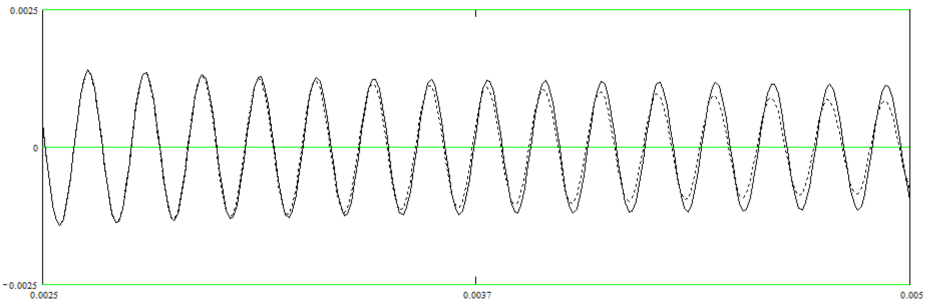

Fig. 3. Forecasting the time series for the imperfect SWG

5. Conclusion

A new method is proposed for estimating the parameters of an imperfect solid-state wave gyroscope based on the linear neural network autoregression model. In addition to the problem of estimating parameters, the problem of predicting the dynamics of oscillations is solved, which is important in optimizing inertial navigation algorithms. The method is flexible and efficient, and also allows one to get an explicit interpretation of the weight coefficients of the forecast model of the two-dimensional time series. The proposed approach can be used to evaluate the parameters of other types of CVGs [3].

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