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Optical Terahertz Lumps of a Yajima–Oikawa–Kadomtsev–Petviashvili System

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Abstract—Rationally localized solutions (lumps) are obtained for a system of nonlinear equations describing the optical generation of terahertz radiation and generalizing the Yajima–Oikawa and Kadomtsev–Petviashvili equations. Conditions and features of the formation of such coupled optical-terahertz structures are discussed.

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INTRODUCTION

Over the last decade, the attention of researchers has been drawn to the generation of terahertz radiation. One reason for this is that terahertz radiation has numerous applications in, e.g., security systems, image recovery, communications, astronomy, medicine, and spectroscopy. One of the most efficient ways of generating such radiation is by optical means [1–3] based on the optical rectification effect observed in quadratically nonlinear media.

Theoretical descriptions of the optical generation of terahertz radiation produce systems of equations that are important for both practical applications and studying their mathematical properties. The possibility of localizing the generated terahertz radiation in space is of special interest. The aim of this work was therefore to investigate localized optical terahertz structures. We consider a case in which the localization of terahertz radiation is of a rational (exponential) character. Such structures are often called lumps.

BASIC EQUATIONS

Let us consider the case where an optical pulse with a wavefront perpendicular to axis z is fed to the input of a nonlinear medium. Let electric field E of the pulse be polarized in the plane of the principal cross section and have the form

$$E = \psi e^{i(\omega t - kz)} + \psi^* e^{-i(\omega t - kz)} + E_{\mathrm{T}}, \tag{1}$$

where ψ is the complex and slowly varying envelope of the optical component, ω and k are the carrier frequency and the longitudinal component of the wave vector of the optical component, and E_{T} is the terahertz component of the pulse.

Using Maxwell's equations and writing the polarization response of a quadratically nonlinear medium as the sum of the optical and terahertz components, we arrive at the system of equations

$$i\frac{\partial \psi}{\partial z} = -\frac{\beta}{2}\frac{\partial^2 \psi}{\partial \tau^2} + \alpha E_{\mathrm{T}}\psi, \tag{2}$$

$$\frac{\partial}{\partial \tau}\left(\frac{\partial E_{\mathrm{T}}}{\partial z} - \gamma\frac{\partial^3 E_{\mathrm{T}}}{\partial \tau^3} + \mu E_{\mathrm{T}}\frac{\partial E_{\mathrm{T}}}{\partial \tau} + \sigma\frac{\partial}{\partial \tau}\left(|\psi|^2\right)\right) = \frac{c}{2}\frac{\partial^2 E_{\mathrm{T}}}{\partial x^2}, \tag{3}$$

where c is the speed of light in a vacuum; $\tau = t - z/v_{\mathrm{g}}$; group velocity v_{g} of the optical component is expressed as $1/v_{\mathrm{g}} = \partial k/\partial\omega$; $k = \omega n/c$; $n = 1 + 2\pi\chi_\omega$ is the optical refractive index; $\chi_\omega = \int_0^\infty \chi(\xi) e^{-i\omega\xi} d\xi$, $\chi(\xi)$ is the temporal linear susceptibility of the medium; $\beta = \partial^2 k/\partial\omega^2$ is the group velocity dispersion (GVD) parameter of the optical component, $\gamma = \pi\left(\partial^2\chi_\omega/\partial\omega^2\right)_{\omega=0}/c$ is the terahertz component dispersion parameter; $\alpha = 4\pi\omega\chi^{(2)}/c$; $\mu = \sigma = 4\pi\chi^{(2)}/c$; and $\chi^{(2)}$ is the nonlinear quadratic susceptibility.

In deriving system (2), (3), we assumed the diffraction was planar, ignored the nonlocality of the nonlinear part of the polarization response of a medium, used the unidirectional propagation approximation

[4], and assumed the dispersion to be weak. In addition, we assumed the Zakharov–Benney condition (ZB) [5] to be valid. (In our case, it has the form $v_g = c/n_T$, where $n_T = 1 + 2\pi\chi_0$ is the terahertz refractive index.) Meeting the ZB condition ensures the most efficient generation of terahertz radiation. In addition, the diffraction of the optical component was ignored when deriving Eq. (2), since its diffraction length is three–four orders of magnitude greater than the corresponding length of the terahertz component.

It should be noted that the functions of the pulse field components in system (2), (3) are different. The terahertz component cannot generate the optical one and propagates in the mode described by Eq. (3) (i.e., the KP-I equation; see below). The optical component in this case generates the terahertz one.

Let us consider particular cases of system (2), (3). In Eq. (3), we ignore diffraction and set $\gamma = \mu = 0$. After integration, we obtain the equation

$$\frac{\partial E_T}{\partial z} = -\sigma \frac{\partial}{\partial \tau}\left(|\psi|^2\right). \tag{4}$$

The system of Eqs. (2), (4) is known as the Yajima–Oikawa (YO) system [6] and is encountered in numerous physical problems (see, e.g., [5]).

If $\sigma = 0$, the dynamics of the optical component is independent of the terahertz component. Equation (3) is then the type-I Kadomtsev–Petviashvili equation (KP-I) [7]. We therefore refer to Eqs. (2), (3) as a Yajima–Oikawa–Kadomtsev–Petviashvili (YO–KP) system.

Note that the YO system and the KP equation are integrable according to the inverse scattering transformation method [5, 7]. The YO system thus has soliton solutions, and the KP equation has solutions in the form of oblique solitons, along with ones in the form of so-called lumps. Lumps are essentially non-one-dimensional solutions (in contrast to solitons), which are localized in a rational (power-low) manner.

Note that if we ignore diffraction in Eq. (3) (i.e., if the right-hand side of the equation is zero), after integration we arrive at a system consisting of the linear Schrödinger and Korteweg–de Vries equations. This system contains the only spatial variable and was studied in detail in [8, 9]. Some of its solutions correspond to the oblique optical terahertz solitons of YO–KP system (2), (3). In the next section, we consider the non-one-dimensional solutions of the YO–KP system.

SOLUTIONS IN THE FORM OF LUMPS

Let us first us consider a case where the YO–KP system (2), (3) is reduced to equation KP-I. We assume $\psi = 0$. The solution to Eq. (3) in the form of a lump is then

$$E_T = E_0 - \frac{12\gamma}{\mu}\frac{\partial^2}{\partial \tau^2}\ln(\Delta_0), \tag{5}$$

where

$$\Delta_0 = (\tau + b_0 x + c_0 z)^2 + d_0\left(x + c b_0 z\right)^2 - \frac{6\gamma}{c d_0},$$

$$E_0 = -\frac{c\left(d_0 - b_0^2\right) + 2nc_0}{2\mu},$$

and b_0, c_0, and d_0 are arbitrary real constants. In these formulas, we can introduce shifts by independent variables τ, x, and z. Without loss of generality, we assume here and below that these shifts are equal to zero.

Solution (5) is nonsingular if $d_0 > 0$ and $\gamma < 0$. It in this case rationally (power-low) approaches constant background E_0 of the terahertz component when the absolute values of variables x and z grow infinitely. Constant background E_0 will be zero if we impose the condition

$$d_0 = b_0^2 - \frac{2c_0}{c},$$

on the lump parameters.

In the plane of variables x and z, the lump moves without changing its shape with constant velocity. The velocity projections in the laboratory system of coordinates are

$$v_x = -c b_0 v_z, \quad v_z = \left(\frac{n_T}{c} + c b_0^2 - c_0\right)^{-1}.$$

We can see that the direction of the lamp velocity vector is determined only by free parameter b_0, while the absolute value of the velocity is determined by two parameters, b_0 and c_0.

We now consider rationally localized solutions of YO–KP system (2), (3). To find such solutions, we use the simplest approach, in which representation (5) is used for the terahertz component. We can then show that if the coefficients of YO–KP system (2), (3) are related as

$$\beta\mu + 12\alpha\gamma = 0, \tag{6}$$

it has a solution in the form

$$\psi = \left(f_0 + \frac{f_1\tau + f_2 z + f_3 x + f_4}{\Delta}\right) \times e^{i(K_x x + K_z z + \Omega\tau)}, \tag{7}$$

$$E_T = E_0 - \frac{12\gamma}{\mu}\frac{\partial^2}{\partial \tau^2}\ln(\Delta), \tag{8}$$

where

$$K_z = -\frac{6\alpha\gamma\Omega^2}{\mu} - \alpha E_0,$$

$$\Delta = (\tau + b_0 x + c_0 z)^2 + d_0 (x + e_0 z)^2 - \frac{6\gamma}{c d_0},$$

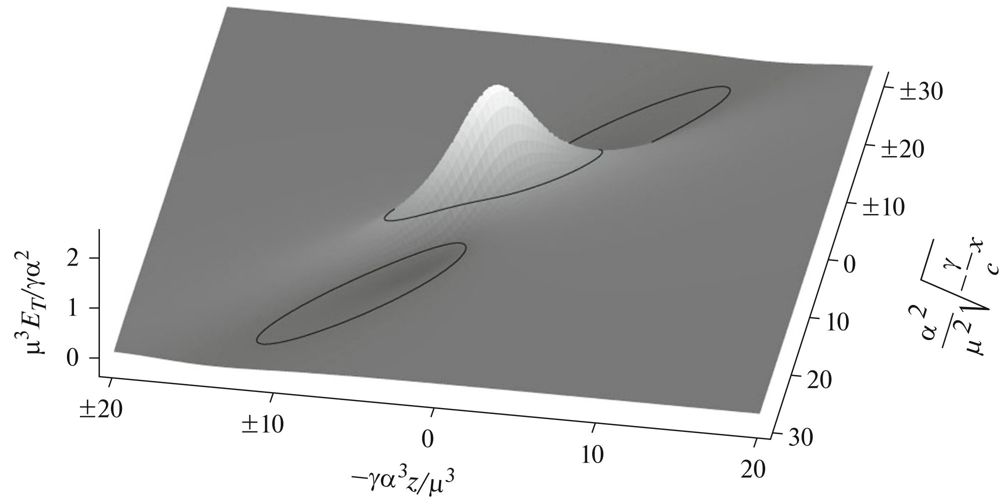

Fig. 1. Profiles of variable E_T of a lump with parameters $b_0 = \frac{\alpha}{\mu}\sqrt{-\frac{\gamma}{c}}$, $c_0 = -\frac{\gamma\alpha^2}{\mu^2}$, $\Omega = 0$ when $\tau = 0$.

$$e_0 = \pm\frac{\alpha}{\mu}\sqrt{-6c\gamma}, \tag{9}$$

$$f_0 = -f_4\frac{r^2 + \mu^2 d_0 e_0^2}{144\alpha^2\gamma^2}, \quad f_1 = if_4\frac{r}{6\alpha\gamma},$$

$$f_2 = if_4\frac{c_0 r + \mu d_0 e_0^2}{6\alpha\gamma}, \quad f_3 = if_4\frac{b_0 r + \mu d_0 e_0}{6\alpha\gamma},$$

$$|f_4|^2 = \frac{864\alpha^2\gamma^3(cb_0 - e_0)}{\sigma\mu^2 e_0 r}, \tag{10}$$

$$E_0 = -\frac{cr + \mu e_0(e_0 - cb_0)}{2\mu r}d_0 + \frac{(e_0 - cb_0)r}{2\mu^2 e_0} - \frac{c_0}{\mu} + \frac{cb_0^2}{2\mu},$$

$$r = -\mu c_0 - 12\alpha\gamma\Omega.$$

Here, b_0, c_0, d_0, K_x, and Ω are the real constants. Since parameter e_0 is considered to be real, condition $\gamma < 0$ must be met. Constant b_0 and the sign of e_0 (see (9)) should then ensure the non-negative right-hand side of Eq. (10). Note that the solution is nonsingular only when $d_0 > 0$.

Using the expressions for β, μ, α, and γ, constraint (6) on the parameters of the medium can be written in the form $\partial^2 k/\partial\omega^2 = -12\pi\omega\left(\partial^2\chi_\omega/\partial\omega^2\right)_{\omega=0}/c$.

With infinite growth of $|x|$ and $|z|$, the absolute value of the optical component and the terahertz component in this solution tend rationally to constant background $|f_0|$ and E_0, respectively. If we assume

$$d_0 = \frac{(e_0 - cb_0)r + \mu e_0\left(cb_0^2 - 2c_0\right)}{\mu e_0(cr + \mu e_0(e_0 - cb_0))}r, \tag{11}$$

constant background E_0 of the terahertz component will be zero. Constant background f_0 of the optical component can be equal to zero only when $d_0 < 0$, which yields a singular rationally decreasing solution.

Solution (7), (8) can also be called a lump. Figure 1 shows the profile of its terahertz component for the case when parameter d_0 is determined from Eq. (11) and the positive sign is chosen in formula (9).

In the plane of variables x and z, the terahertz component and the absolute value of the optical component of lump (7), (8) move with constant velocity without changing their shape. The velocity projections in the laboratory system of coordinates are

$$v_x = \mp\frac{\alpha}{\mu}\sqrt{-6c\gamma}v_z, \quad v_z = \left(\frac{n_T}{c} \pm \frac{\alpha}{\mu}\sqrt{-6c\gamma}b_0 - c_0\right)^{-1}.$$

In contrast to the above, the direction of the lump velocity vector is independent of its parameters and takes one or two values. The tangent of angle θ between the z axis and the velocity vector is

$$\tan\theta = \mp\frac{\alpha}{\mu}\sqrt{-6c\gamma}.$$

The absolute value of the velocity is determined here by parameters b_0 and c_0. The sign in this relation should be such that the right-hand side of formula (10) is non-negative. Depending on the value of b_0, we can accept one or both variants of the sign.

Let us discuss the interplay between the investigated solutions in the form of lumps. In limit

$b_0 \to e_0/c$, lump (7), (8) becomes a special case of lump (5) when

$$b_0 = \pm \frac{\alpha}{c\mu}\sqrt{-6c\gamma}.$$

This is because the expressions for the terahertz component of the lumps contain the same number of free parameters in both cases.

CONCLUSIONS

We obtained rationally localized solutions in the form of so-called lumps for the system of YO–KP equations, which describes the generation of terahertz radiation with allowance for its dispersion and intrinsic nonlinearity. It was shown that in this case, terahertz radiation is generated most efficiently in two selected directions.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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