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# ON TYPICAL AND ATYPICAL ASYMPTOTIC BEHAVIOR OF SINGULAR SOLUTIONS TO EMDEN–FOWLER TYPE EQUATIONS

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Consider the equation

$$y^{(n)} = p(x, y, y', \dots, y^{(n-1)})|y|^k \operatorname{sign} y, \tag{1}$$

where $n \geq 2$, $k > 1$, $p$ is a positive, continuous and Lipschitz continuous in the last $n$ variables function. Consider also a special case of (1), namely

$$y^{(n)} = p_0\,|y|^k \operatorname{sgn}\, y \tag{2}$$

with $p_0 > 0$.

We discuss the problem posed by I.Kiguradze (see [1, Problem 16.4]) on asymptotic behavior of all positive non-extensible (so-called "blow-up") solutions to this equation. It appears for $n = 2$ (see [1]), $n = 3, 4$ (see [2]), that if $p(x, y_1, y_2, \dots, y_{n-1})$ tends to $p_0$ as $x \to x^* - 0, y_0 \to \infty, \cdots, y_{n-1} \to \infty$, then all such solutions have the power-law asymptotic behavior

$$y(x) = C(x^* - x)^{-\alpha}(1 + o(1)), \quad x \to x^* - 0, \tag{3}$$

with $\alpha = \frac{n}{k-1}$, $C = \left(\frac{\alpha(\alpha+1)\dots(\alpha+n-1)}{p_0}\right)^{\frac{1}{k-1}}$.

The same is true for weakly super-linear equations.

**Theorem 1.** ([3]) *Suppose $p \in C(\mathbf{R}^{n+1}) \bigcap Lip_{y_0,\dots,y_{n-1}}(\mathbf{R}^n)$ and $p \to p_0 > 0$ as $x \to x^*, y_0 \to \infty, \dots, y_{n-1} \to \infty$. Then for any integer $n > 4$ there exists $K > 1$ such that for any real $k \in (1, K)$, any solution to equation (1) tending to $+\infty$ as $x \to x^* - 0$ has the power-law asymptotic behavior (3).*

In the case $n \geq 12$ even if we deal with equation (2), another type of asymptotic behavior of singular solutions appears (see [3, 4, 5]).

If we have more strong nonlinearity, then the power-law asymptotic behavior becomes atypical. The following theorem generalizes the results of [6]:

**Theorem 2.** *If $12 \leq n \leq 100000$, then there exists $k_n > 1$ such that at any point $x_0 \in \mathbb{R}$ the set of initial data of asymptotically power-law solutions to equation (2) has zero Lebesgue measure whenever $k > k_n$.*

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