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Scattering rates of inner belt protons by EMIC waves: A comparison between test particle and diffusion simulations

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[1] Inner belt energetic protons are a hindrance to development of space technologies. The emission of electromagnetic ion cyclotron (EMIC) waves from spaceborne transmitters has been proposed as a way to solve this problem. The interaction between particles and narrowband emissions has been typically studied using nonlinear test particle simulations. We show that this formulation results in a random walk of the inner belt protons in velocity space. In this paper we compute bounce-averaged pitch angle diffusion rates from test particle simulations and compare them to those of quasi-linear theory for quasi-monochromatic EMIC waves interacting with inner belt protons. We find that the quasi-linear solution is not sensitive to the frequency bandwidth for narrow distributions. Bounce-averaged diffusion coefficients from both approaches are in good agreement for all energies and pitch angles. The interaction with inner belt protons, therefore, can be addressed using quasi-linear diffusion codes, which allows faster exploration of parameter space. **Citation:** de Soria-Santacruz, M., K. G. Orlova, M. Martinez-Sanchez, and Y. Y. Shprits (2013), Scattering rates of inner belt protons by EMIC waves: A comparison between test particle and diffusion simulations, *Geophys. Res. Lett.*, *40*, doi:10.1002/grl.50925.

1. Introduction

[2] The inner Van Allen radiation belt traps highly energetic protons up to 300 MeV sourced from solar storms, cosmic rays, and other processes. These particles can rapidly damage solar panels, electronics, and other space systems orbiting the inner region, limiting access to Low Earth Orbit [*Baker*, 2000, 2001]. Decades of modeling and observations, however, show that naturally generated ultralow frequency/very low frequency (VLF) waves have the capability of precipitating energetic trapped electrons as well as ring current protons [e.g., *Abel and Thorne*, 1998; *Albert*, 1999; *Jordanova et al.*, 2001; *Loto'aniu et al.*, 2006]. This fact suggests that there could be human control over the stable inner belt proton population by artificially transmitting electromagnetic ion cyclotron (EMIC) waves from spaceborne antennas. These waves are naturally generated by the cyclotron instability of ring current ions of tens of keV located at $L > 3$ [*Meredith et al.*, 2003], which explains the absence of EMIC power at lower L-shells. Consequently, the precipitation of MeV protons requires artificial generation of EMIC waves into the inner zone. It must be noted that many previous missions did not have the ability to observe EMIC waves at $L < 3$, which may also account for the absence of EMIC wave observations at these L-shells. The recently launched Van Allen Probes, however, are equipped to detect such waves at very low L values. The controlled removal of energetic outer belt electrons by man-made whistler waves has been widely studied [*Inan et al.*, 1984, 2003; *Kulkarni et al.*, 2008]. Contrarily, the interaction between inner belt protons and EMIC waves radiated from in situ transmitters is an unexplored situation that should be addressed given its relevance to the scientific and engineering communities.

[3] The interaction between particles and monochromatic waves is commonly calculated with computationally intensive test particle simulations of the nonlinear equations of motion. This formulation is capable of reproducing narrowband coherent waves characteristic of VLF transmitters [*Inan et al.*, 1978; *Bell*, 1984; *Tao and Bortnik*, 2010]. On the other hand, the particles' scattering by natural emissions is frequently addressed using quasi-linear theory [*Kennel and Engelmann*, 1966], which dramatically reduces the required CPU time. Quasi-linear theory finds diffusion coefficients [*Lyons and Thorne*, 1972; *Lyons et al.*, 1972] to solve the Fokker-Planck equation for the distribution function of energetic particles perturbed by the wave. This approach traditionally assumes a Gaussian distribution of wave frequencies, and for this reason, it is commonly used to reproduce the effect of natural broadband incoherent emissions [e.g., *Lyons*, 1974; *Albert*,2003; *Li et al.*, 2007].

[4] *Tao et al.* [2011] recently compared quasi-linear diffusion coefficients and test particle simulations for energetic electrons scattered by whistler waves. They showed that the electron's scattering induced by small amplitude, broadband, incoherent whistler waves is stochastic, and demonstrated that test particle simulations are in good agreement with quasi-linear diffusion theory. In a later paper, *Tao et al.* [2012] used nonlinear test particle simulations to model the interactions between electrons and chorus subpackets with realistic amplitude modulation taken from observations. While previous research based on single-wave elements lead to nonlinear phase trapping and bunching [*Albert*, 2002; *Bortnik et al.*, 2008]. *Tao et al.* [2012] showed that realistic chorus amplitude modulation could significantly modify wave-particle interactions predicted by single-wave models.

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[5] In this paper, and for the first time, we discuss both formulations for the case of inner belt protons interacting with monochromatic field-aligned EMIC waves generated by spaceborne antennas. The simulations assume that the man-made EMIC waves cover a broad region in Magnetic Local Time (MLT), thus allowing phase trapping of the rapidly drifting protons. Using a test particle simulation, we study the linearity of the EMIC-proton interaction. We show that protons perform a random walk in velocity space in a similar way to diffusion processes for fairly large wave amplitudes of up to 2 nT. We next compare the solution of the nonlinear equations of motion with a computationally efficient result from quasi-linear theory.

2. Description of the Models

[6] The interaction between EMIC waves and energetic protons is calculated at L=1.5 using a dipole model for the Earth's magnetic field. The code assumes cold plasma in diffusive equilibrium with an equatorial electron density of $n_e = 3.05 \cdot 10^4$ cm^{-3} and thermal ion composition given by 85% H^+, 10% He^+, and 5% O^+ [*Meredith et al.*, 2003]. The heavy ion composition has been assumed constant within the plasmasphere [*Horwitz*, 1987]. These concentrations, however, are difficult to measure due to the high spacecraft potentials, which may lead to inaccurate measurements. The waves are considered field aligned with the Earth's magnetic field, with a mean amplitude of 2 nT and a frequency of 4 Hz. This frequency falls within the EMIC oxygen band, and it is much below the oxygen gyrofrequency at the equator, f_{O^+} = 9 Hz. The parameters above result in an equatorial proton resonant energy of 3.77 MeV for loss cone particles (α_{eq} = 28°) at L=1.5, which corresponds to an atmospheric height for particle loss of 100 km.

[7] We calculate the solution of the nonlinear gyroaveraged equations of the motion of protons interacting with field-aligned EMIC waves. These equations were initially derived by *Inan et al.* [1978] for whistlers and electrons, and we rewrote them for the case of protons and EMIC waves. The particles are started at their turning point in the Southern Hemisphere, and we calculate the single-pass wave-particle interaction along half of their bouncing period. The simulation assumes azimuthal spreading of the waves over a broad MLT region at L=1.5, which translates into interaction times larger than the proton's gyroperiod such that the equations of motion can be gyroaveraged. Due to the fast drifting period of inner belt protons, however, very short interactions could dominate the scattering in the case of beam-like waves that have very small MLT spreading. Nevertheless, the scattering resulting from interactions shorter than the particle's gyration is out of the scope of this paper, the latter targeting the comparison between the gyroaveraged nonlinear formulation and the quasi-linear diffusion approach.

[8] In the case of linear interactions, bounce-averaged diffusion coefficients $\langle D_{\alpha\alpha}\rangle$ may be estimated from the test particle simulation as follows

$$\langle D_{\alpha\alpha}\rangle = \frac{\langle \Delta\alpha^2\rangle}{2\tau_b}, \tag{1}$$

where $\Delta\alpha$ is the protons' pitch angle scattering calculated between mirror points, τ_b is the protons' bounce period [*Lenchek et al.*, 1961], and the brackets denote the average over initial Larmor phase.

[9] Additionally, we use the inhomogeneity parameter R to characterize the linearity of the interaction. This parameter represents the competition between adiabatic and wave-induced motions of the protons interacting with field-aligned waves [*Omura et al.*, 2008; *Albert and Bortnik*, 2009]

$$R=\left|\frac{B_0}{B_w}\frac{n^2}{n^2-1}\frac{c}{v_\perp}\frac{1}{k}\left(\gamma\frac{\omega}{\Omega_{H^+}}\frac{v_\parallel^2}{c^2}\frac{\partial n}{\partial s}+\frac{1}{B_0}\frac{\partial B_0}{\partial s}\left(\frac{v_\parallel}{c}-\frac{n\gamma}{2}\frac{\omega}{\Omega_{H^+}}\frac{v_\perp^2}{c^2}\right)\right)\right|, \tag{2}$$

where B_0 and B_w are the Earth's magnetic field and wave mean magnitude, respectively. k is the wave number magnitude, $n = ck/\omega$, γ is the relativistic factor, s is the position along the field line, Ω_{H^+} is the proton cyclotron frequency, and $v_\parallel$ and $v_\perp$ are the parallel and perpendicular velocities of the particle, respectively. All the quantities are evaluated at the resonance point. $R \gg 1$ means that the interaction behaves linearly, while $R \ll 1$ implies that nonlinear effects dominate.

[10] For comparison with quasi-linear theory, we use the code of *Shprits et al.* [2006] that follows the formulation of *Summers* [2005] to estimate local pitch angle diffusion coefficients, reduced by a factor of 2 due to *Albert* [2007]. The bounce averaging is performed using the approach from *Lyons et al.* [1972]. The waves are assumed to have a Gaussian spectral density given by

$$W(\omega) = \frac{(B_w)^2}{8\pi}\frac{1}{\rho}\frac{1}{\delta\omega}exp\left[-\left(\frac{\omega-\omega_m}{\delta\omega}\right)^2\right], \tag{3}$$

inside the range of frequencies from ω_1 = 1 rad/s to ω_2 = 100 rad/s. ρ is a normalization factor given by $\rho = \sqrt{\pi}/2\,[erf((\omega_m-\omega_1)/\delta\omega)+erf((\omega_2-\omega_m)/\delta\omega)]$, ω_m and $\delta\omega$ are the frequency of maximum wave power and bandwidth, respectively, and *erf* is the error function.

3. Results

[11] Integration of the nonlinear equations of motion of a sheet of 3.77 MeV protons with α_{eq} = 28° interacting with field-aligned EMIC waves is presented in Figures 1a and 1b; the color code represents a uniform distribution in Larmor phase defined with 12 test particles. Figure 1a shows the scattering as a function of latitudinal location, and the black dashed line corresponds to its root-mean-square (RMS) value. The oscillations observed away from the equator are generated by the protons' off-resonant interaction with the wave fields. These oscillations are not seen at the equator during the cyclotron resonant interaction because the particle's gyration stays locked with the wave fields' rotation for a significant amount of time, which extends along a latitude range of ±8° around the equator. Figure 1a shows that particles perform a random walk in velocity space in a similar way to diffusion processes. Furthermore, the sinusoidal dependence of the scattering on initial Larmor phase in Figure 1b is also characteristic of linear interaction processes and can be derived by linearizing the equations of motion [*Inan et al.*, 1978]. The EMIC-electron interaction in the outer radiation belt is nonlinear for wave amplitudes around 2 nT and larger [*Albert and Bortnik*, 2009]. For the same wave amplitude, however, we show that the EMIC-proton interaction in the inner belt behaves linearly, which is due to the larger proton mass

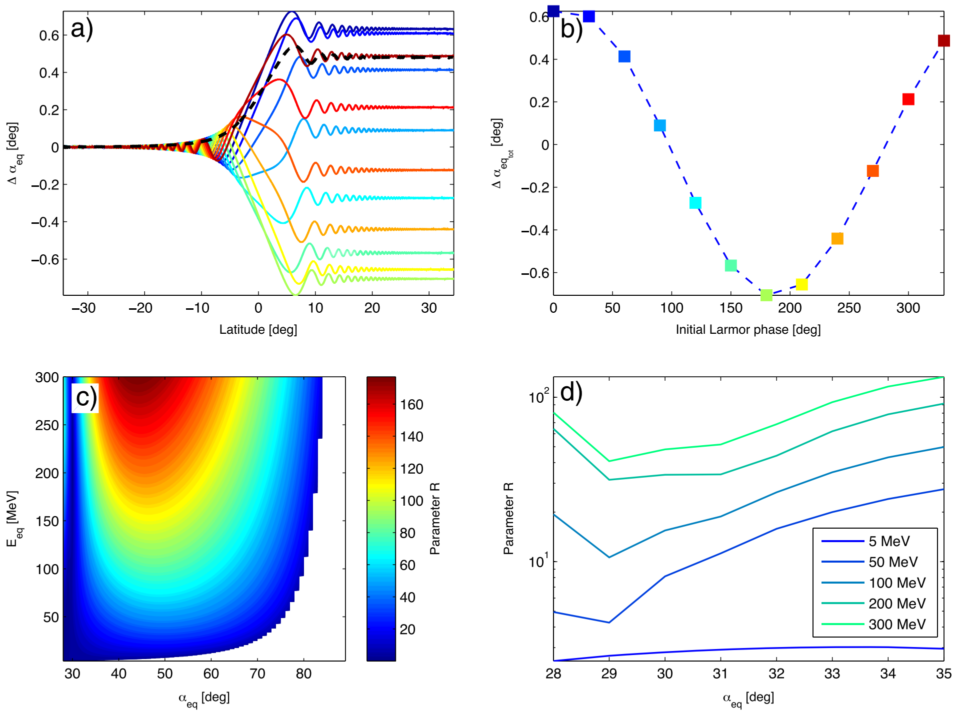

Figure 1. (a) Pitch angle scattering of equatorially resonant loss cone protons interacting with 4 Hz field-aligned EMIC waves. The trajectory corresponds to half of the proton's bouncing motion. The color code represents a uniform distribution in Larmor phase, and the black dashed line corresponds to the RMS value of the scattering. (b) Total pitch angle scattering as a function of initial Larmor phase for the case in Figure 1a. (c) Inhomogeneity parameter R as a function of proton energy and equatorial pitch angle. (d) Inhomogeneity parameter R of equatorial resonant particles as a function of equatorial pitch angle.

compared to electrons and the stronger inner belt background magnetic field. The behavior of the wave-particle interaction can also be determined from Figure 1c, which presents the inhomogeneity parameter in equation (2) as a function of protons' energy and equatorial pitch angle. R is larger than unity in most of the range under consideration, suggesting that nonlinear effects are negligible thus quasi-linear approximations of the bounce-averaged diffusion coefficients (equation (1)) should adequately describe the system. For clarification, Figure 1d shows line plots at different energies of the inhomogeneity parameter as a function of equatorial pitch angle. R is close to one for the smaller energy values, which could represent a limiting case between linear and nonlinear phenomena. These conclusions will be examined below with the results from the test particle simulations.

[12] In our test particle simulations, we consider monochromatic waves. Comparison with quasi-linear bounce-averaged diffusion coefficients, therefore, requires a narrow $\delta\omega$ of the Gaussian wave frequency distribution in equation (3). Figure 2a presents a sensitivity analysis of the quasi-linear $\langle D_{\alpha\alpha} \rangle$ on bandwidth, $\delta\omega$, where $\delta\omega = C \cdot \omega_m$ and C is a scaling factor. Diffusion coefficients are calculated for three different energies, $E = 4$, 50, and 200 MeV. It can be observed that the quasi-linear $\langle D_{\alpha\alpha} \rangle$ is independent of bandwidth for $C \leq 10^{-2}$. Quasi-linear scattering rates, therefore, are likely insensitive to this parameter for narrow distributions.

[13] The comparison of bounce-averaged diffusion coefficients calculated using both test particle and diffusion approaches is finally presented in Figure 2b and 2c. The Gaussian wave frequency distribution used in quasi-linear theory is taken to be narrow with $\omega_m = 8\pi$ rad/s (= 4 Hz) and $\delta\omega = 0.01 \cdot \omega_m$. Figure 2b presents $\langle D_{\alpha\alpha} \rangle$ of loss cone protons as a function of energy, where it can be observed that test particle and quasi-linear formulations give similar results. The oscillating behavior of the test particle solution is due to the phase coherence between the two resonance points at both sides of the equator. Figure 2c presents $\langle D_{\alpha\alpha} \rangle$ of 100 MeV protons as a function of equatorial pitch angle. In this case, test particle and quasi-linear formulations give similar results for $\alpha_{eq} < 80°$; protons with $\alpha_{eq} > 80°$ have minimum resonant energies larger than 100 MeV and are therefore out of resonance at any point along their bouncing motion. The quasi-linear approach does not allow to quantify diffusion coefficients for nonresonant particles with $\alpha_{eq} > 80°$. These particles are out of resonance with the waves and can only be scattered due to nonresonant interactions. The test particle solution of the equations of motion is capable of capturing this weak off-resonant scattering, which results in a smooth transition to zero diffusion at large pitch angles. The bounce-averaged diffusion coefficients estimated using a quasi-linear code adequately describe the test particle simulations.

[14] Despite the fact that the waves under consideration are monochromatic, the analysis above shows that quasi-linear theory with a narrow frequency distribution can accurately reproduce the results from test particle narrowband simulations and significantly reduce the required computational time.

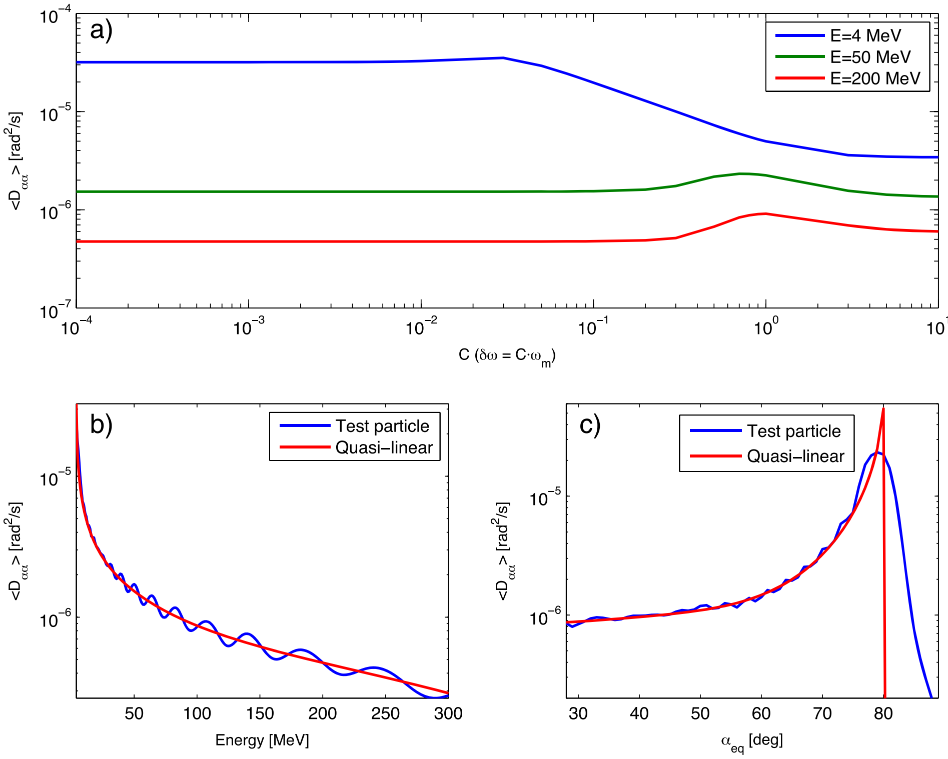

Figure 2. (a) Quasi-linear bounce-averaged diffusion coefficient for three different proton energies as a function of the bandwidth of the frequency distribution. (b) Comparison between quasi-linear and test particle diffusion coefficients for loss cone particles (α_{eq} = 28°, corresponding to an atmospheric height for particle loss of 100 km) as a function of proton energy. (c) Comparison between quasi-linear and test particle diffusion coefficients of 100 MeV protons as a function of protons' equatorial pitch angle.

4. Discussion and Summary

[15] The radiation of electromagnetic ion cyclotron waves from in situ transmitters has been proposed as a way to remediate the energetic proton radiation in the inner Van Allen belt. This technology could enable safe access to new orbits that would open up possibilities for space missions and unprecedented science. Using a test particle simulation of the nonlinear gyroaveraged equations of motion, we have shown that energetic inner belt protons respond linearly to field-aligned monochromatic EMIC waves radiated from spaceborne antennas that cover a broad MLT region. Test particles perform a random walk in velocity space in a similar way to diffusion processes. Based on this result, we have compared the bounce-averaged pitch angle diffusion coefficients estimated from the nonlinear equations of motion to those calculated from quasi-linear theory. Narrowband coherent signals characteristic of VLF transmitters are commonly reproduced using test particle simulations. On the other hand, quasi-linear theory is typically used for natural broadband incoherent emissions. We have shown that the quasi-linear diffusion approach can be used with a narrow frequency distribution to accurately reproduce the results from test particle narrowband simulations and significantly reduce the required computational time. Furthermore, the quasi-linear solution is not sensitive to the frequency bandwidth for relatively narrow distributions. Bounce-averaged diffusion coefficients estimated from both approaches are in good agreement for all energies, pitch angles, and for fairly large wave amplitudes up to 2 nT. The only difference appears at large pitch angles where particles are out of resonance at any point along their bouncing motion. Quasi-linear theory does not allow to quantify diffusion coefficients for nonresonant particles, but the test particle solution is capable of capturing the small off-resonant scattering, which results in a smooth transition to zero diffusion at large pitch angles. Our simulations also agree with the conclusions of *Albert* [2007], evident as well in *Shprits et al.* [2006], showing that *Summers* [2005] should be corrected by a factor of 2. The analysis suggests that, although monochromatic, the interaction between man-made EMIC waves and inner belt protons need not be addressed using computationally intensive test particle simulations but instead are adequately described by CPU efficient quasi-linear diffusion codes. Future work will examine obliquely propagating EMIC waves, which are a more realistic approximation to the radiation pattern of a spaceborne transmitter. This more realistic approximation using oblique waves is expected to increase the linearity and inhomogeneity parameters further.

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