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Valentin Skvortsov

ON SOME PROBLEMS IN FOURIER ANALYSIS ON COMPACT GROUPS

One of the main field in abstract harmonic analysis is the theory of representation of functions, defined on a compact abelian group, by series with respect to the sequence of characters of this group (see [1]). If the coefficients of a series are uniquely defined by the function, represented by the series (convergent or summable in a certain sense), and it is known that this function is Lebesgue integrable, then it is natural to expect that the coefficients are Fourier coefficients. It is a particular case of the problem of recovering the coefficients of orthogonal series from its sum.

A difficult situation arises if the sum of the series fails to be Lebesgue integrable. For example, in the classical case of characters of one-dimensional torus (system of exponents) it is known that the series

$$\Big(\sum_{k=-\infty}^{-1}+\sum_{k=1}^{\infty}\Big)\frac{\exp(ikx)}{\ln k}$$

converges everywhere but fails to be the Fourier-Lebesgue series. This kind of examples can be given for many other systems of characters as well.

Two typical (in a certain way opposite) cases of compact abelian groups are the above mentioned one-dimensional torus, on one side, and the family of so-called zero-dimensional groups (Cantor dyadic group or group of p-adic integers), on the other side (see [1], [2]). For systems of characters of those groups a complete solution of the problem of recovering the coefficients of point-wise convergent series is found by constructing some generalization of Lebesgue integral on the group, so

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that any such a series is the Fourier series of its sum, if this integral is used in the Fourier formulas.

In the classical case the solution was found by Denjoy and later, in another way, by D. Preiss and B. Thomson (see [3]). For some particular zero-dimensional groups see [4].

We discuss in our talk various ways for solution of this problem. One of the most useful construction of a generalized integral to solve this problem is Henstock-Kurzweil-type integrals (see [5]), with respect to specially chosen basis on the group (see [4]).

A generalization of the above problems can be considered also in the case of a locally compact groups. In this case the problem of recovering coefficient is getting reduced to obtaining an inversion formula for integral transforms with kernel expressed in terms of characters of the corresponding group (see [4]).

Some of our results obtained in [4] can be extended to the non-abelian case (see [6]). In this case the role of the dual object of a group G is played by the set of equivalence classes of irreducible unitary representations of G (see [1]), instead of the system of characters. In the compact case, an analog of a series in the system of characters is a formal series in the system of representations. We find a condition necessary and sufficient for the operator coefficients of such a formal series to be the Fourier-Stieltjes coefficients of an additive measure on the group and prove that if this series is everywhere convergent to a finite function f then f is integrable on G in the sense of some generalization of Henstock-Kurzweil integral and the series is the Fourier-Stieltjes series of the measure $\mu = \int f$.

References

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Lomonosov Moscow State University
Email address: vaskvor2000@yahoo.com