Application of the KAN theorem to two coupled nonlinear oscillators

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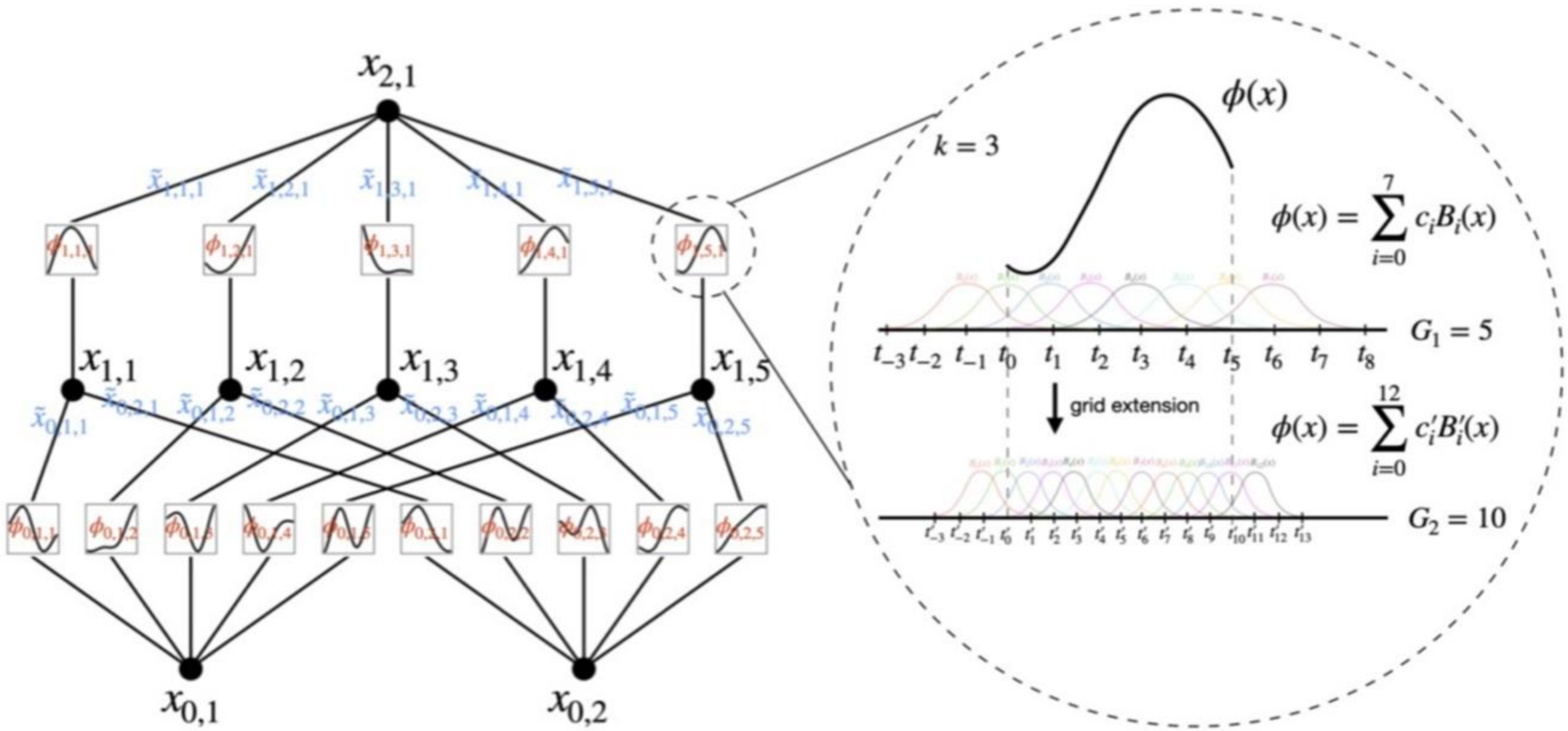


N Novgorod 2024

This article examines the influence of the hysteresis link on the behavior of a system of coupled nonlinear oscillators with internal resonance. The sinusoidal KAN model for two interacting oscillators is chosen as the model of the hysteresis link. In earlier works, the Bouk-Ven model was chosen to describe hysteresis [1].

$$\dot{z} = \sigma\dot{x} - \beta\dot{x}|z|^n - \gamma z|\dot{x}||z|^{n-1}$$

Where: σ, β, γ, n - parameters characterizing the Bowk-Wen model (n $\in$ N).

The hysteresis loops of adjacent cycles differ due to the irreversibility of the processes accompanied by energy dissipation. Analytical matching of hysteresis curves of adjacent cycles is a difficult task. Such mathematical models often turn out to be not only complex, but also very limited for describing real processes, and the KAN theorem can be used to describe hysteresis during non-stationary oscillations of a mechanical system [2].

Discrete models.

The study of the effect of periodic influence on relaxation oscillations arising in a Van der Pol-type oscillator greatly influenced the development of mathematics in this direction. Early studies of such models demonstrated both bistability (the ability to observe one of two modes depending on the initial conditions) and aperiodic dynamics.

Studies of periodic effects on nonlinear oscillators have shown that multistability and aperiodic dynamics can be explained by considering one-dimensional circle mappings (functions that map the circumference of a circle onto itself). It has also been shown that such mappings can exhibit aperiodic dynamics as a result of a sequence of period doubling bifurcations.

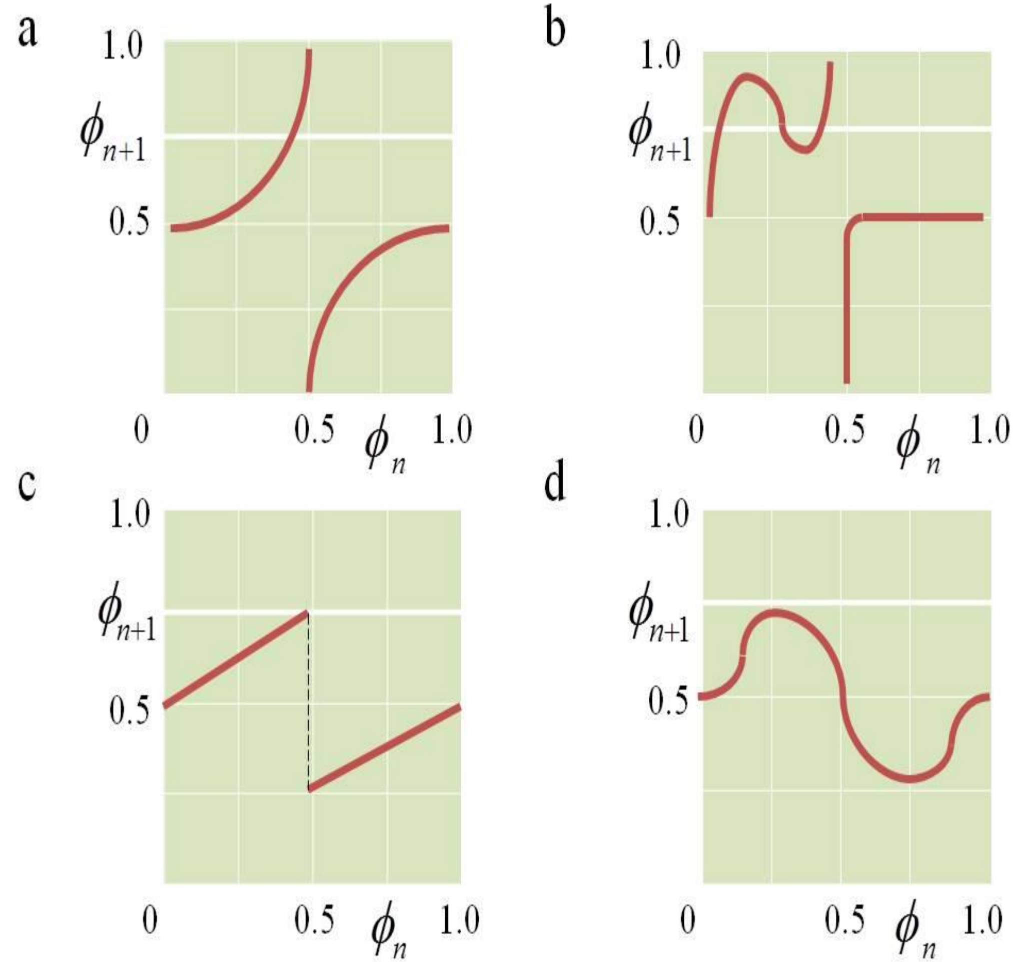
Circle displays can also be discontinuous; The bifurcations of these maps have not been well studied. Among the discontinuous mappings studied, one can single out the piecewise linear monotonically increasing irreversible mapping of a circle. The dynamics of irreversible discontinuous mappings that do not have fixed points have been studied for piecewise linear models as applied to the study of neural networks and analog-to-digital converters. But we are primarily interested in the applications of circle mapping to the study of cardiac arrhythmias.

Bub and Glass considered the possible dynamics of a generalized class of discontinuous irreversible circular mappings without fixed points and applied the results to a mathematical model of the ventricular or ventricular parasystole. Attempts were made to model the parasystole; the first person to think of using a circle display to study the heart was Acad. IN AND. Arnold.

$$\varphi = t/T_0 \ (\text{mod}1),$$

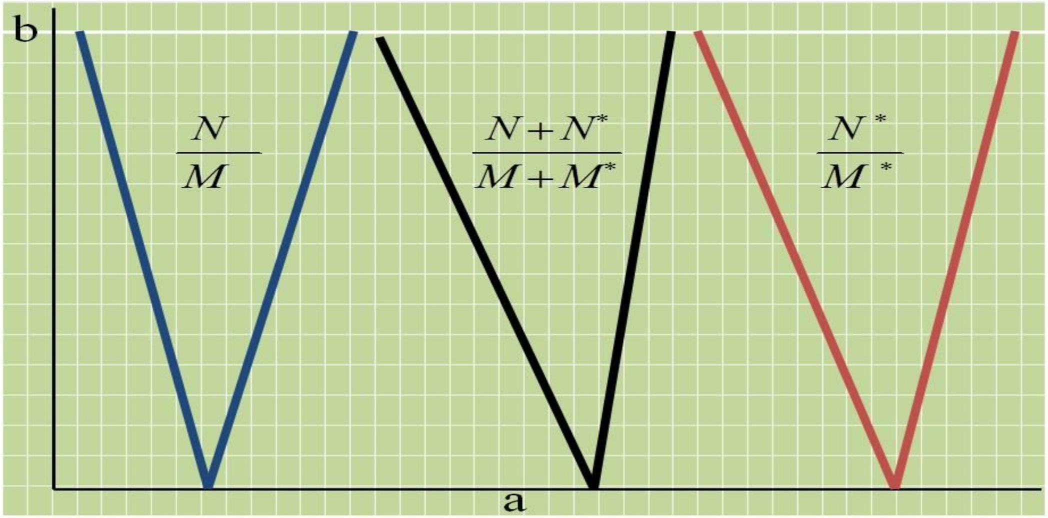
$$0 < t < T_0,$$

$$\varphi_{n+1} = \varphi_n + f(\varphi_n) \ (\text{mod}1).$$



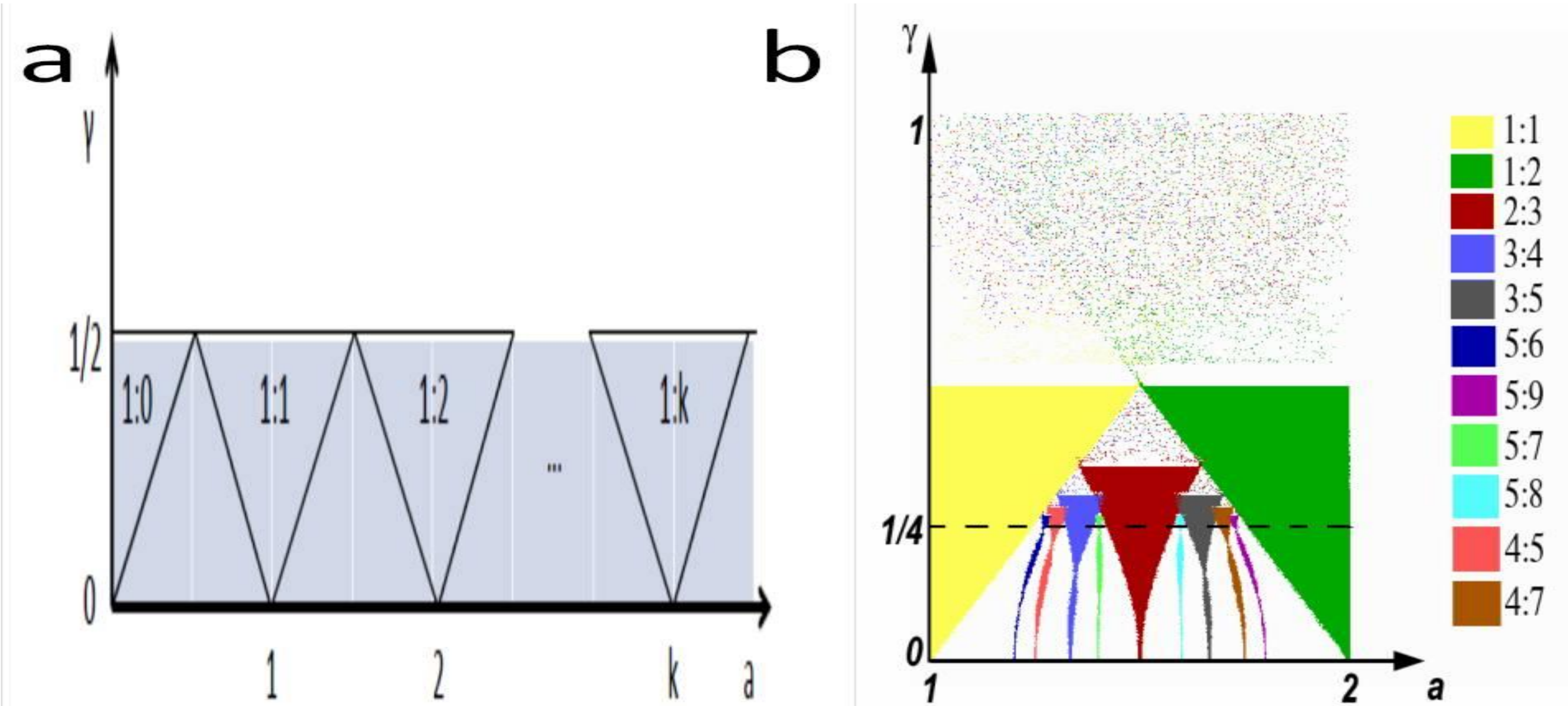
Different types of circle mappings: (a) invertible, topological degree 1; (b) irreversible, topological degree 1; (c) piecewise continuous; (d) topological degree 0 display.

$$\varphi_{n+1} = f(a, b, \varphi_n) = \varphi_n + a + b\sin 2\pi\varphi_n \pmod 1.$$



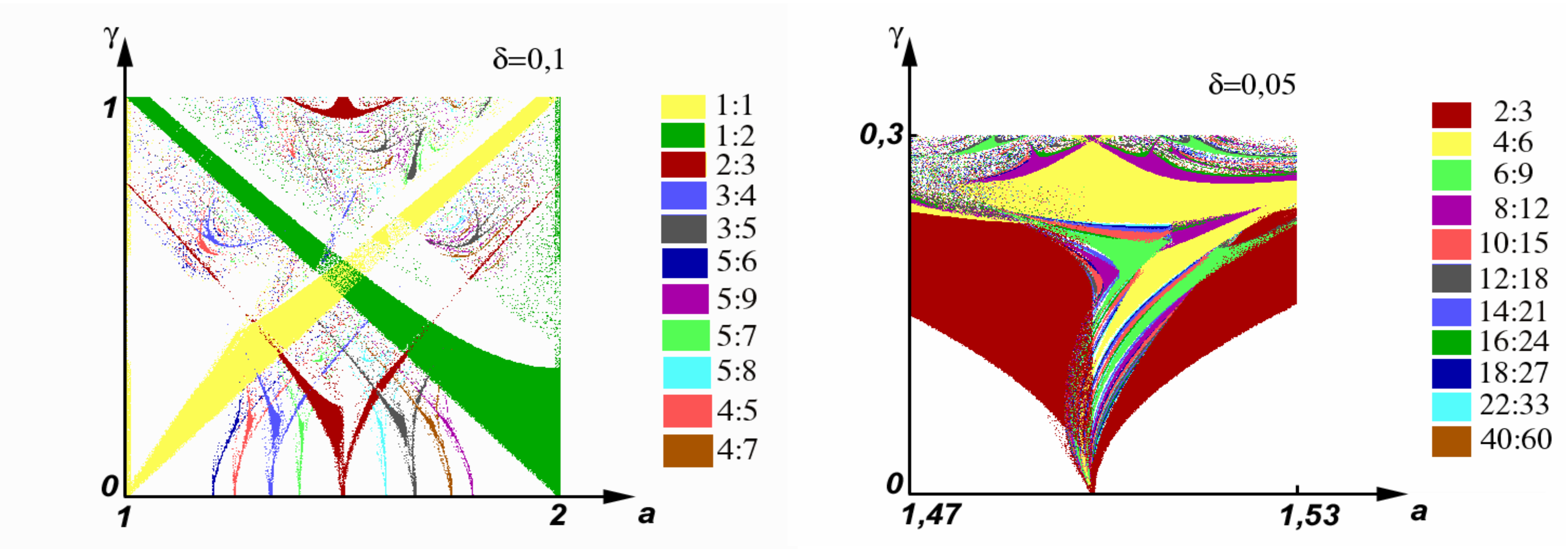
Schematic diagram of Arnold's languages. In the shaded areas there is stable phase locking. Between any two capture zones there are always other zones.

$$x_{n+1} = x_n + a - \gamma h(x_n) \qquad (\mathrm{mod}\,1).$$



The simplest case of the presence of a refractory period

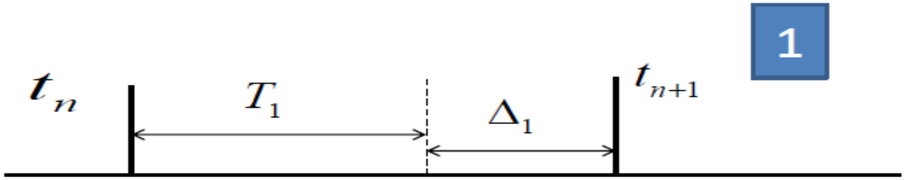
$$x_{n+1} = \begin{cases} x_n + a, & 0 \le x_n \le \delta \quad (\mathrm{mod}\,1) \\ x_n + a - \gamma \sin(2\pi x_n), & \delta < x_n \le 1 \quad (\mathrm{mod}\,1). \end{cases}$$



Phase diagram for sine mapping of a circle taking into account the refractory period (6) and stable phase captures of the sine mapping (6) inside the 2:3 capture splitting region.

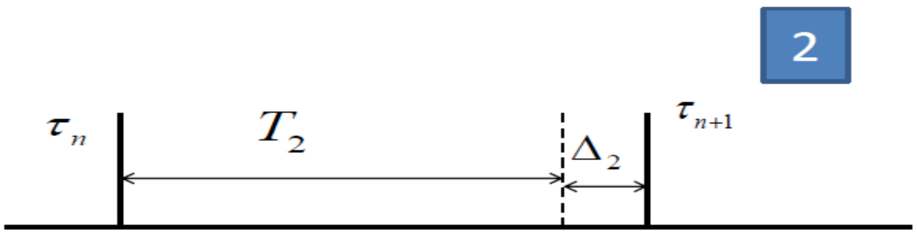
Model of two interacting pacemakers taking into account refractory time

$$t_{n+1} = t_n + T_1,$$
$$\tau_{n+1} = \tau_n + T_2 .$$



$$t_{n+1} = t_n + T_1 + \Delta_1((\tau_n - t_n)/T_1).$$

$$\tau_{n+1} = \tau_n + T_2 + \Delta_2((t_{n+1} - \tau_n)/T_2).$$



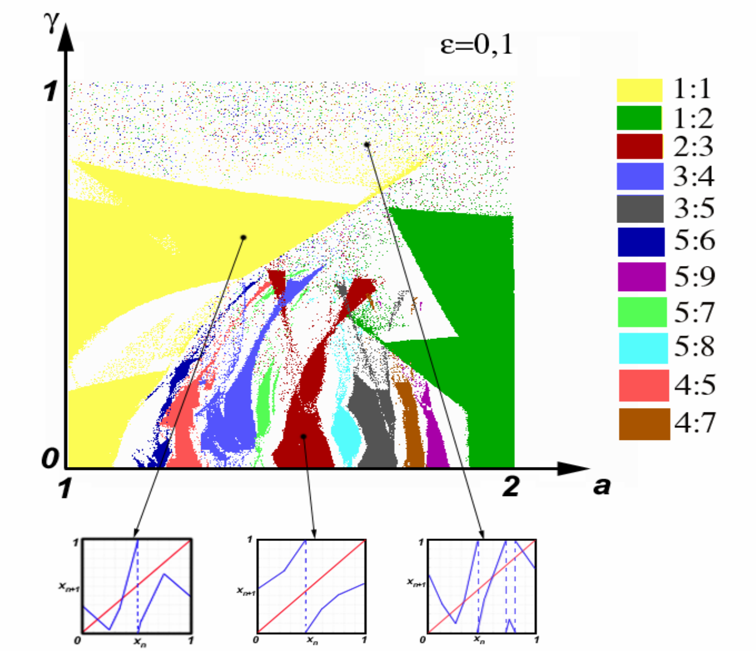
Scheme of constructing a model describing a system of two interacting nonlinear oscillators.

$$\begin{cases} \varphi_{n+1} = \varphi_n + \dfrac{1}{T_1}\Delta_1(\delta_n - \varphi_n), \\ \delta_{n+1} = \delta_n + \dfrac{T_2}{T_1} + \dfrac{1}{T_1}\Delta_2\left(\dfrac{t_n}{T_2} + \dfrac{T_1}{T_2} + \dfrac{1}{T_2}\Delta_1(\delta_n - \varphi_n) - \dfrac{\tau_n}{T_n}\right). \end{cases}$$

$$\begin{cases} \varphi_{n+1} = \varphi_n + f_1(\delta_n - \varphi_n), \\ \delta_{n+1} = \delta_n + a + f_2\left(\dfrac{1}{a}(\varphi_n + 1 + f_1(\delta_n - \varphi_n) - \delta_n)\right). \end{cases}$$

$$x_{\mathrm{n+1}} = x_{\mathrm{n}} + a + f_2[(1/2)(1 + f_1(x_{\mathrm{n}}) - x_{\mathrm{n}})] - f_1(x_{\mathrm{n}}) \quad (\mathrm{mod}1),$$

$$x_{\mathrm{n+1}} = x_{\mathrm{n}} + a + \varepsilon h[(1/\mathrm{a})(1 + \gamma h(x_{\mathrm{n}}) - x_{\mathrm{n}})] - \gamma h(x_{\mathrm{n}}) \quad (\mathrm{mod}1),$$

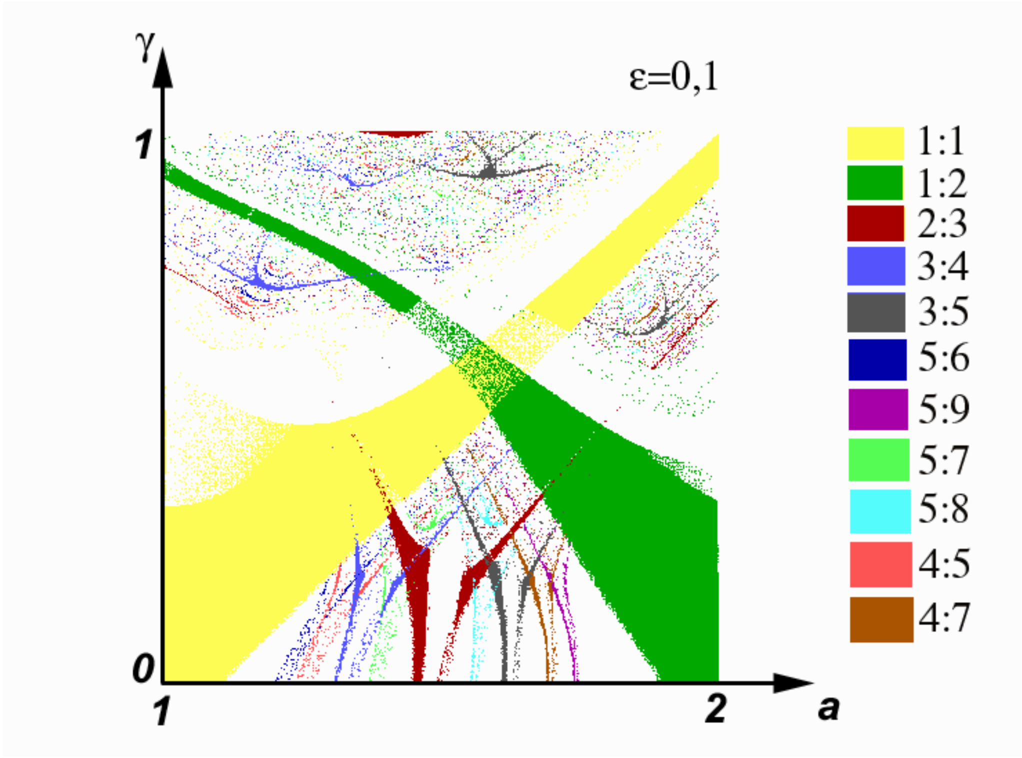


Regions of stable phase locks for piecewise linear mapping of a circle (2) with QFO of the form (4) taking into account the mutual influence of oscillators.

Sine model

$$h(x) = \sin(2\pi x), \quad f'_1(x) = \gamma \sin(2\pi x), \; f'_2(x) = \varepsilon \sin(2\pi x).$$

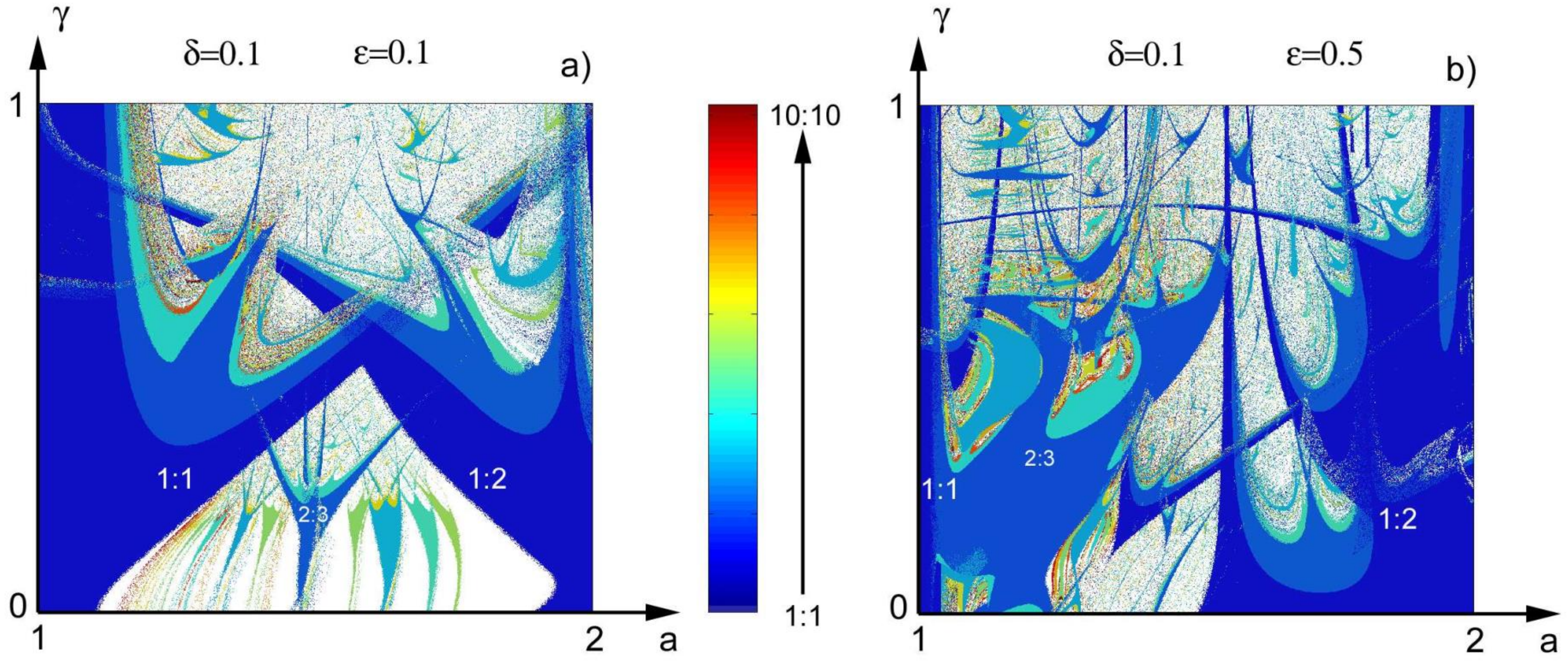
$$x_{n+1} = x_n + a + \varepsilon \sin[(1/a)(1 + \gamma \sin(2\pi x_n) - x_n)] - \gamma \sin(x_n) \quad (\text{mod}1),$$



Structure of some phase locking regions for piecewise linear mapping of a circle.

Areas of phase locking in space (a, γ)

$$\begin{cases} x_{n+1} = x_n + a & 0 \le x_n \le \delta \quad (\mathrm{mod}1), \\ x_{n+1} = x_n + a + \varepsilon \sin\left(\dfrac{1}{a}(1+\gamma\sin(2\pi x_n) - x_n)\right) - \gamma \sin(2\pi x_n) & \delta \le x_n \le 1 \quad (\mathrm{mod}1). \end{cases}$$



Phase locking areas of the oscillator system with two-way communication (δ = 0,1): (a) ε = 0,1: (b) ε = 0,5.

The merit of Kolmogorov and Arnold is that they proved that the approximation of a continuous bounded function of a set of variables is reduced to finding a polynomial number of one-dimensional functions [1] (1):

$$f(x) = f(x_1, \ \dots \ , \ x_n) = \sum_{q=1}^{2n+1} \Phi_q\left(\sum_{p=1}^{n} \phi_{q,p}(x_p)\right)$$

1. Matthias F. SplineCNN: Fast Geometric Deep Learning with Continuous B-Spline Kernels, Department of Computer Graphics TU Dortmund University, arXiv: 1711.08920v2 [cs.CV] 23, 5, 2018, P. 1-9.

For a network with two (n = 2) input parameters, we obtain a two-layer (since the composition depth in the theorem is two) neural network with five (since the theorem involves 2n + 1 = 5 functions) neurons on the hidden layer. In our case, we consider with two neurons, based on the KAN theorem, the equation [2] (2) is obtained:

$$x_{n+1} = x_n + a + f_2\left(\frac{1}{a}\left(1 + f_1(x_n) - x_n\right)\right) - f_1(x_n) \pmod 1$$

2. Loskutov A.U. A model of cardiac tissue as a conductive system with interacting pacemakers and refractory time, Int. J. Bif and Chaos. 2004. vol. 14, P. 2457-2466.

Parameter a = T2 ⁄ T1, the ratio of the natural periods of the neuron oscillators. Functions f1(x), f2(x) are called phase response curves, which generally do not coincide with each other. However, we believe that since, according to the meaning of the problem, both oscillators are sources of action potentials in one tissue, they have a similar nature, and we can consider the functions f1(x) and f2(x) to be practically the same. The oscillator's response to an external stimulus depends only on the phase of the stimulus and its amplitude, the CFO changes its shape when the amplitude of the external influence changes. This means that the functions that define the shape of the phase response curves must depend on one parameter that determines the amplitude value, this dependence can be considered multiplicative.

Then the phase response curves will be written as: $f1 = \gamma h(x)$, $f2 = \varepsilon h(x)$, where $h(x)$ is a periodic function, $h(x+1) = h(x)$. In this case, formula (3) will take the form:

$$x_{n+1} = x_n + a + \varepsilon h\left(\frac{1}{a}(1 + \gamma h(x_n) - x_n)\right)\Bigg) - \gamma h(x_n) \pmod 1$$

3. Belyakin S.T. Discrete models of active media in attached to the activities of cardiac arrhythmia, Biomedical J. of Scientific & Technical Research. 2019. vol. 20, n. 3, P. 16730-16737.

Sine model

As a model of two nonlinearly interacting excitation sources. Let us consider two coupled oscillators, taking the role of h(x) as a sine function without taking into account refractoriness and assuming the magnitude of the influence of the first oscillator on the second. Then the mapping (3) will take the form [4] (4):

$$x_{n+1} = x_n + a + \varepsilon \sin\left(\frac{2\pi}{a}\left(1 + \gamma \sin(2\pi x_n) - x_n\right)\right) - \gamma \sin(2\pi x_n) \pmod 1$$

The sinusoidal model is most often used to study the behavior of two interacting oscillators.

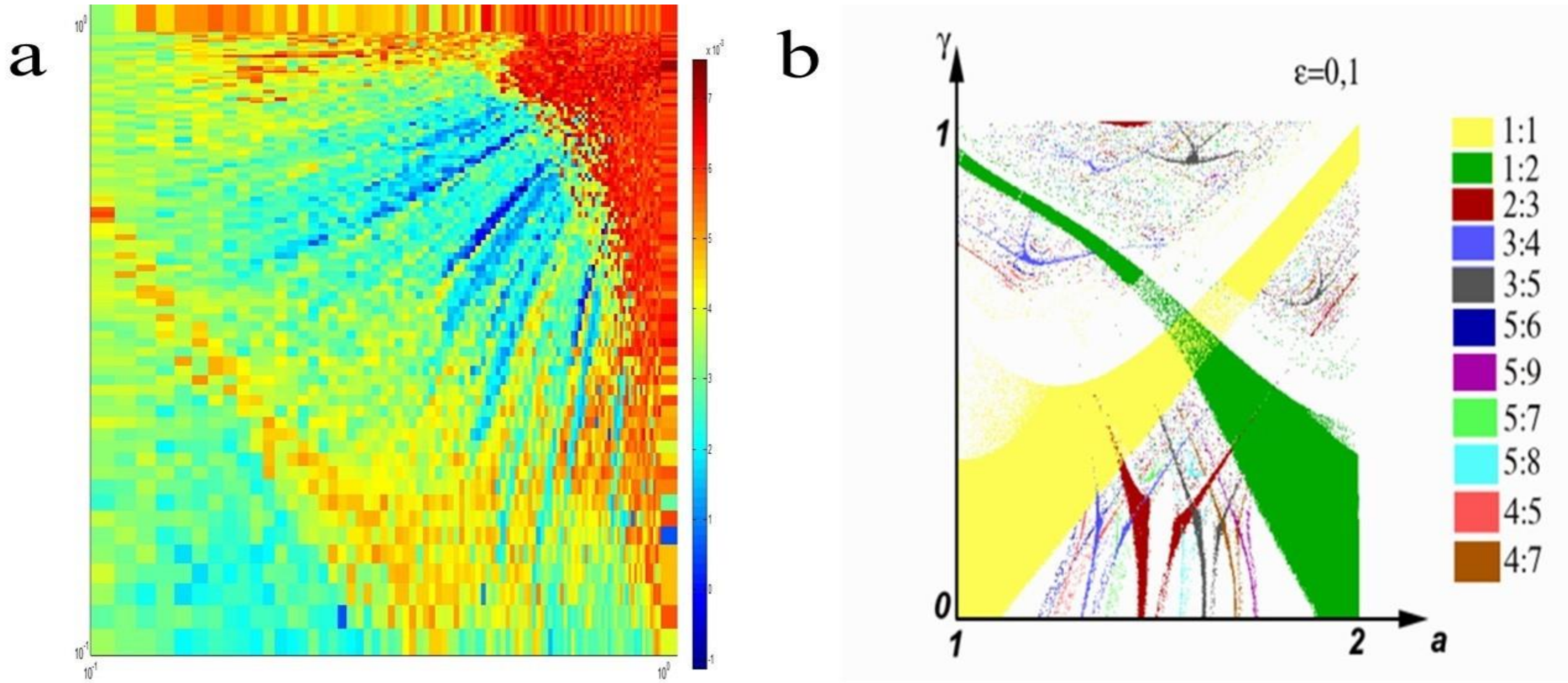


Fig.3. a) Sine model. Lyapunov exponents, (a_1 =1.45, a_2 =1.55), along the horizontal axis ε (0.1 $\rightarrow$ 1.0), along the vertical axis γ (0.1 $\rightarrow$ 1.0).

b) Phase locking regions of a two-way oscillator system(ε = 0.1).

In Fig.3(a) capture areas with temporary ($\delta = a_2 - a_1 = 0.1$) refractoriness. The colors correspond to the Lyapunov indices. Dark blue (λ = – 1.0) stable steady state, blue (λ = 0.0) nonequilibrium state, red (λ = 5.0) unstable unstable chaotic state, brown (λ = 7.0) stochastic state.

The arrangement of the phase-locking regions obtained as a result of the numerical study (7) is shown in Fig. 3(b). Similar to other cases of phase response curve approximation, taking into account the mutual influence of two discrete systems leads to the curvature of the phase-locking regions, their superposition on each other at small γ, and splitting of the main tongues. Within the split regions, locks arise that are multiples of the main one.

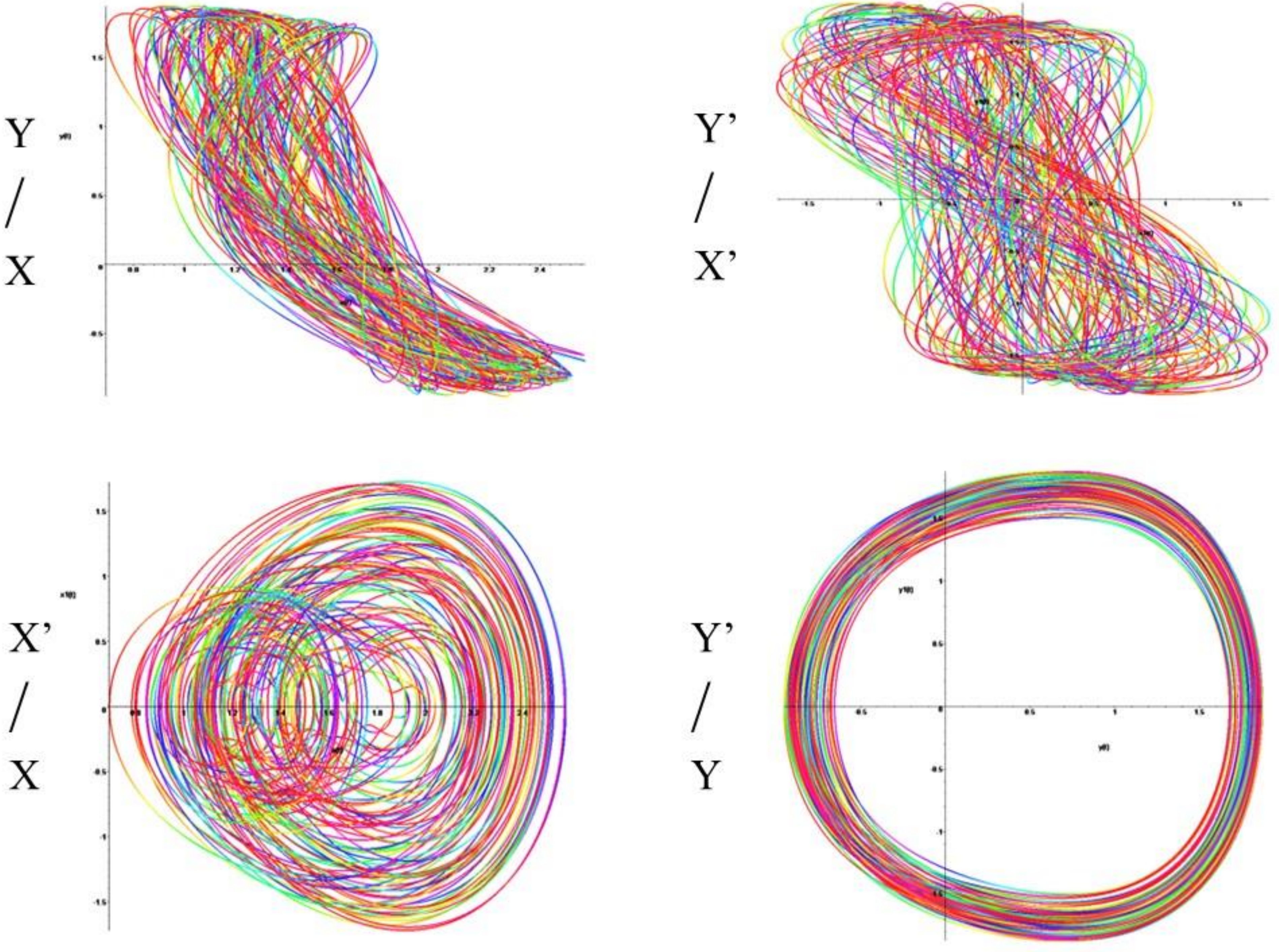
Let us consider two interconnected nonlinear oscillators with frequencies ω_1, ω_2, which are described by the Dufing equations, μ is the coefficient of viscous friction, where z is the hysteresis link, for its description a sinusoidal model of two nonlinearly interacting excitation sources was chosen (5) :

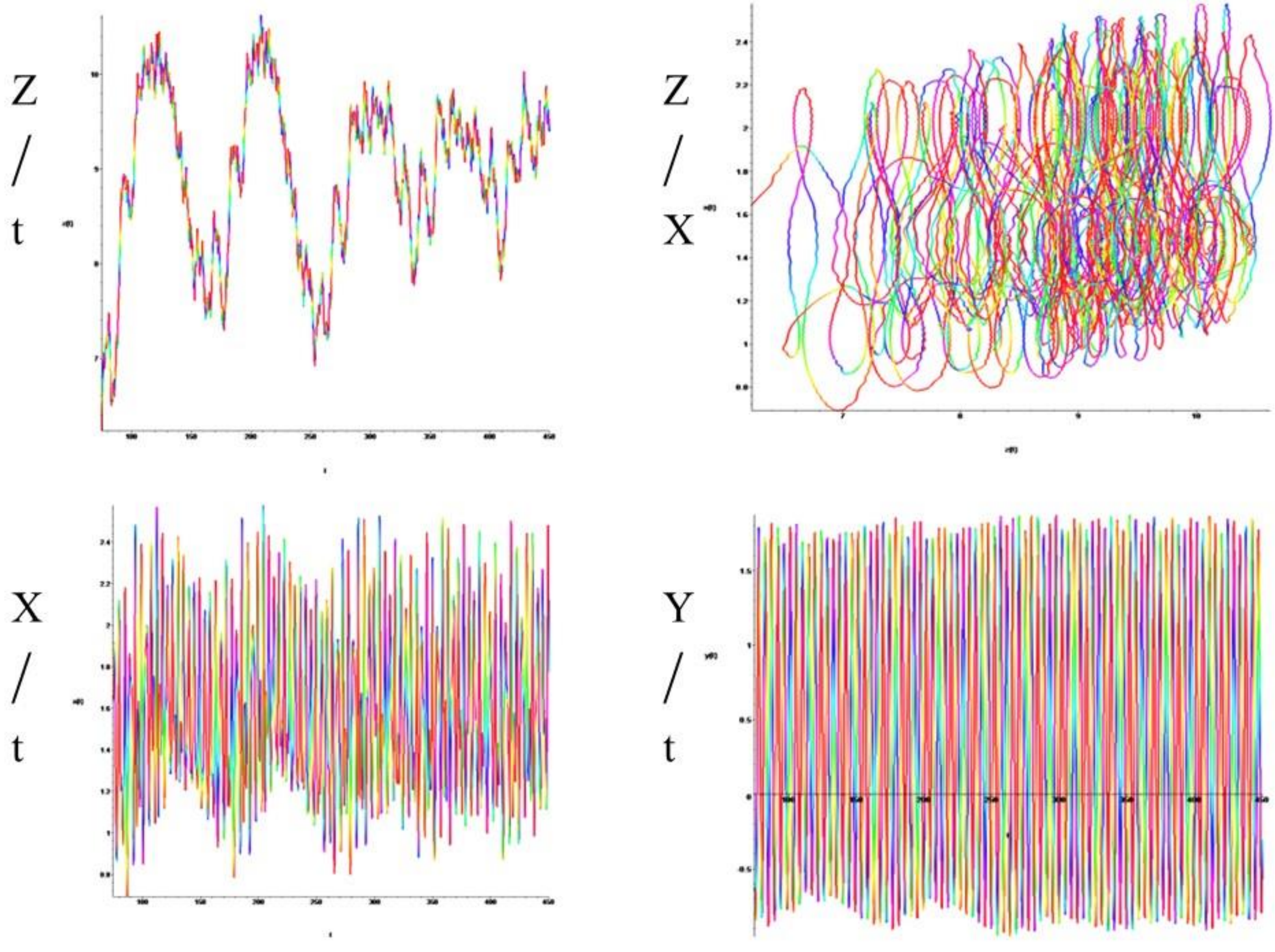
$$\ddot{x} = -\omega_1^2 (1+\varepsilon y)^2 x - \alpha z - 2\mu \dot{x} - \alpha_1 x^3 + \alpha_2 y,$$

$$\ddot{y} = -\omega_2^2 (1+\gamma x)^2 y - 2\mu \dot{y} - \alpha_1 y^3 + \alpha_2 x,$$

$$\dot{z} = a + \alpha_2 \sin\left(\frac{2\pi}{a} (1+\alpha_2 \sin(2\pi x) - y) \right) - \alpha_2 \sin(2\pi x).$$

Where a are the parameters characterizing the KAN model [3], α reflects the influence of the hysteresis term, (ε, γ) is a small parameter. Numerically solving the above-described system for the values $\alpha = -0.5$, $\alpha_1 = 0.5$, $\alpha_2 = 0.5$, $\omega_1 = 2$, $\omega_2 = 1$, $a = 0.1$, $\mu = 0.002$, $\varepsilon = \gamma = 0.5$ (in practice, it has been found that with such a frequency ratio, an internal resonance occurs in the system [4]).

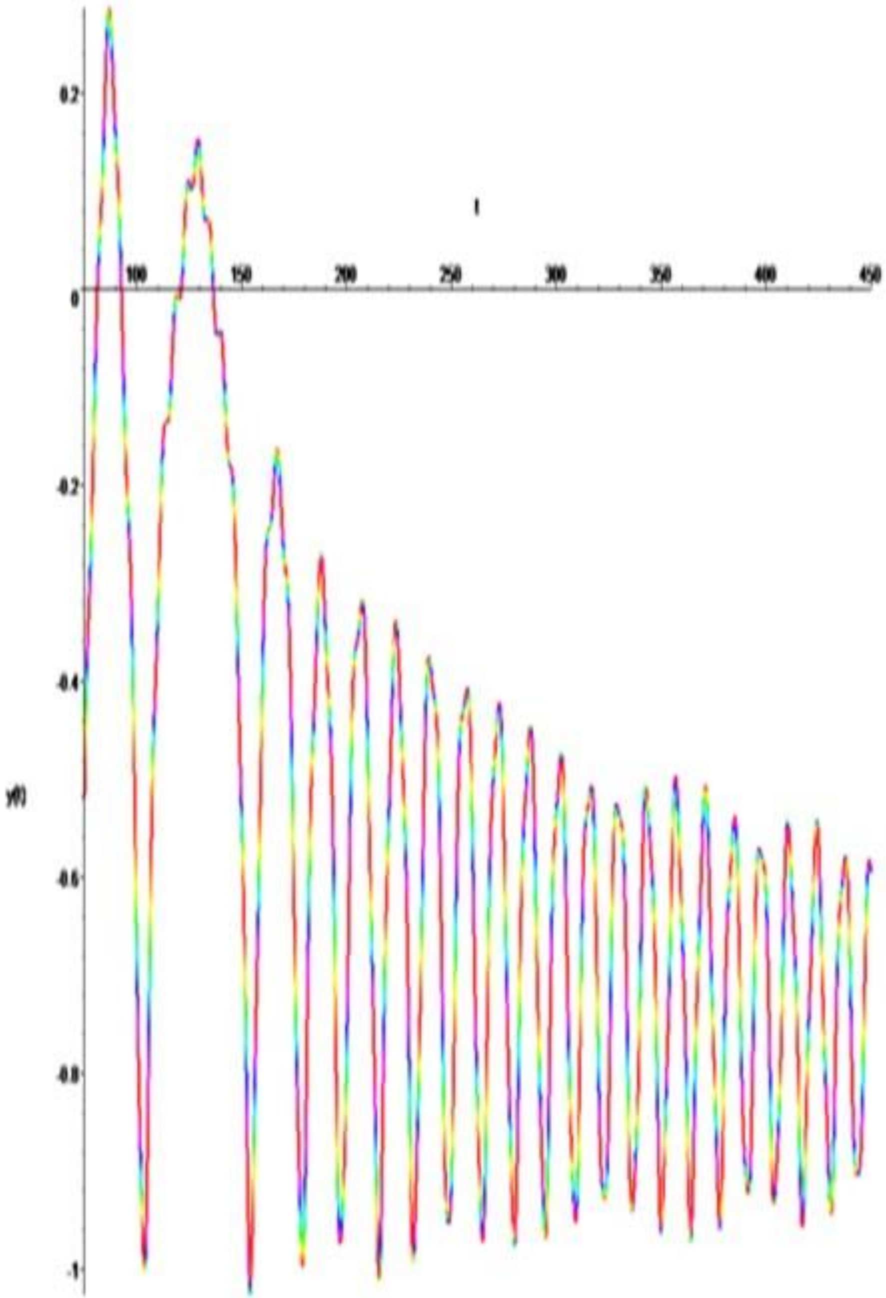

Y / X
Y' / X'
X' / X
Y' / Y


Z / t
Z / X
X / t
Y / t

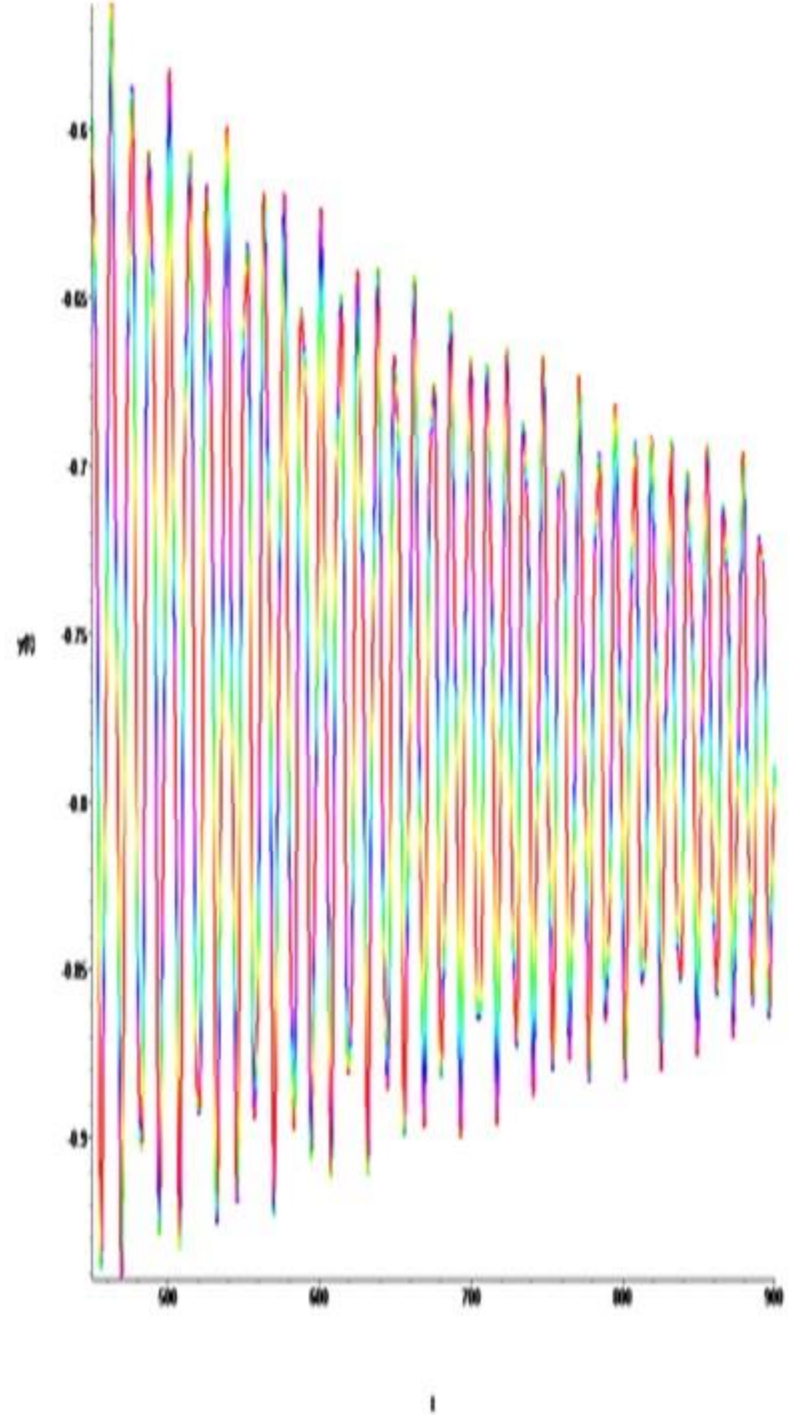
$$\alpha = 0$$

$$t = 0.00 - 450 - 900$$

y
/
t



0.2
t
100
150
200
250
300
350
400
450
0
-0.2
-0.4
y(t)
-0.6
-0.8
-1

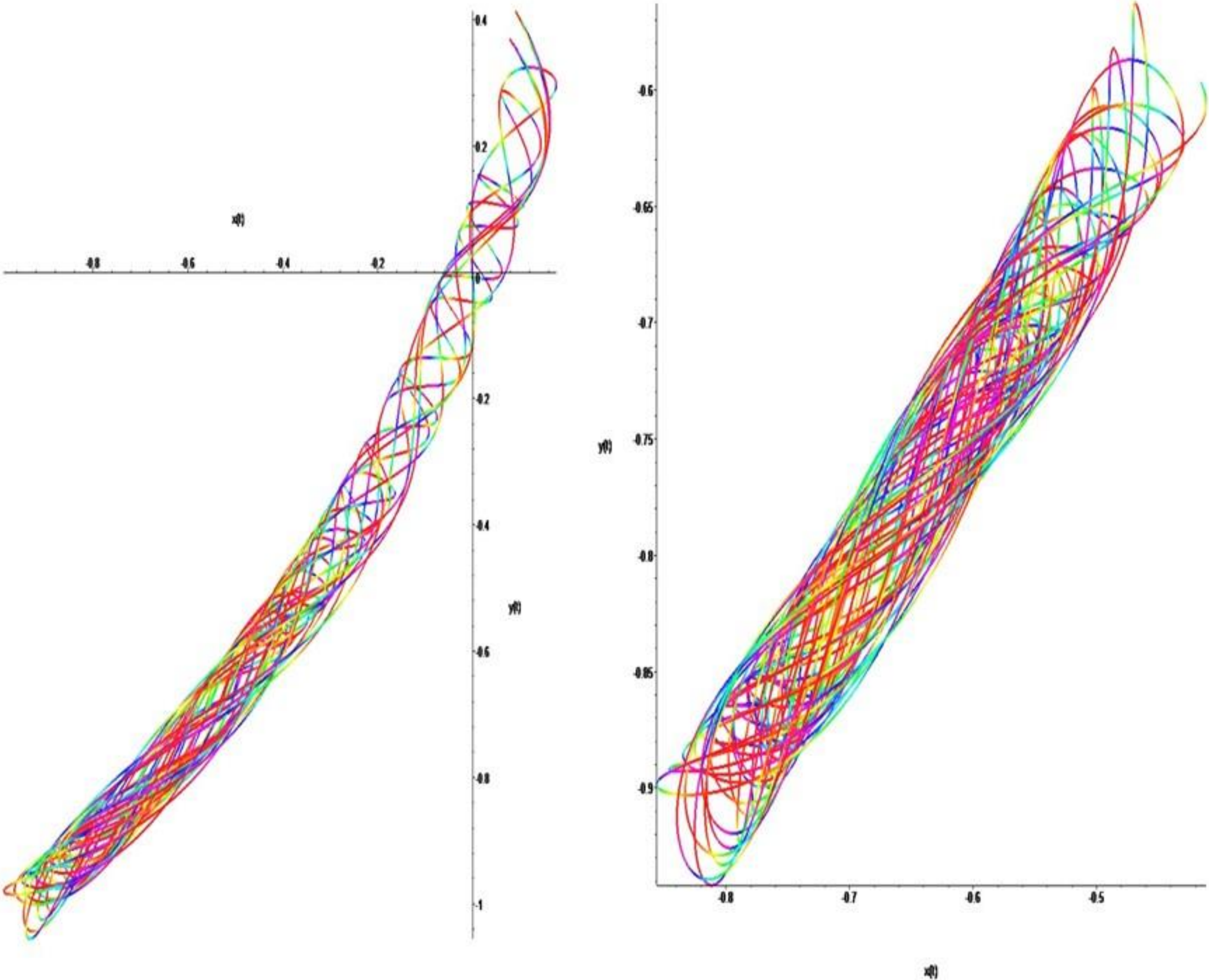


-0.6
-0.65
-0.7
y(t)
-0.75
-0.8
-0.85
-0.9
500
600
700
800
900
t

Y

/

X

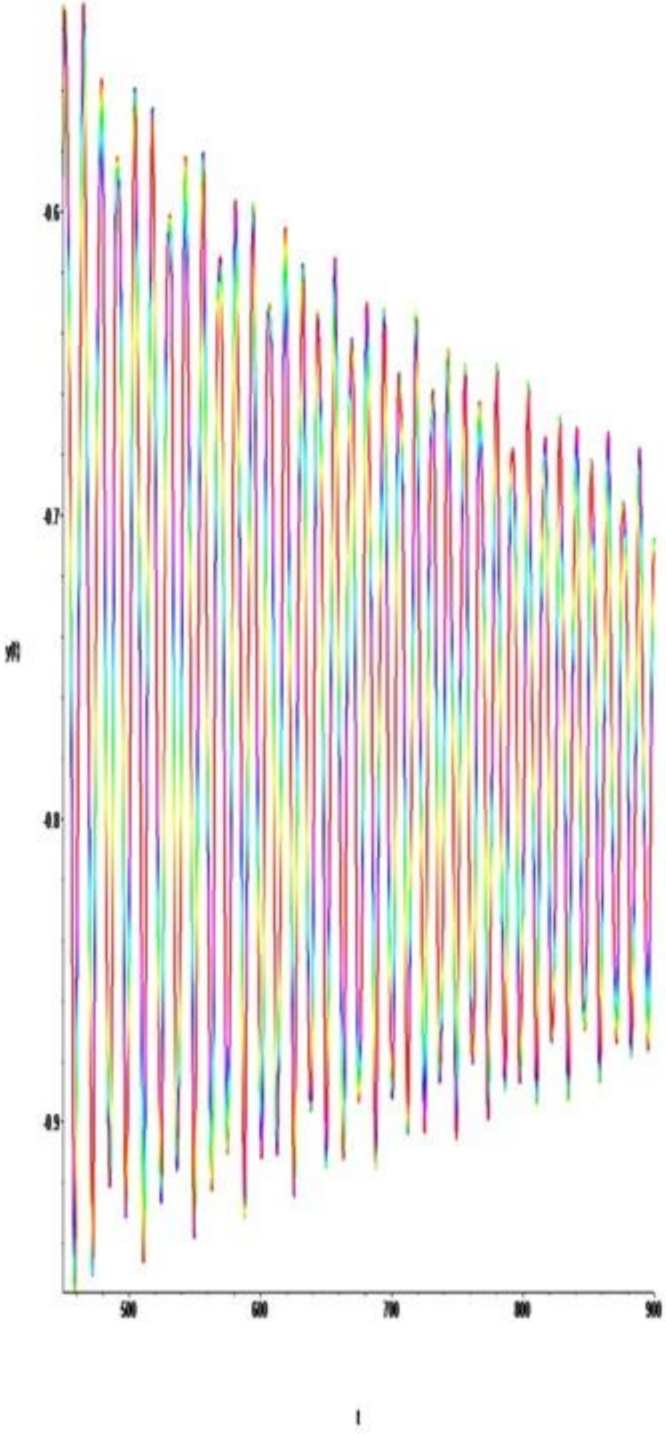


x(t)
-0.8
-0.6
-0.4
-0.2
0
0.4
0.2
-0.2
-0.4
-0.6
-0.8
-1
y(t)
-0.6
-0.65
-0.7
-0.75
-0.8
-0.85
-0.9
y(t)
-0.8
-0.7
-0.6
-0.5
x(t)

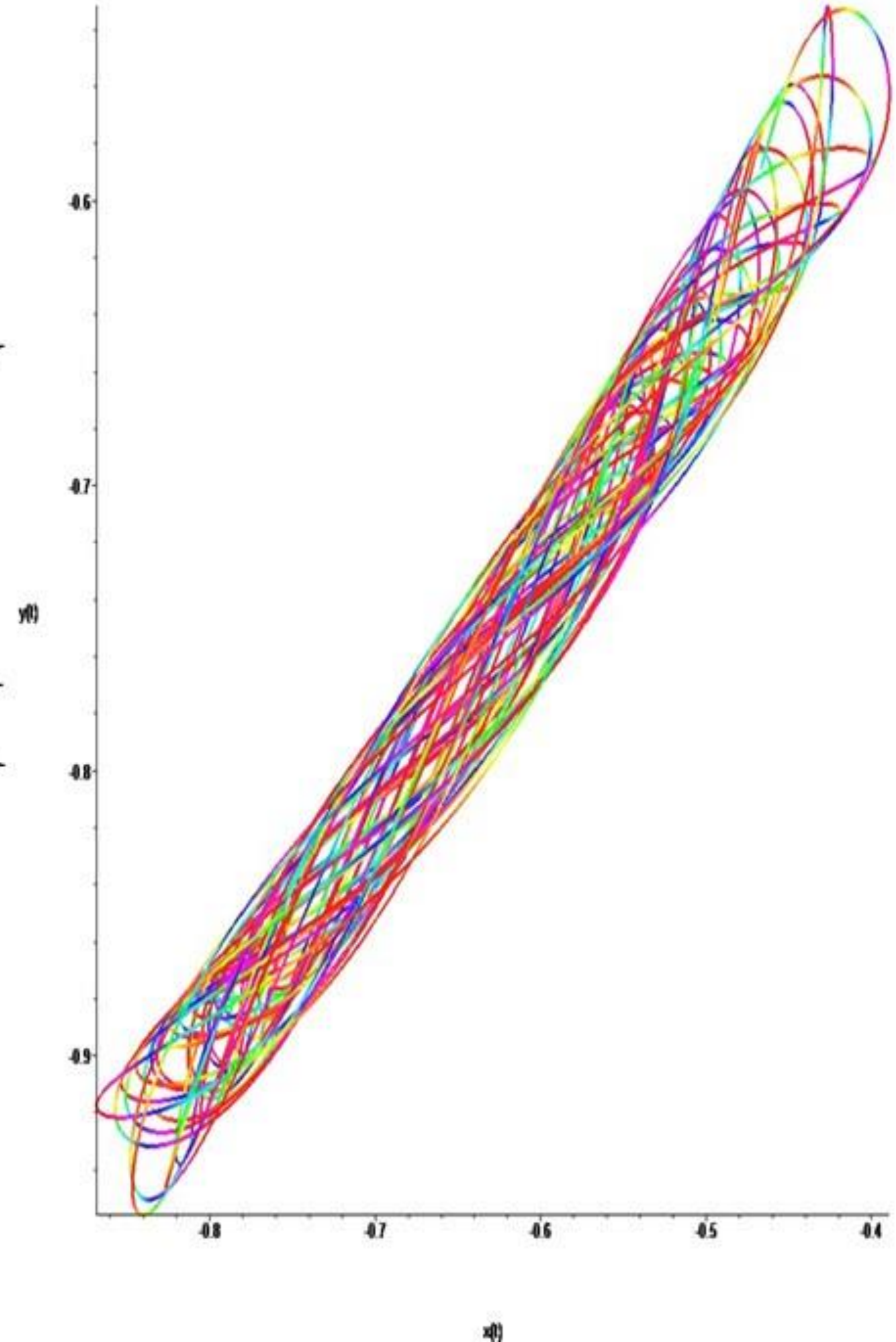
$$Y'' = F_2(x, y) + a \sin(2\pi t/n)$$

$$a = 0.0001,\ n = 12.5,\ t = 450 - 900$$

Y
/
t



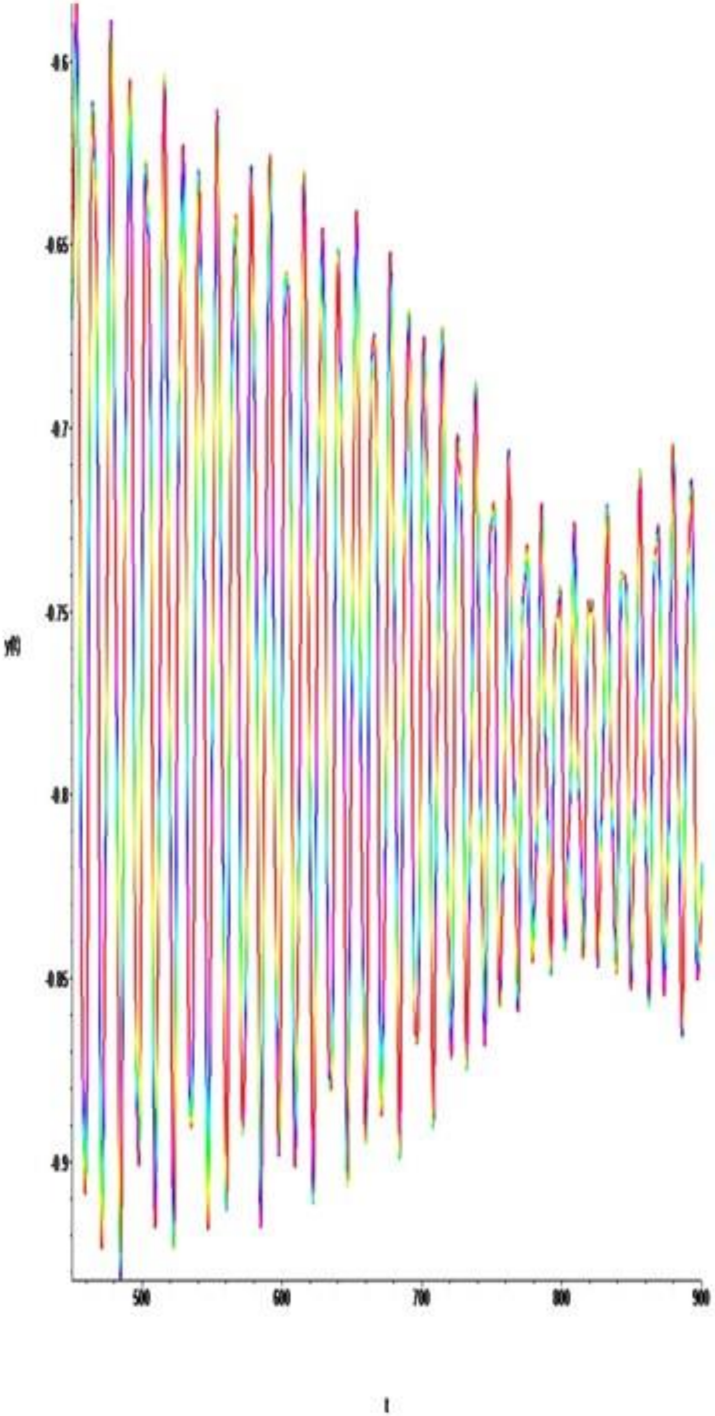
Y
/
X



$$Y'' = F_2(x, y) + a \sin(2\pi t/n)$$

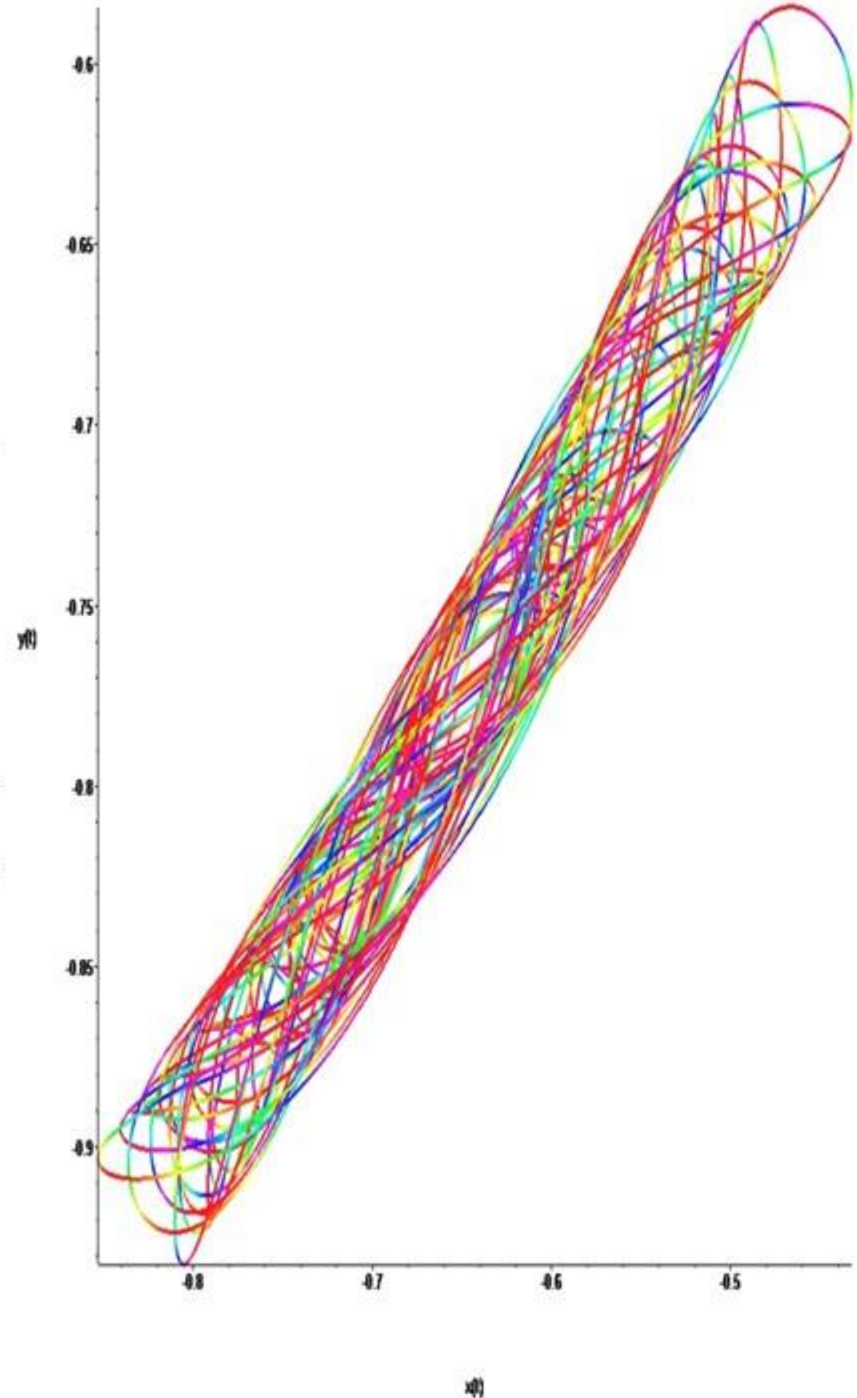
$$a = 0.001,\ n = 12.5,\ t = 450 - 900$$

Y / t



-0.6
-0.65
-0.7
-0.75
-0.8
-0.85
-0.9
y(t)
500
600
700
800
900
t

Y / X

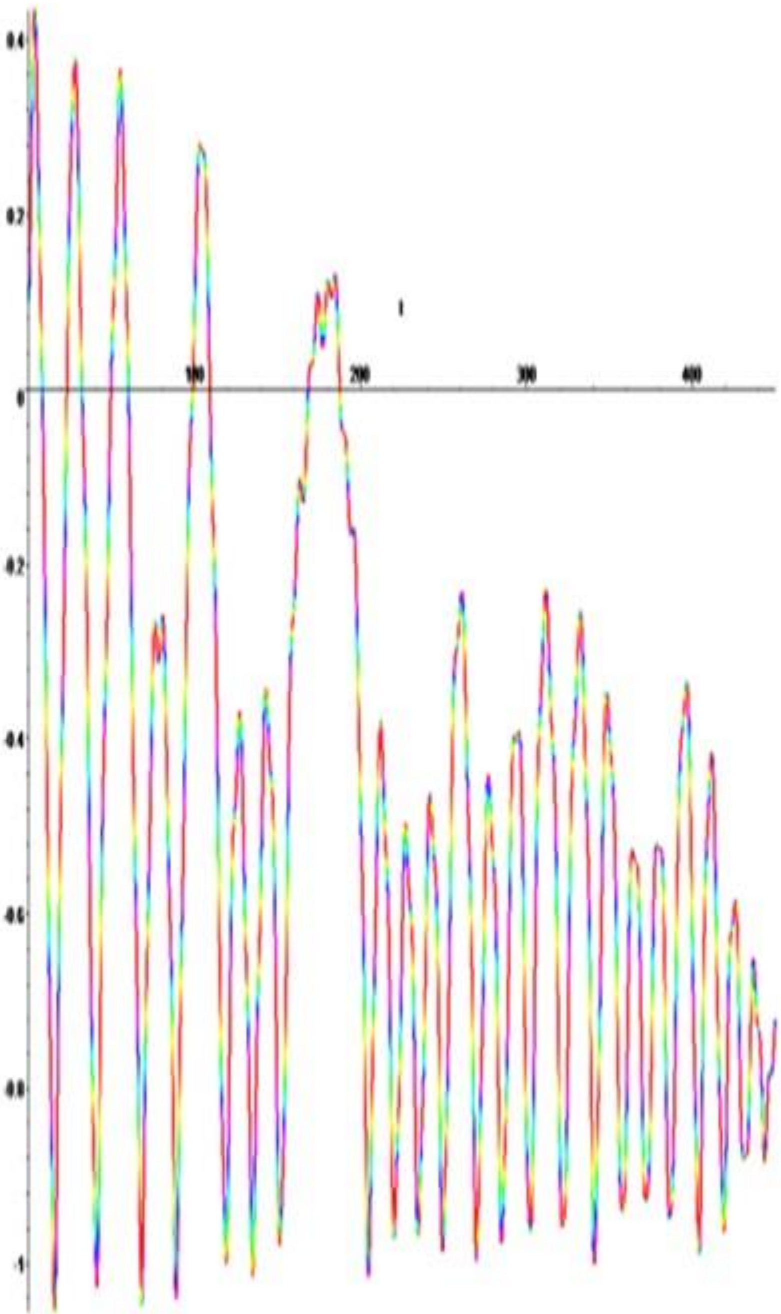


-0.6
-0.65
-0.7
-0.75
-0.8
-0.85
-0.9
y(t)
-0.8
-0.7
-0.6
-0.5
x(t)

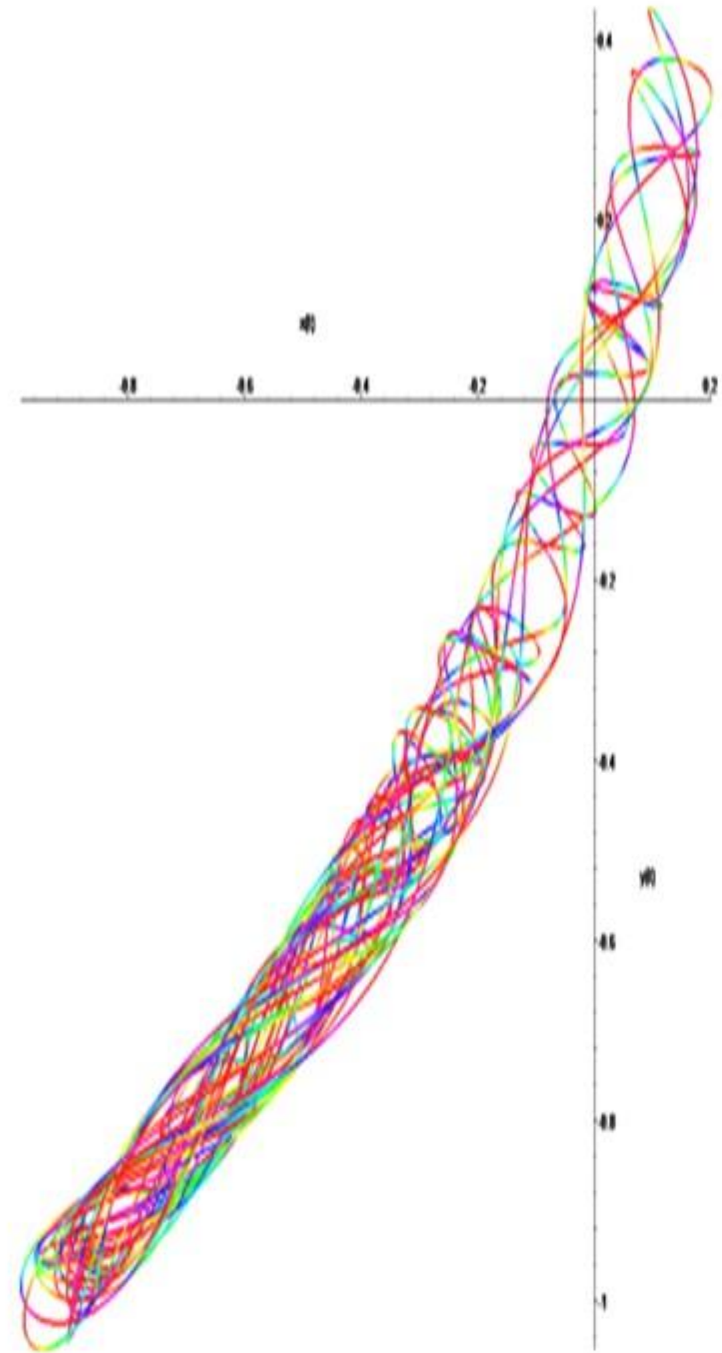
$$Y'' = F_2(x, y) + a \sin(2\pi t/n)$$

$$a = 0.01,\ n = 12.5,\ t = 0.0 - 450$$

Y / t



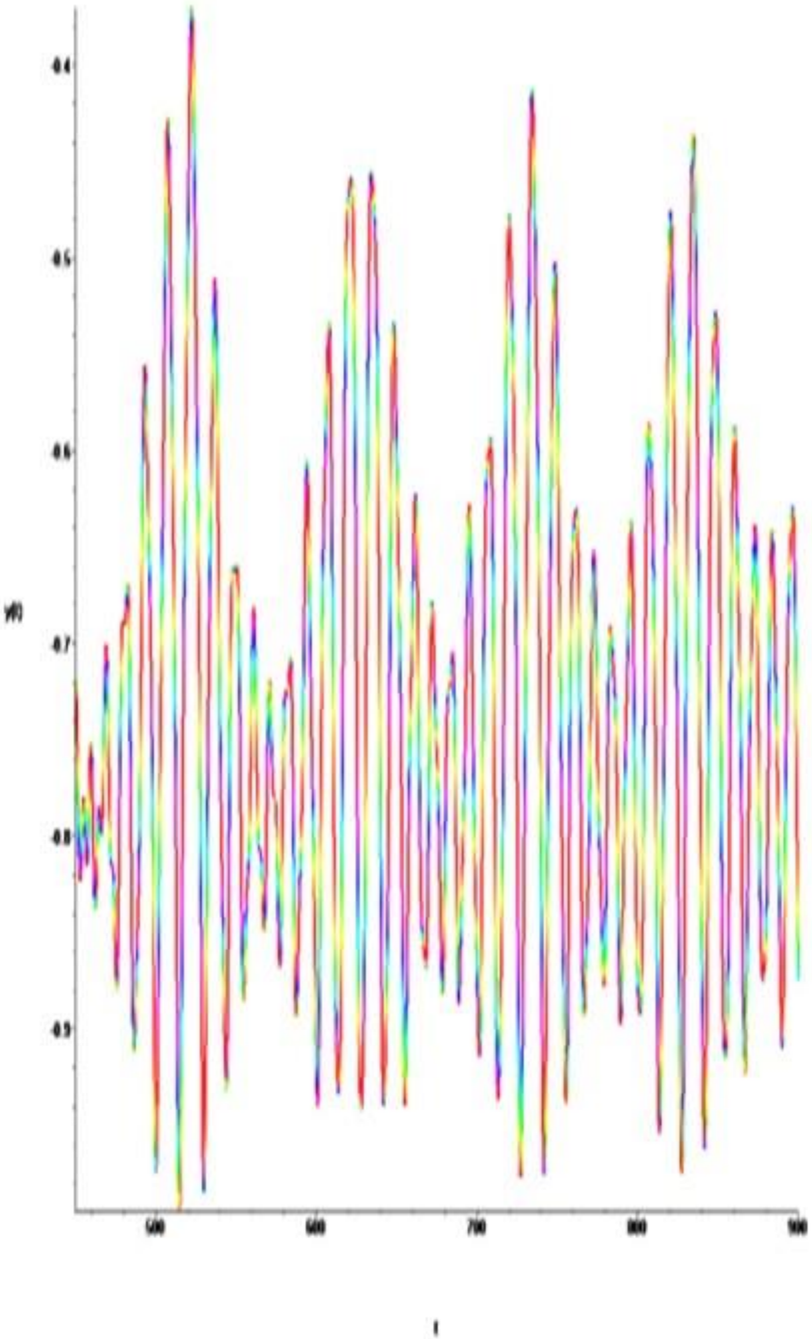
Y / X



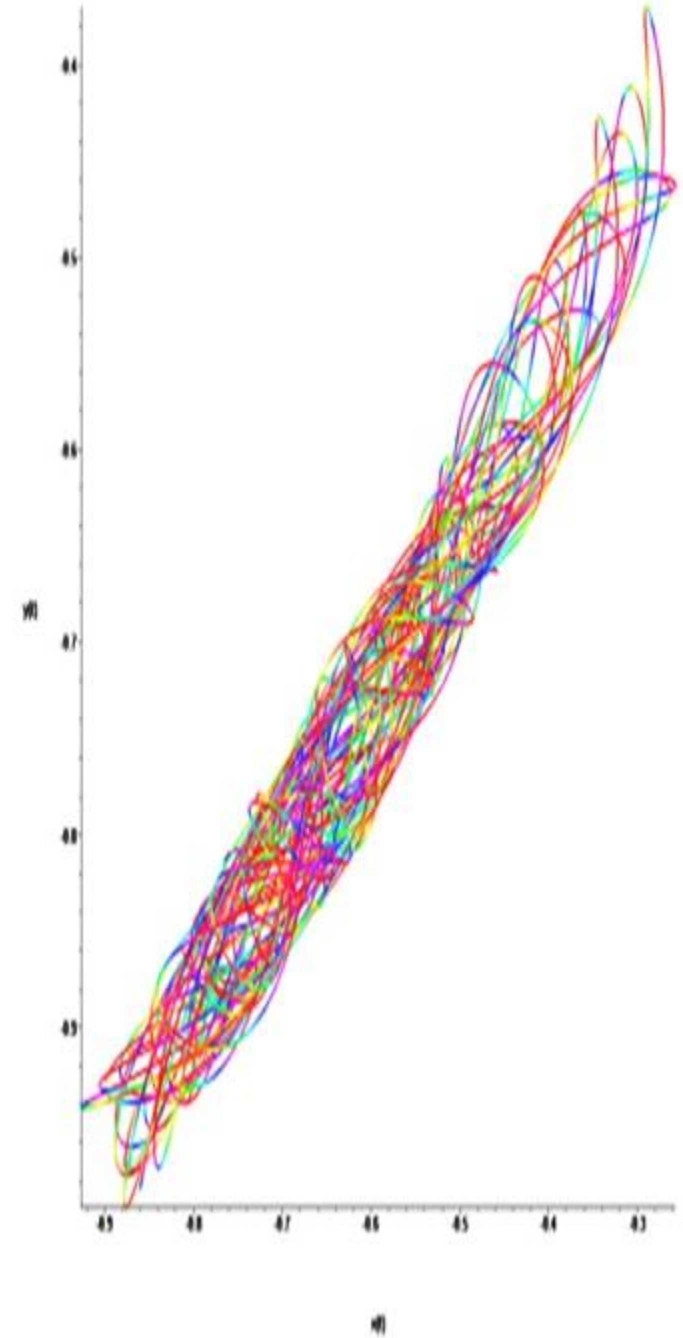
$$Y'' = F_2(x, y) + a \sin(2\pi t/n)$$

$$a = 0.01,\ t = 450 - 900$$

Y
/
t



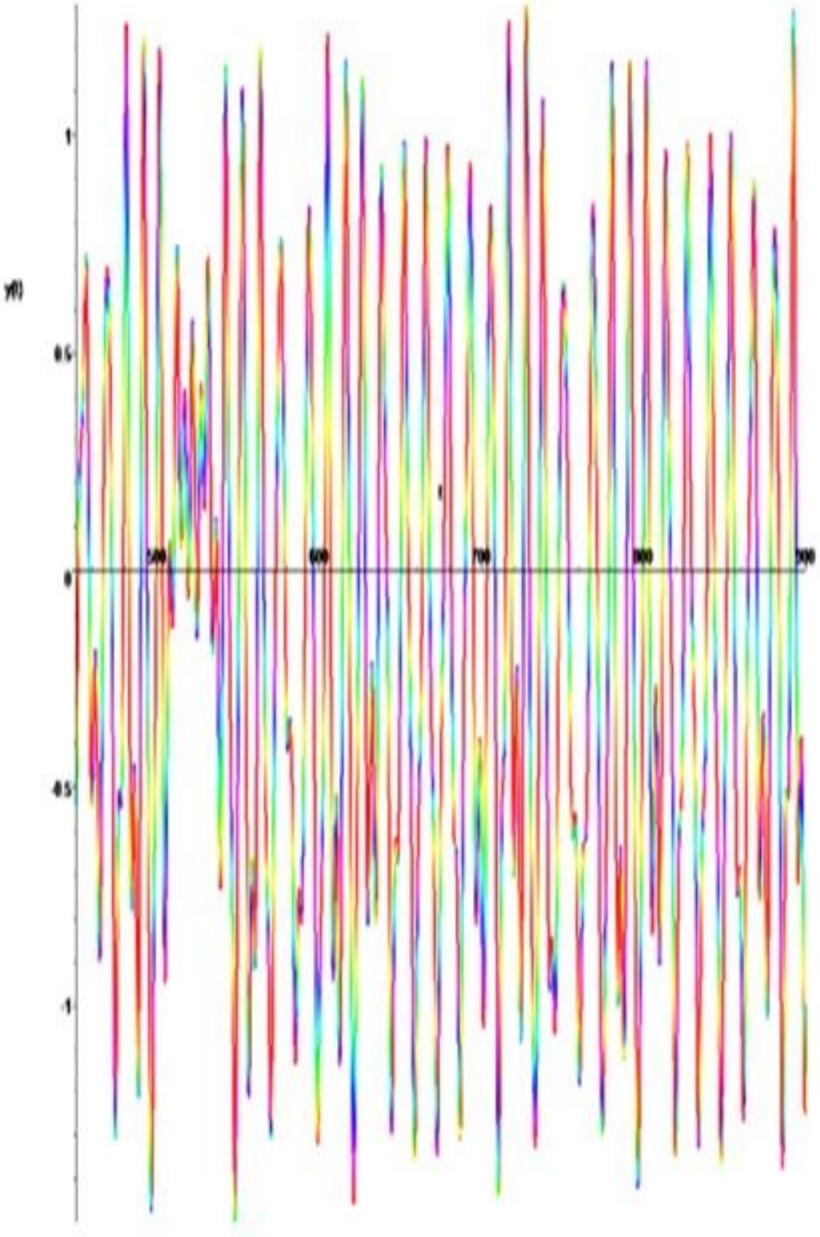
Y
/
X



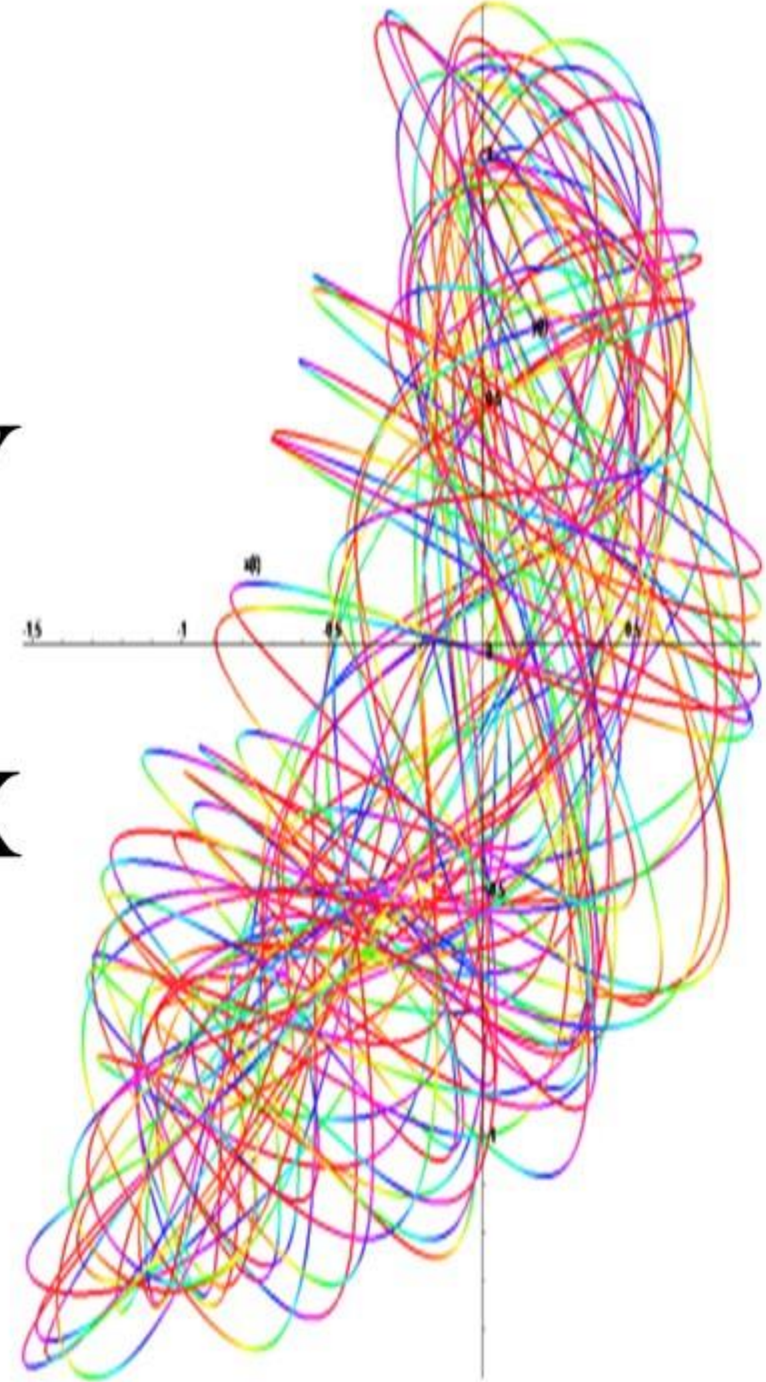
$$Y'' = F_2(x, y) + a \sin(2\pi t/n)$$

$$a = 0.05$$

Y
/
t



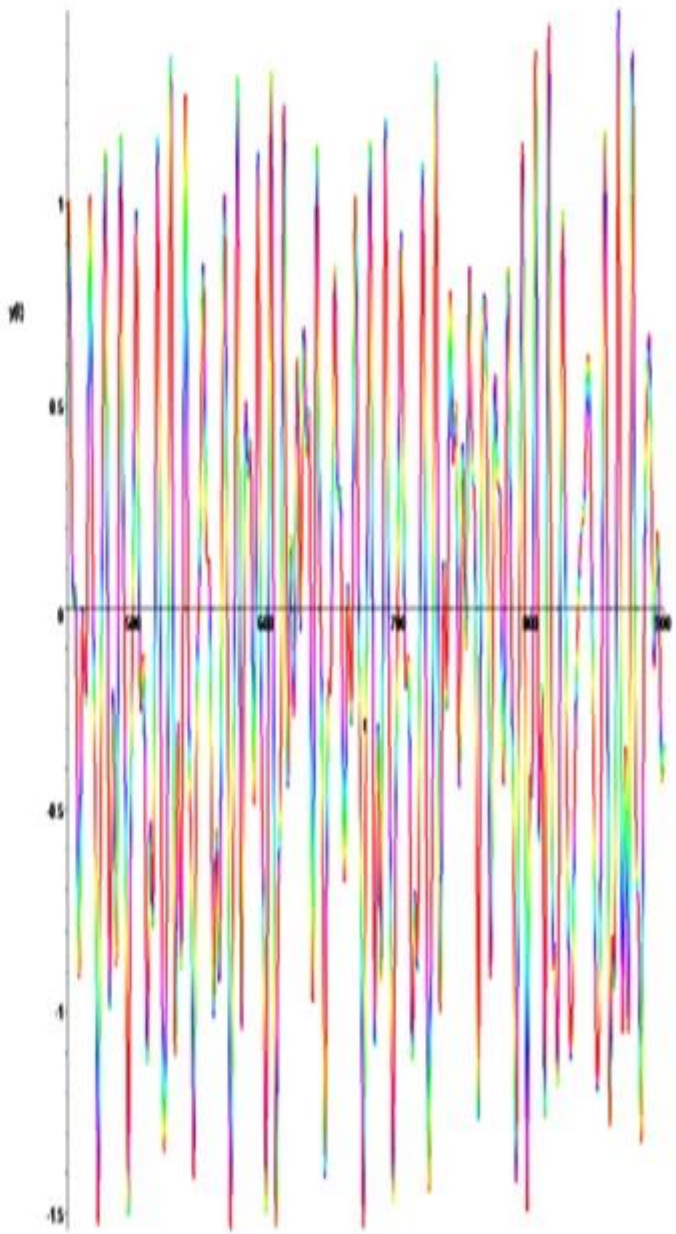
Y
/
X



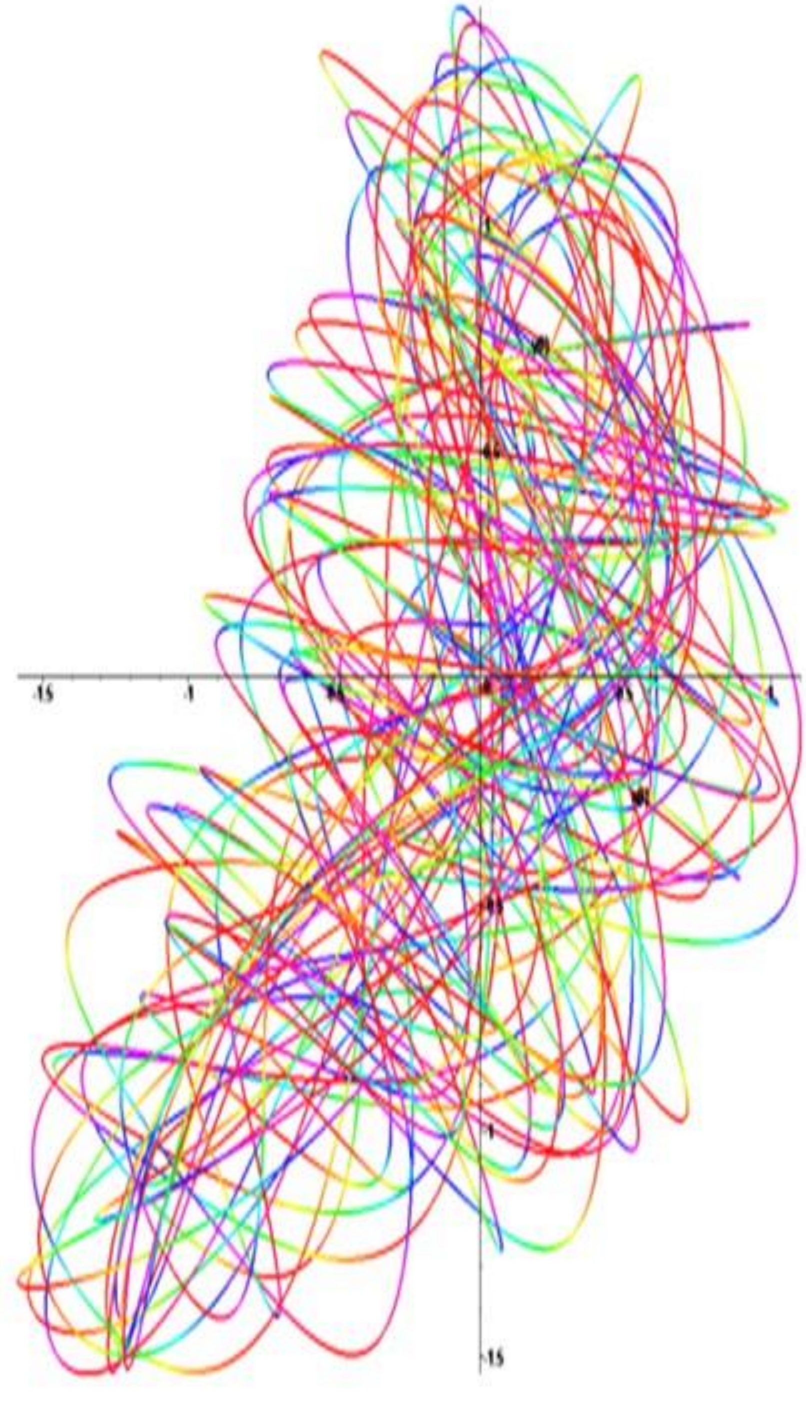
$$Y'' = F_2(x, y) + a \sin(2\pi t/n)$$

$$a = 0.1$$

Y / t



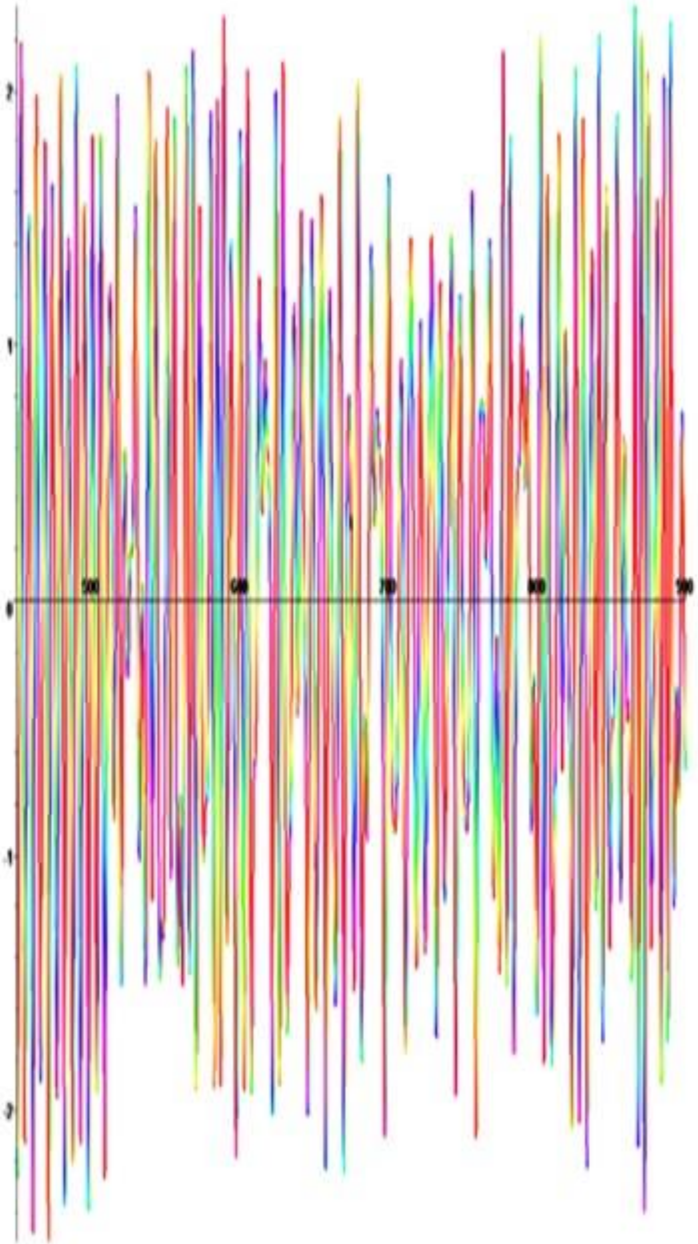
Y / X



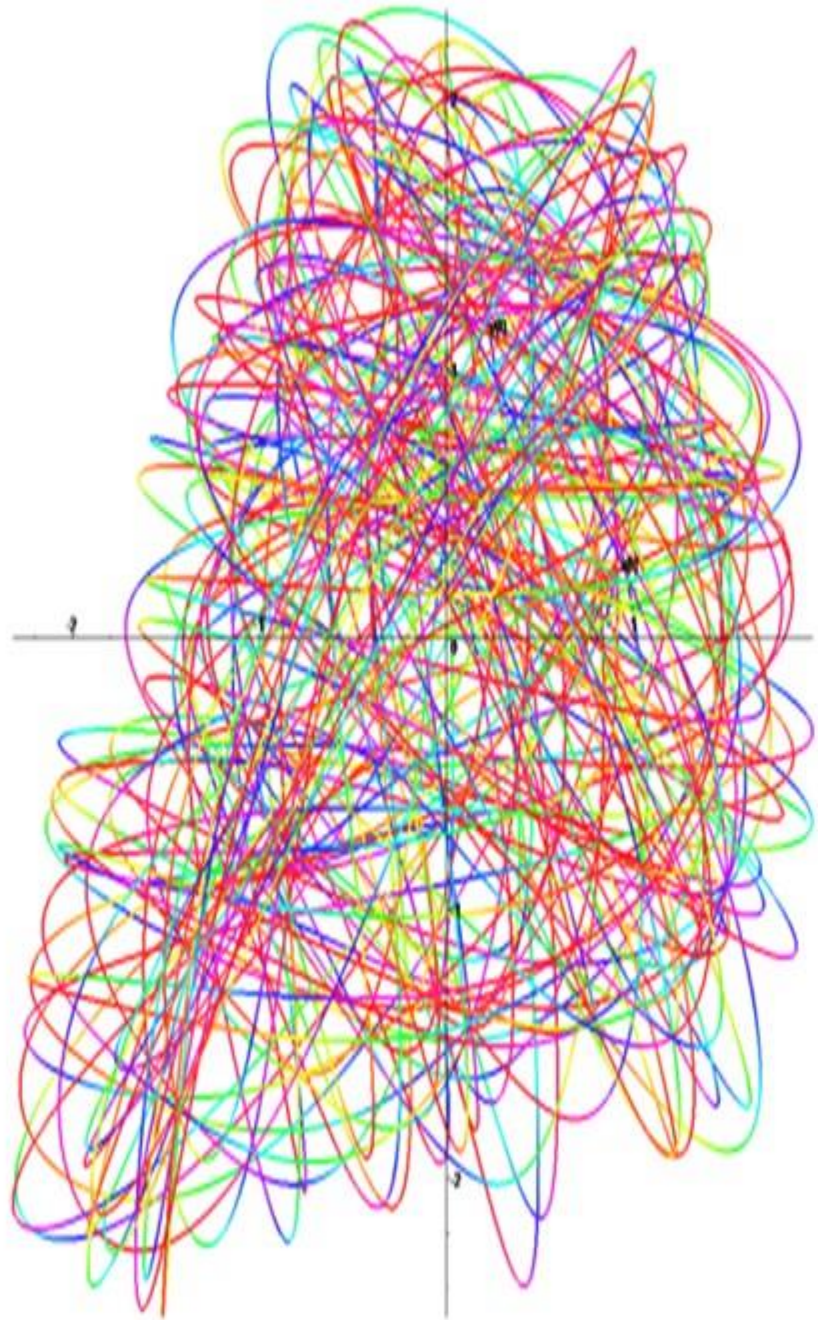
$$Y'' = F_2(x, y) + a \sin(2\pi t/n)$$

$$a = 0.5$$

Y
/
t

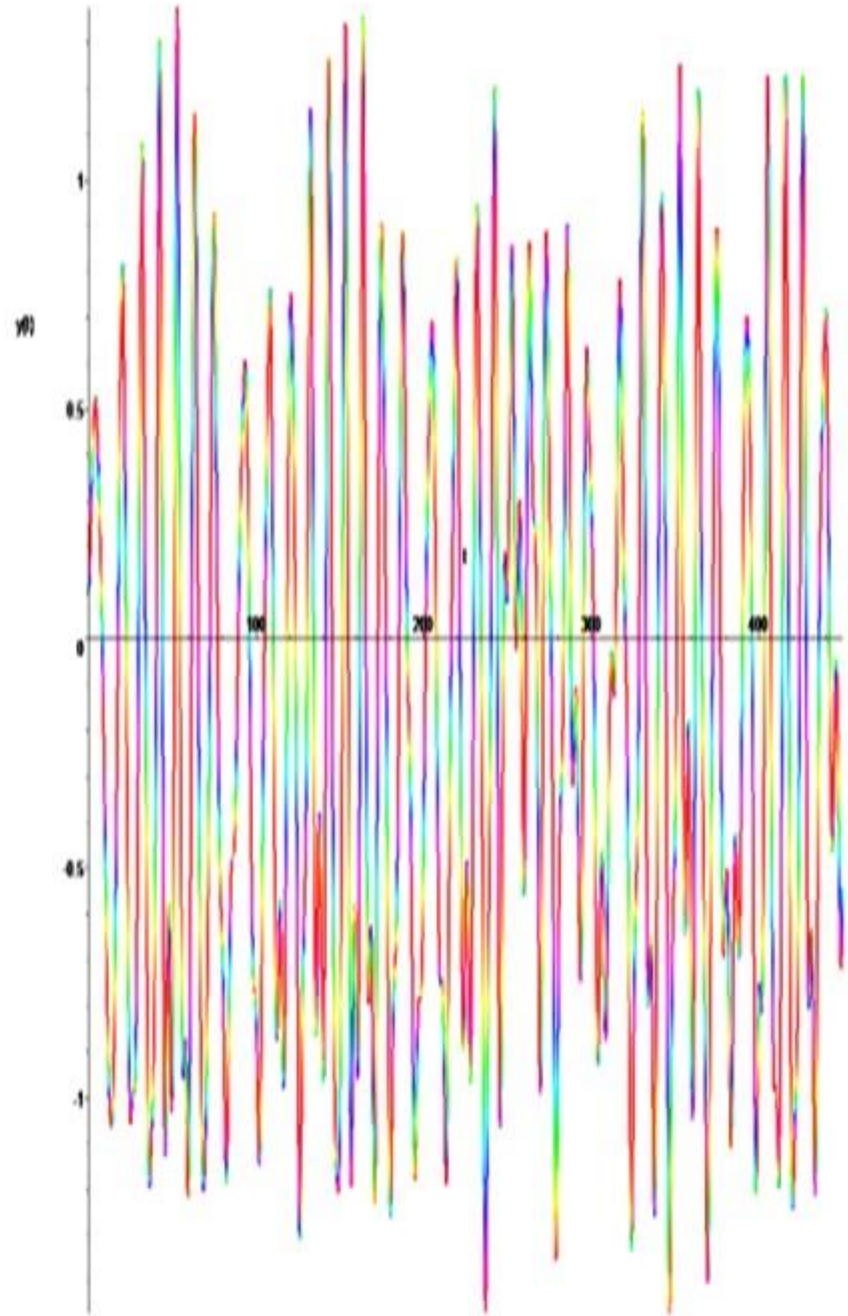


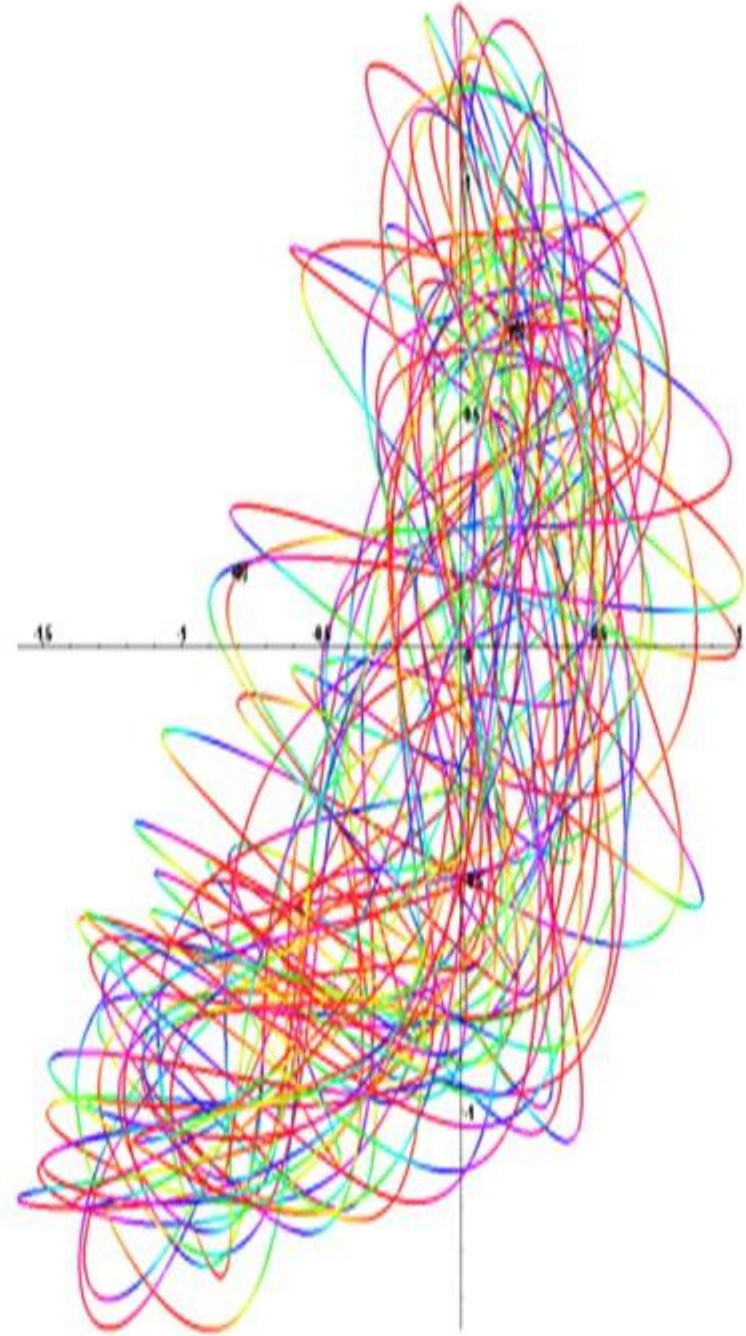
Y
/
x



$$Y'' = F_2(x, y) + a \sin(2\pi t/n)$$

$$a = 1.0$$

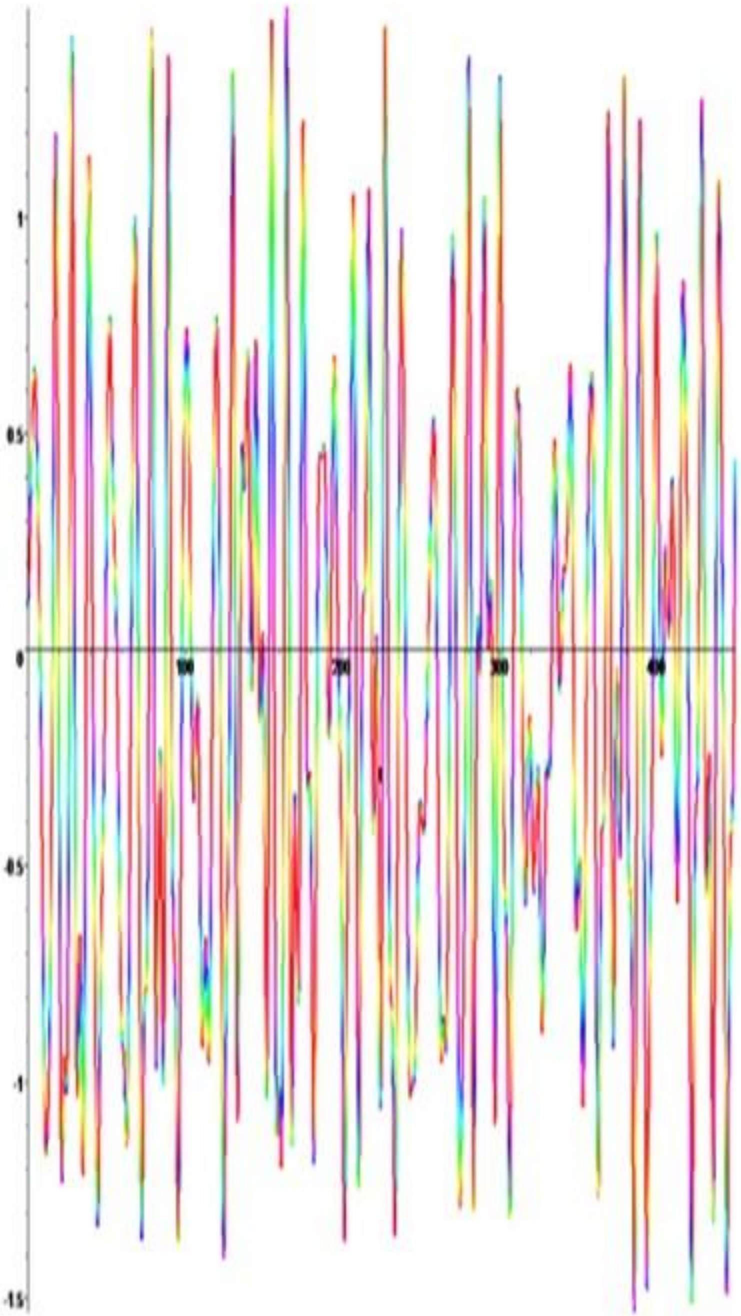

y(t)
1
0.5
0
-0.5
-1
100
200
300
400


-1.5
-1
0.5
1

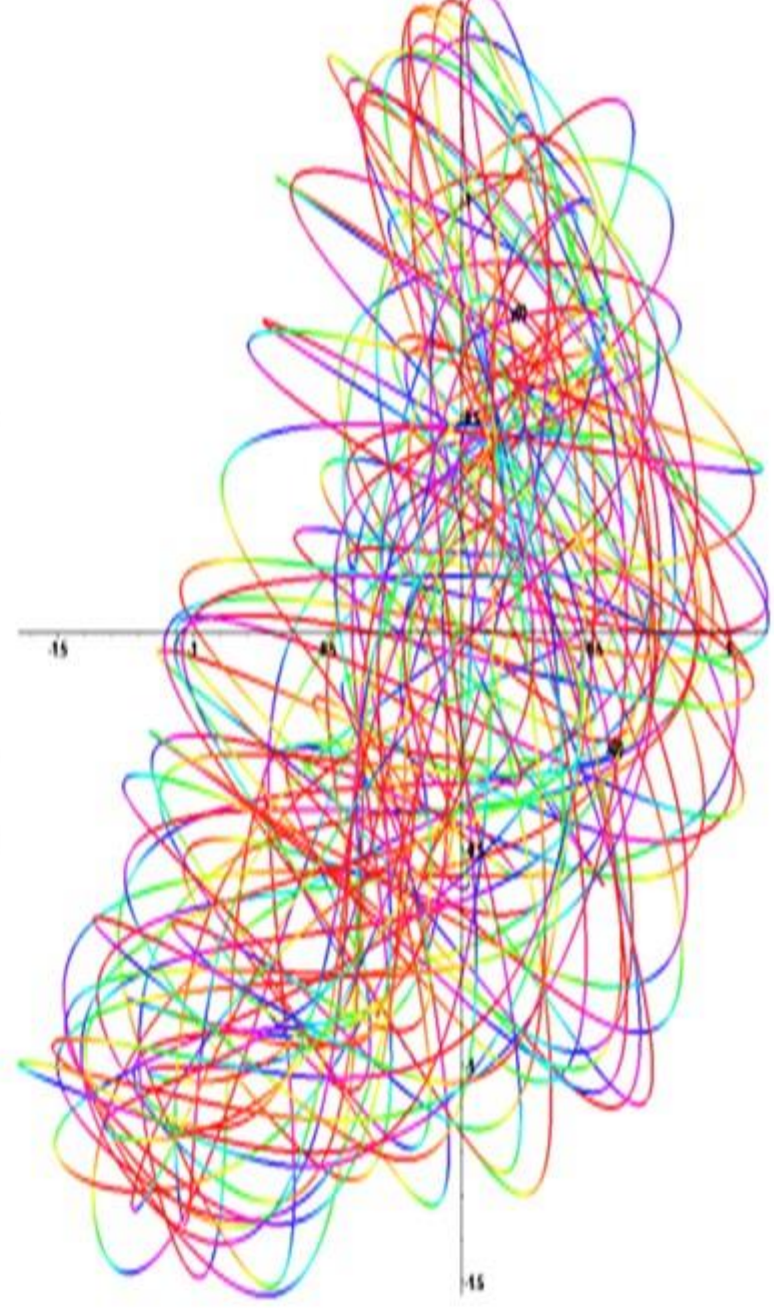
$$Y'' = F_2(x, y) + a \sin(2\pi t/n)$$

$$a = 1.5$$

Y
/
t



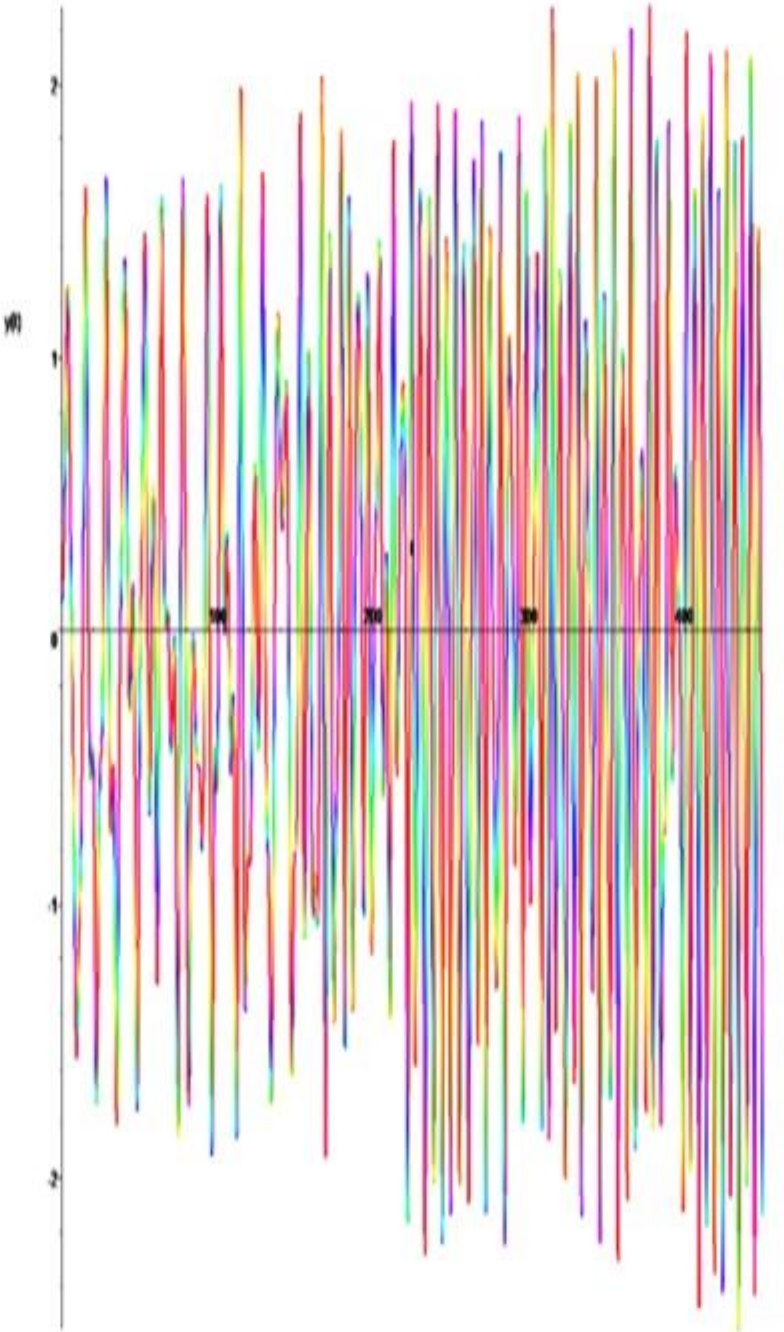
Y
/
X



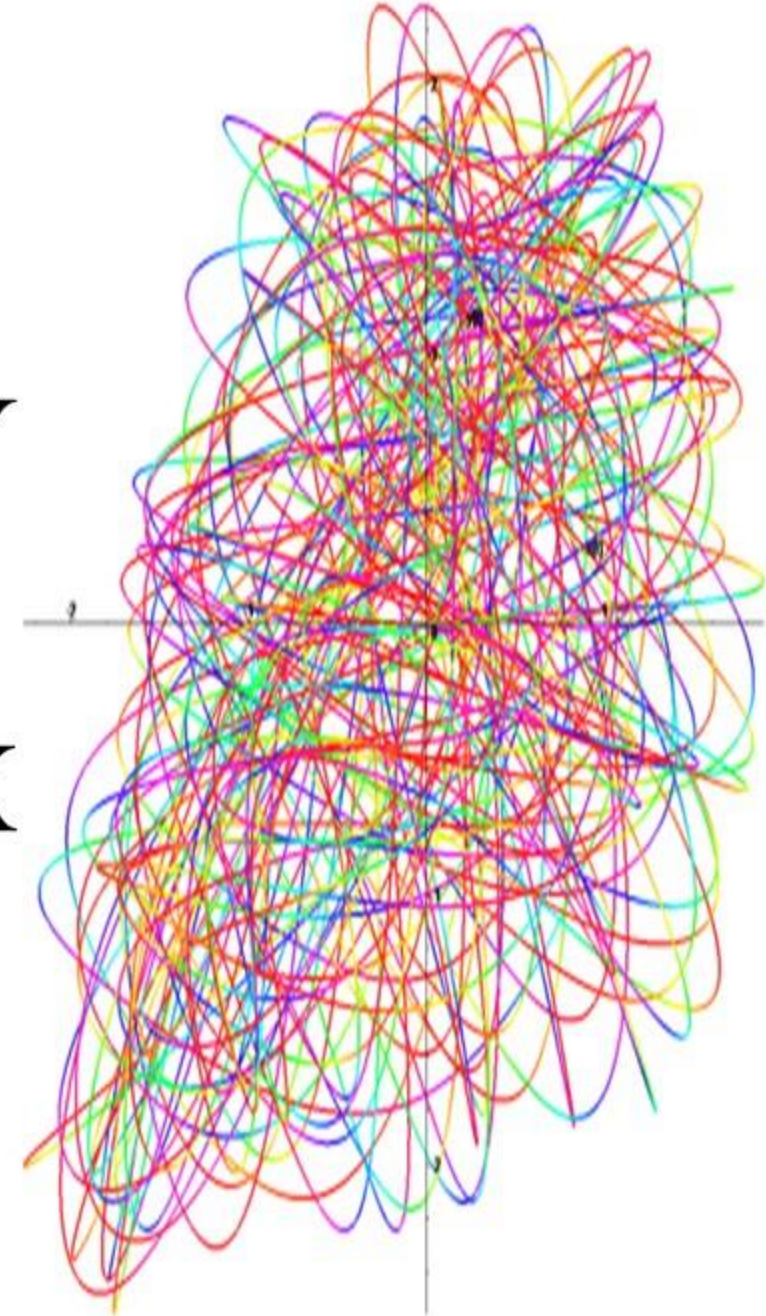
$$Y'' = F_2(x, y) + a \sin(2\pi t/n)$$

$$a = 2.0$$

Y
/
t



Y
/
X



Using this substitution, the sine model of the KAN, for two interconnected nonlinear oscillators, we obtain stable hysteresis. The two interconnected nonlinear oscillators themselves, in the presence of parametric action (ε, γ), in the absence of external influence, have hysteresis.

With small external periodic influences, the system has hysteresis. With large external periodic influences, the destruction of hysteresis occurs in the system.

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Thank you very much for your attention!

I would like to thank the Organizers for the wonderful opportunity to attend your wonderful VII INTERNATIONAL CONFERENCEVII, Topological methods in dynamics and related topics.

And report to the sections

- Bifurcations and chaos.