

# The schematization of soil properties in mathematical modeling in engineering geology and geotechnics

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**ABSTRACT:** Modeling of natural ground conditions will always require approximations due to their spatial variability. Often limited datasets describing material behavior are available resulting in the estimation of material properties. In many cases only deterministic estimates of material strength are reported or used in geotechnical analysis, despite the natural variation and uncertainty of material strength in the area under investigation. Failure to take into account the random nature and variation of material mechanical strengths can result in misleading stability calculations. This paper shows the difference in slope stability results that are calculated when soil properties are modelled as deterministic values, interpolated from known sample sites, described by a range of values (probabilistic analysis) and varied spatially and stochastically.

**Keywords:** engineering-geological schematization, engineering-geological element, models of property distribution, calculation of slope stability

## 1 INTRODUCTION

One of the main stages of building a geotechnical model encompassing the engineering-geology properties of natural ground conditions is schematization. Schematization can be categorized as generalized or special.

Generalized schematization, in this context, can be described as the process of simplifying a real natural object, with an infinite degree of complexity, to a conceptual model, limited by the framework of scientific knowledge.

Special schematization presupposes simplification of a conceptual model to a specialized scheme that maximizes simplification with minimum loss of adequacy. Special schematization can be divided into several interrelated stages: behavior schematization, structure schematization, properties schematization, and modeling object state schematization [1].

The special schematization is based on the concept of an engineering-geological element (EGE). According to Russian standards (SP 446.1325800.2019) [2] EGE is the main soil unit used to create a geotechnical model of a soil massif, including a certain volume of soil of the same type (subtype), type (subspecies) and variety when the values of soil characteristics within an element change randomly (irregularly) or with an observed pattern of changes in soil characteristics with a coefficient of variation for physical soil characteristics  $\leq 0.15$ , for mechanical  $\leq 0.30$ .

Despite the fact that the random nature of the distribution of properties in the EGE is present in the definition, until recently, not enough attention was paid to this fact in the

construction of geotechnical models. Although statistical properties describing the variation in soil properties were calculated, strength parameters applied in geotechnical models were deterministic.

However, the situation has changed in recent years. This is due to the increased computation of probabilistic analysis [3], which considers the uncertainty of material properties in geotechnical models, and can also be used in risk-based design analysis [4].

Currently, there are four main types of models that describe the distribution of soil properties geotechnical models. These are deterministic models, interpolation models, probabilistic models and models applying spatial variability.

Example applications of all four methods will be shown using a slope case from Moscow. Stability was assessed using 2D (plain strain) limit equilibrium analysis. Optimized Cuckoo search methods were applied.

## 2 OBJECT OF STUDY

The work site is located in Lapshinka, Moscow (Figure 1)

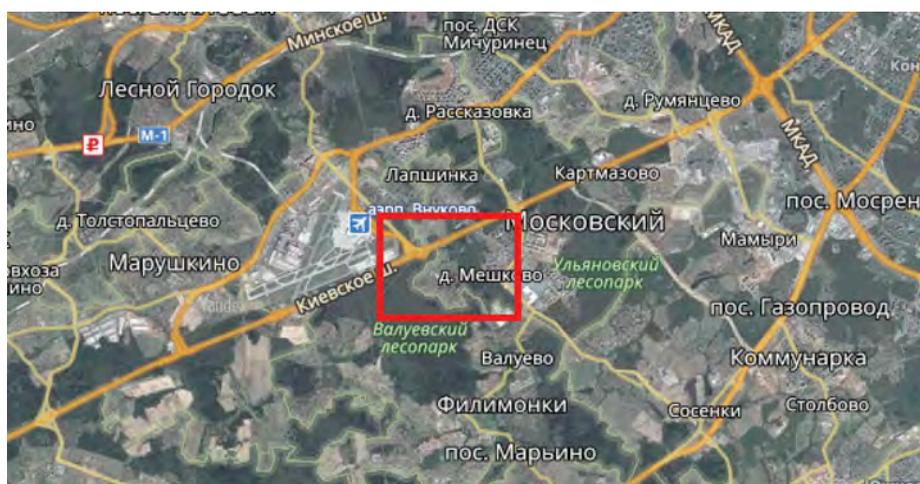


Figure 1. Location of the study area (red rectangle).

Geomorphologically, the study area is located within the gently undulating moraine plain, with well-developed river valleys. The relief is characterized by absolute surface elevations of the order of 180.35 - 182.05 m.

Quaternary deposits, of technogenic displaced soils, alluvial, cover, glacial and Cretaceous deposits, form the geological structure to the explored depth of 20.0 m. Hydrogeological conditions are characterized by the close occurrence of groundwater, as well as the presence of a large number of aquifers, hydraulically connected with each other.

Within the study area, the technogenic-displaced soils that compose the dump body are brown loam, slightly compacted, and refractory. The thickness of technogenic soils varies from 2.7 m to 12.0 m.

Field observations of the study area noted surficial cracking on the edge of the embankment (Figure 2).



Figure 2. Cracks on the edge of the embankment.

### 3 DESIGN SCHEMES AND INITIAL DATA

The physical and mechanical properties of technogenic soils occurring within the study area are very different from other soils formed in nature. The distribution of properties in technogenic soils is characterized by great heterogeneity. For this reason, the selection of layers of different physical and mechanical properties and their assignment in geotechnical models presents a certain difficulty. To assess the variation in results that can be calculated with and without consideration of the heterogeneity of the technogenic soil, slope stability was assessed using deterministic, interpolation, probabilistic and spatial variation analysis methods.

#### 3.1 *Deterministic model*

When modelling using deterministic inputs, soil strength parameters included in the geotechnical model are the single (scalar) values for each EGE: density, cohesion, angle of internal friction. The critical slip surface and factor of safety (FS) calculated using deterministic soil properties is displayed in Figure 3.

#### 3.2 *Interpolation model*

In interpolation models, the initial parameters are numerical arrays defined for each soil property (density, specific cohesion, angle of internal friction) and coordinates of the sample sites. Based on this information, soil properties are interpolated for the remainder of the model extents using interpolation algorithms built into the analysis software. Different results may be obtained using different interpolation methods.

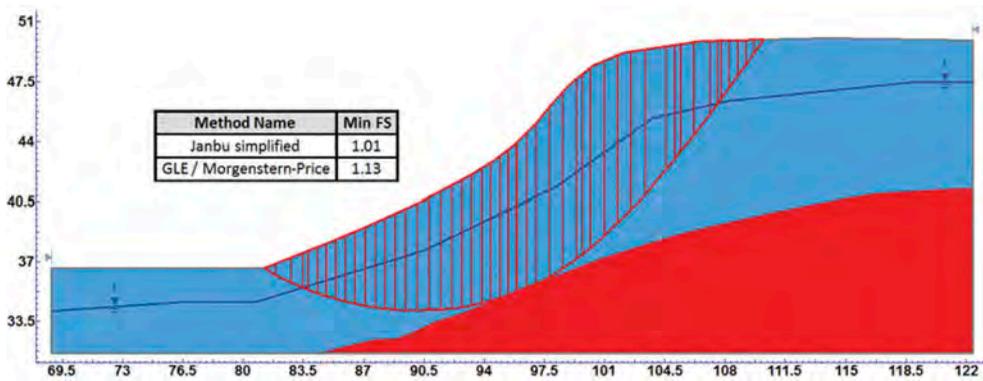


Figure 3. Model results using deterministic inputs of soil properties.

The critical slip surface and FS calculated using interpolated soil properties is displayed in Figure 4.

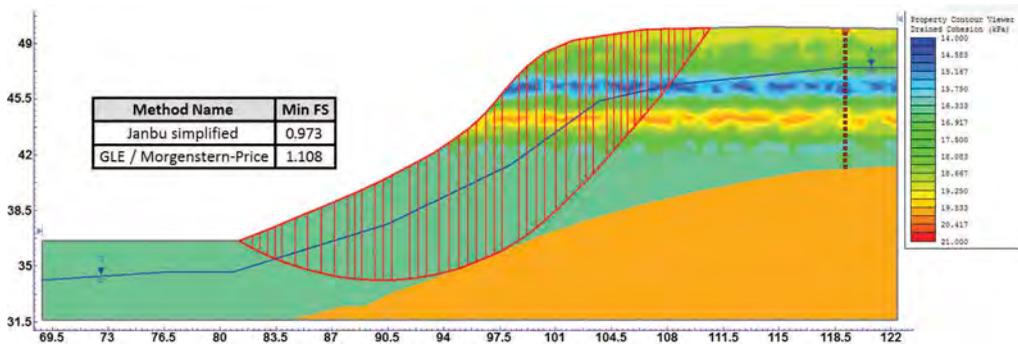


Figure 4. Model results using interpolated soil properties. Variance in soil cohesion displayed.

### 3.3 Probabilistic model

In contrast to the models described above, the initial parameters in probabilistic models are not scalar quantities, but functions of the probability distribution of soil properties [11].

The main problem in constructing a probabilistic model is to determine the distribution of material properties. In the case of a normal distribution, two parameters are required to construct a probability function: mean and standard deviation.

The output of probabilistic models is the distribution of FS, Figure 5, where several FS have been calculated using different combinations of soil parameters along the selected distribution.

The variability of the properties of soils affects the value of FS. Together with probabilistic analysis, sensitivity analysis can be completed to determine the sensitivity of FS to one unit change in material property (e.g. one unit change in cohesion over the specified range). Sensitivity analyses can assist to determine if the FS is more sensitive to changes in certain materials and/or properties in the model.

It should also be noted that the critical strength parameters are not a single pair of values (e.g. a single angle of internal friction and cohesion), but could be any combination over a range of values, Figure 6.

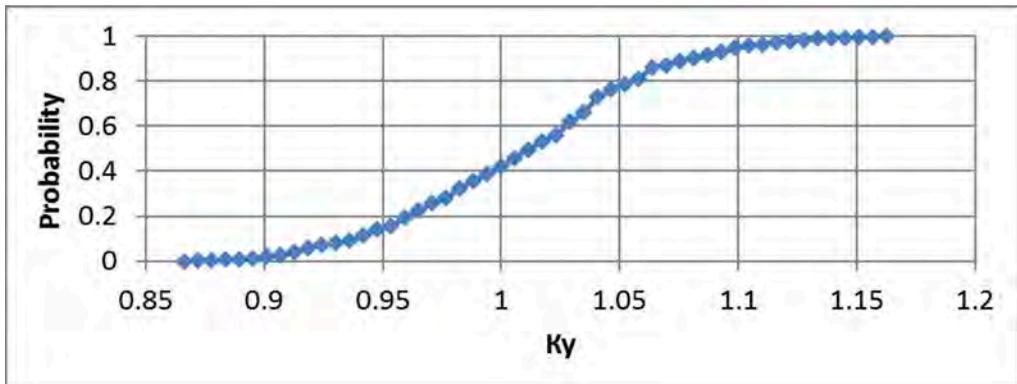


Figure 5. Cumulative distribution of FS as calculated from probabilistic model (42% of simulations had a FS less than 1).

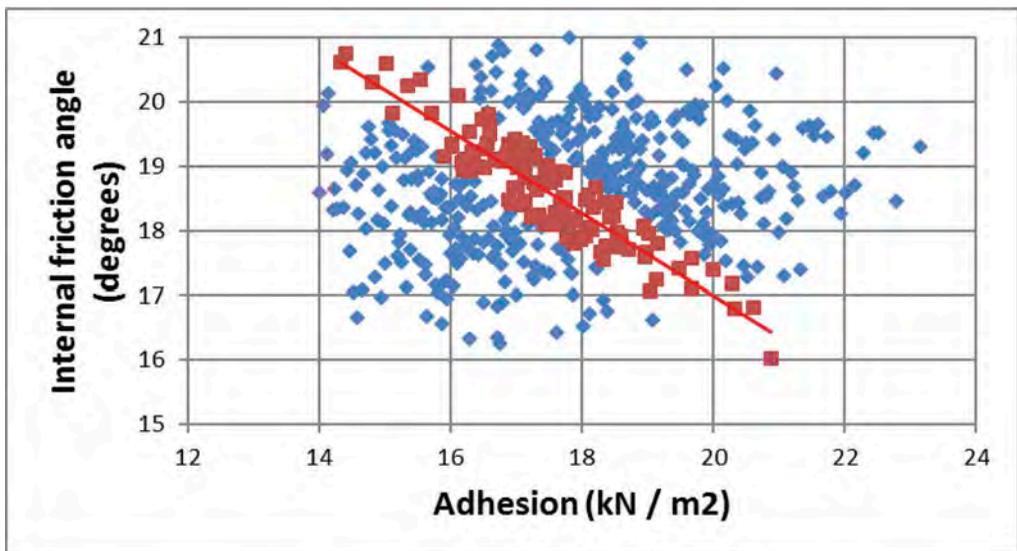


Figure 6. Two-way sensitivity analysis for the simulated slope. The red line is the dependence of the angle of internal friction on cohesion at which the FS slope is 1. Any combination of friction angle and cohesion highlighted in red will result in FS = 1.

### 3.4 Spatial variation

#### 1. Model of variability

Not taking into account the random component in the structure of numerical estimates of the geological parameter (using purely deterministic models) often leads to erroneous results [10].

Spatial variability of soil properties can be modeled using the theory of random fields [6, 12, 13], according to which, in any area of the geotechnical model, soil properties are a random variable characterized by a probability distribution function and correlating with the values of soil properties in adjacent areas [7]. Spatial correlation of soil properties is determined based on the autocorrelation function, which can be estimated from the results of measurements of the parameter at various points according to the results of field or laboratory tests [8].

The correlation structure of a random Gaussian domain can be determined using the Markov correlation coefficient function:

$$R(\tau_x, \tau_y) = \exp \left\{ -\sqrt{\left(\frac{2\tau_x}{\theta_x}\right)^2 + \left(\frac{2\tau_y}{\theta_y}\right)^2} \right\}$$

where  $R(\tau_x, \tau_y)$  is the autocorrelation coefficient,  $\tau_x$  and  $\tau_y$  are the absolute distances between two points in the horizontal and vertical directions, respectively,  $\theta_x$  and  $\theta_y$  are the correlation distances in the horizontal and vertical directions, respectively [9].

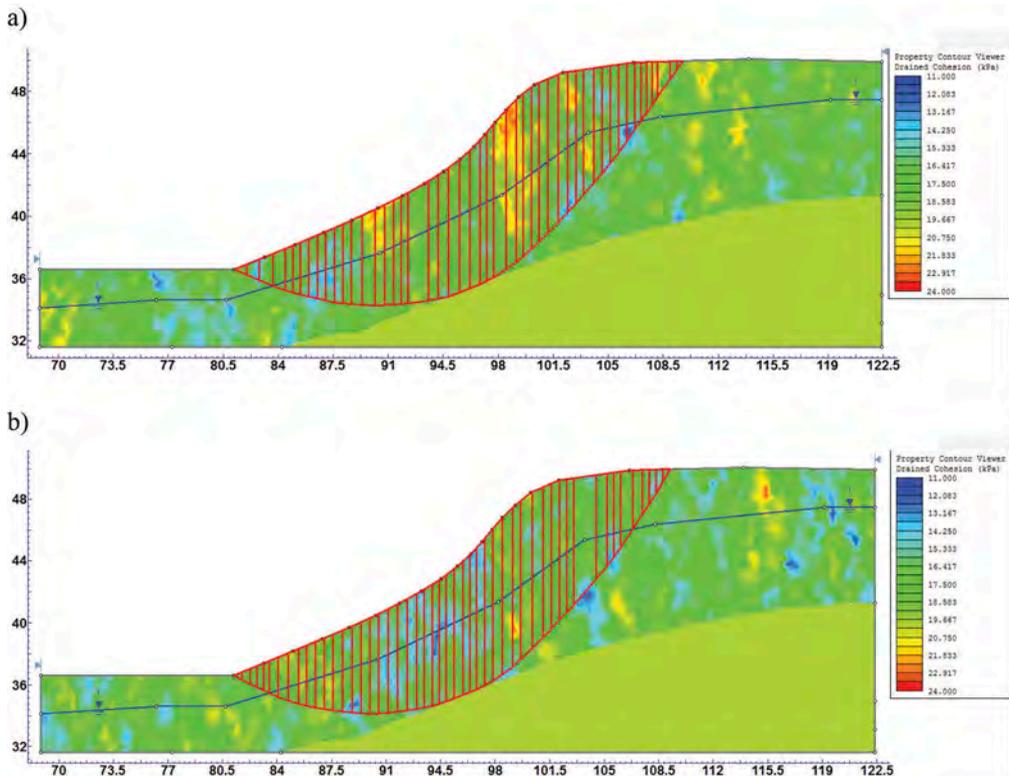


Figure 7. The model of the specific cohesion distribution for: a) the average value of  $K_u$  -1.01; b) the minimum value of  $K_u$  is 0.97. The horizontal correlation distance is 1m, the probability of a landslide process is 53.6%.

Thus, the necessary initial parameters for the variability model, in addition to the probability distribution function of soil properties, are the values of the correlation distance,  $\theta_x$  and  $\theta_y$ .

Stability results of the case study when spatial variation of soil properties is applied is displayed in Figure 7.

Model results show that variation in  $\theta_x$  has a limited impact on FS, for all cases the average FS is 1.01). However, the probability of failure increases with decreasing  $\theta_x$ , Figure 8.

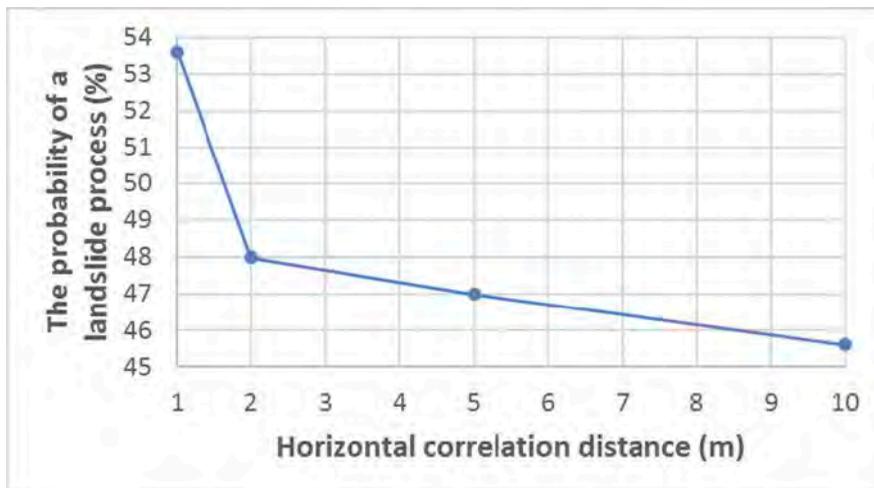


Figure 8. Variation in probability of failure with  $\theta_x$ .

#### 4 CONCLUSION

The mechanical properties of natural ground conditions are inherently variable. Often only limited is available to describe the mechanical behavior of soils, leaving much uncertainty in the true material behavior. Geotechnical models should consider this inherent variability in strength.

This paper has shown how the FS and probability of failure can vary when slope stability is calculated with and without consideration of material property heterogeneity.

A case slope from near Moscow was assessed using deterministic, interpolated, probabilistic and spatially variable analysis methods. Different results were calculated for each method for the same slope case.

Probabilistic analysis and the application of spatial variability in slope stability models can assist to account for the spatial variability and uncertainty in natural slopes. In the case study described in this paper, probability of failure was shown to increase with decreasing horizontal spatial correlation.

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