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Excitation of the Modes of an Optical Resonator by a Tunable Laser Beam

O. M. Vokhnik[a], **P. V. Korolenko**[a, b, *], **and R. T. Kubanov**[a]

[a] *Moscow State University, Moscow, 119991 Russia*
[b] *Lebedev Physical Institute, Russian Academy of Sciences, Moscow, 119991 Russia*
**e-mail: pvkorolenko@rambler.ru*

Abstract—An investigation is performed of the mechanism of modes of interaction in the analytical resonator of a weak absorption spectrometer during fast tuning of the frequency of the reference laser. It is shown that an improvement in resolving power is matched by a definite drop in spectrometer sensitivity at a high rate of change in frequency and the overlapping of fields of longitudinal modes.

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INTRODUCTION

Studies of light fields in optical resonators continue to be of relevance [1], even though the excitation of an open resonator by an external laser beam has been widely covered in the literature. Such resonators are used in a variety of optical devices (e.g., Fabry–Perot scanning interferometers [2], coupled laser systems, and regenerative amplifiers) [3]. Despite the many works on this topic, the problem of resonator excitation by a laser beam with fast tuning of its frequency remains virtually unstudied. It is nevertheless of great importance, due to the creation and wide use of diode lasers with fast frequency-tuning emissions. Such lasers are especially used in low-absorption spectrometers to excite analytical resonators containing the material of interest [4, 5]. The aim of this work is to identify patterns of the formation of an intracavity light field under conditions where the time interval between the excited modes is comparable to the period of radiation attenuation in the resonator. Special attention is given to determining the effect intermode optical coupling during fast frequency scanning has on the sensitivity of spectral measurements. The approach used in this work is based on adding the amplitudes of partial beams obtained from multiple reflections of input radiation off the mirrors of the resonator at its output [6]. In contrast to many works where this approach was used to determine the characteristics of interferometers and resonators, we consider the interaction between excited modes in an analytical resonator when making calculations.

CALCULATION SCHEME

It is assumed in our calculations that the width of the laser emission line is negligible, compared to that of the absorption lines and the bandwidth of the resonator. The phase overrun of the beam on pass θ is determined using the expression

$$\theta = 2\pi L\left(f_0 + vt\right), \tag{1}$$

where frequency $f_0 = 1/\lambda_0$; L is the length of the resonator; λ_0 is the initial wavelength of the laser; and $v = \mathrm{d}f/\mathrm{d}t$ is the rate of change in period t of laser frequency f. It was assumed that $2\pi L f_0 = 2\pi N$, where N is a large integer. We introduce quantity $\delta = 2\pi L v$, which denotes the rate of the phase increment of the laser beam per pass and has dimensionality [rad/time]. Assuming the rate of the phase increment is proportional to the period (the frequency usually varies by a law close to linear), we can write $\delta = x\,k$, where k is the number of significant points on the timescale associated with the degree of its discretization; and x is the proportionality factor. The value of this coefficient affects the rate of change in frequency. The summation of partial beams to determine total output amplitude S of radiation is done according to a formula of descending geometric progression. Using this formula, we can write the temporal dependence of amplitude for the symmetrical resonator as

$$S\left(k\right) = \frac{a(1 - R^2)e^{i[\delta(k)d+\Phi]}p(k)}{1 - R^2 e^{2i[\delta(k)d+\Phi]}p(k)^2}, \tag{2}$$

where R is the coefficient of reflection of the resonator mirrors, according to amplitude; a is the amplitude of an incident wave; Φ is the phase of the wave; $k = 0, 1, 2 \ldots K$; d is the interval of time scale sampling; and $p(k)$ is the coefficient of radiation transfer for the intracavity medium (losses in mirrors are considered to be negligible). Note that the range of change in significant points k can characterize both time T of one scan

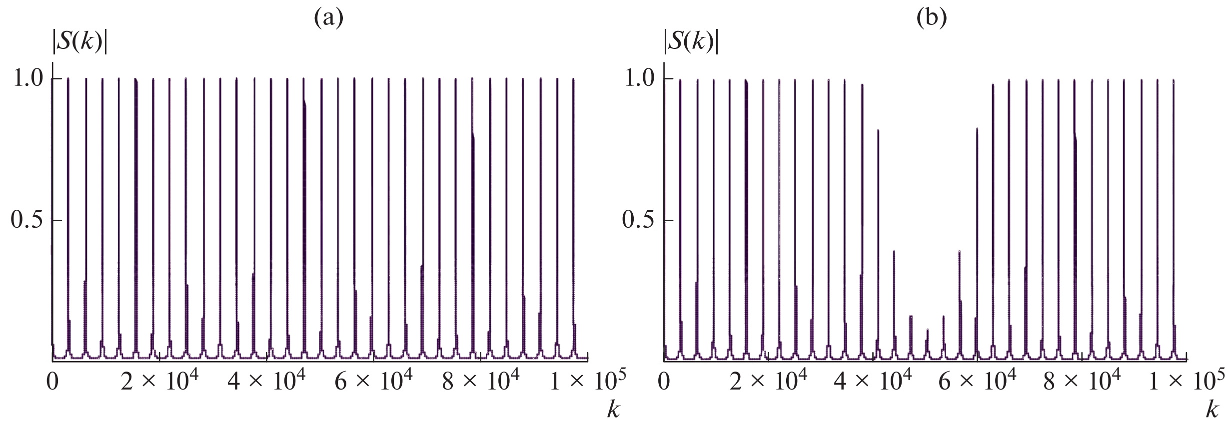

Fig. 1. Structure of resonant peaks: (a) $\alpha = 0$, (b) $\alpha = 0.075$.

of a change in laser frequency and range F of frequency tuning during this period.

An analysis of changes in the structure of the output radiation depending on the speed of laser frequency scanning, performed using formula (2), shows that an increase in speed produces certain effects that require additional study and consideration. From the viewpoint of practical importance, they should be attributed mainly to the reduction in the intervals of time and frequency between the resonance peaks that form at the resonator output and their simultaneous narrowing, and the possible complication of the structure of the output radiation at an arbitrary degree of sampling of the signal. Finally, the effect of intermode coupling amplification, which results in superpositioning of the fields of neighboring modes of the resonator starting from a certain rate of frequency scanning, should be noted. Calculations show that this effect becomes noticeable when the interval of the temporal intermode is comparable to the period of radiation decay in the resonator.

A general view of the distribution of resonant peaks corresponding to the excited longitudinal modes of the resonator is presented in Figs. 1a,b. The plots of the dependence of modulus $S(k)$ in Fig. 1a are for parameters $a = 1$, $\Phi = 0$, $R = 0.99$, $K = 10^5$, $x = 10^{-3}$, and $p(k) = 1$ of a resonator with no absorbing of matter. The peaks in Fig. 1b correspond to when an absorbing medium is in the resonator. Its coefficient of transfer was determined using the relation $p(k) = \exp[-p'(k)\alpha - i\varphi_n(k)]$, where $p'(k) = e^{-\frac{(k-k_0)^2}{\Delta^2}}$ is the form factor of the absorption line; α is the coefficient of absorption at the center of the line; k_0 determines the moment when the laser frequency passes through the center of the line; Δ characterizes its width; and $\varphi_n(k)$ is an additional phase overrun caused by the change in the refractive index in the region of the absorption line. Figure 1b shows the sequence of resonances corresponding to values $a = 1$, $\Phi = 0$, $k_0 = 5 \times 10^4$, $\Delta = K/20$, $K = 10^5$, $\alpha = 0.075$, and $x = 10^{-3}$. Our estimates show that $\varphi_n(k)$ plays no appreciable role for the given parameters, and it is therefore not considered below. The presence of the absorption line can be judged from the drop in the amplitudes of the resonance peaks in its area at the center of the figure.

Raising the rate of the frequency scan reduces the intermode interval. The value of initial phase Φ, which is responsible for temporal shifts of the resonance sequence as a whole, is also important. When parameter x, which characterizes the rate of frequency change, is greater than 10^{-2}, there is a considerable scatter in the peak values of the resonances when the signal sampling procedure is not changed. This scatter is caused by the altered distribution of the phases of interfering partial beams, and thus by other conditions of the formation of temporal interference maxima.

From the general tendency of the complexity of the radiation output's structure growing along with x, calculations show the points at which the values of x appear to be multiples of the value π disappear. At such values of x upon an increase in the rate of frequency tuning, the amplitudes of resonant peaks at the resonator output do not change. They remain equal to 1, except for the dip caused by absorption in the medium. The form of the dip qualitatively repeats the form factor of the absorption line.

MECHANISM OF MODE INTERACTION

When analyzing the intracavity field, we must also consider that abrupt changes occur in the phase of light oscillations, due to narrowing of the resonance peaks in their vicinity. This can be seen from Fig. 2, where the amplitude–phase structure of the output beam is shown on an enlarged scale. For better comparison of the plots of changes in the field amplitude and phase, they are given for $R = 0.9$. The abrupt

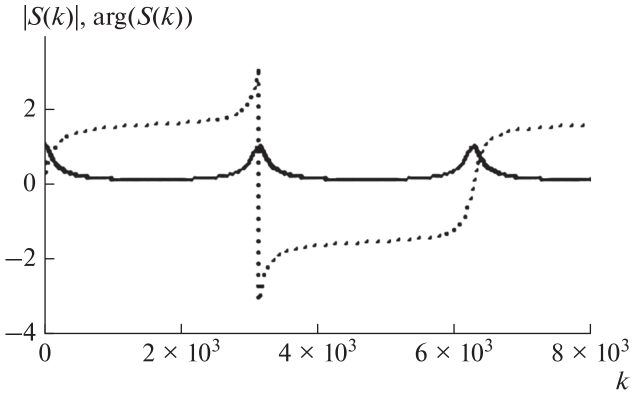

Fig. 2. Amplitude and phase distribution in the resonator's output beam. Solid lines are the field amplitude; dotted lines are the phase ($x = 10^{-3}$, $\Phi = -\pi/10$).

changes in phase in the vicinity of resonances are especially great. When analyzing the phase behavior on the left side of the resonance shown in the figure, we must consider aspects of the representation of phases in our calculation program (it reduces by 2π the values of a phase if they exceed π). Rapid changes in the phase of light oscillations near the points of resonance on the π scale could reduce the amplitude of resonance peaks when superimposing neighboring modes.

Let us consider this effect in detail. For the sake of clarity, we assume the quality factor of the resonator ensures a period τ of radiation attenuation that is comparable to time interval m between the excited modes. The period of attenuation is known to be directly related to the parameters of the resonator and is usually determined using the relation

$$\tau = 2L/c(1 - R_1R_2), \quad (3)$$

where L is the length of the resonator, and R_1, R_2 are the mirrors' coefficients of reflection (according to power). At the above relationship between τ and m, there is partial superpositioning of each field of resonance on the one that follows. This effect can be considered by replacing expression (2) with a sum that describes the effect preceding resonances have on another with order number k:

$$S(k) = \sum_{s=0}^{Q} \frac{a(1 - R^2)e^{i\delta(k+sm)d}}{1 - R^2e^{2i\delta(k+sm)d}} \frac{p(k + sm, \alpha)}{p^2(k + sm)} e^{-\frac{sm}{\tau}}. \quad (4)$$

The exponents by which the peak amplitudes are multiplied characterize the intensity of interaction between neighboring modes. Considering relaxation in the excitation of longitudinal modes introduces certain changes in the structure of the output radiation. This can be seen from Fig. 3a, which shows the breaking of symmetry upon a change in the resonance peaks in the central spectral region at $Q = 1$, $m = \tau$, and $\alpha = 0.075$. Such structural transformation of longitudinal resonator modes can deform the measured contour of the absorption line, the initial shape of which is also shown in Fig. 3a. Calculations show that at approximate equality of values m and τ in expression (4), we need only limit the number of terms determined by value $Q = 2$.

Figure 3b shows the dependence of the reduction of the peak in the center of the absorption line on coefficient of absorption α. The reduction is estimated relative to the maximum values in the frequency comb for different values of Q. We therefore introduce value $Y(\alpha) = S(0) - S(k_0)$, which characterizes the sensitivity of the output radiation field's transformation to the effect of intracavity absorption. It is noteworthy that the minimum reduction for a fixed value of α is observed when the interaction between two neighboring modes ($Q = 1$) with the corresponding phase rela-

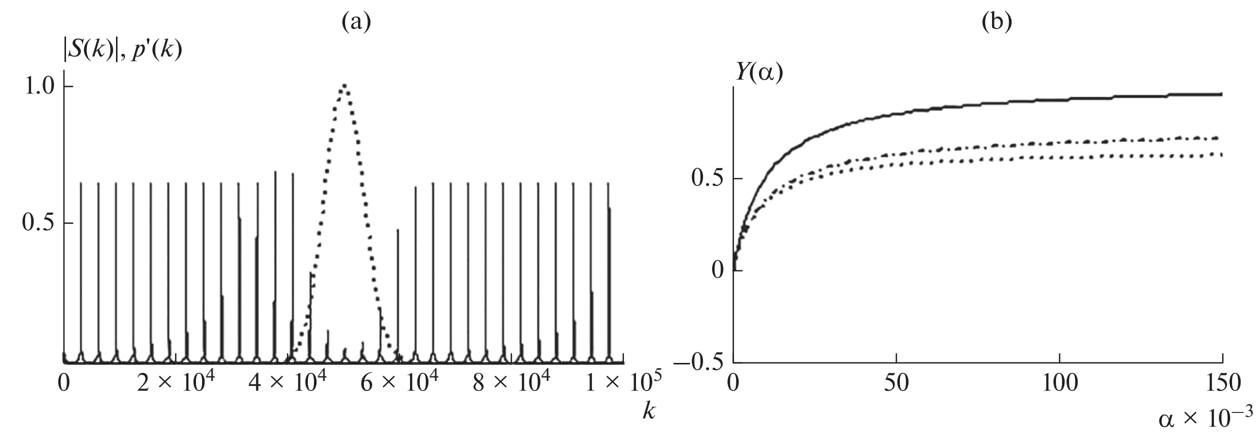

Fig. 3. Effect of intermode interaction on the structure of the output radiation at $m = \tau$: (a) solid lines show the amplitude of resonant peaks; dotted lines, the original shape of the absorption line. (b) Reduction of the peak amplitude in the central region of the absorption line upon a change in the coefficient of absorption: solid line, $Q = 0$; dashed line, $Q = 1$; dashed-and-dotted line, $Q = 2$.

tion between them is considered. This shows that the sensitivity of the system to variations in absorption inside the resonator falls upon such interaction. The situation is improved somewhat by considering the effect of yet another mode ($Q = 2$), since part of its field is superimposed on that of the chosen mode in the phase.

CONCLUSIONS

It was established that during fast scanning of the frequency of a master laser, we must consider the effect associated with the superpositioning of the fields of neighboring longitudinal modes of the analytical resonator. When analyzing this effect from the viewpoint of its influence on the sensitivity of spectral measurements, we must consider the specificity of phase relations between the excited resonances. Since interaction between modes can reduce the sensitivity of a spectrometer, we must ensure the possibility of limiting the rate of frequency scanning when optimizing its parameters, so that the time interval between modes exceeds the period of radiation attenuation in the resonator.

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