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Soliton Regime of Generation of Terahertz Radiation Taking into Account the Phase of an Optical Pulse

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An integrable generalization of the Yajima–Oikawa system of equations has been obtained to describe the optical generation of broadband terahertz radiation in a quadratically nonlinear medium with allowance for the effect of the phase of an optical pulse on this process. The corresponding Lax pair has been found. A soliton solution has been constructed. The analysis has indicated, in particular, that the role of the phase of the optical pulse increases significantly in the vicinity of zero value of the second-order group dispersion parameter. This behavior can result in a significant increase in the efficiency of generation in the soliton regime.

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The optical method of generation of terahertz radiation is based on the generation of a wave packet, which consists of Fourier components at difference frequencies of the spectrum of an optical pulse introduced into a nonlinear medium [1–4]. Such a generation occurs under the condition of the closeness of the group velocity v_g of the optical pulse to the phase velocity v_T of the terahertz signal. The simplest case of this generation takes place when the spatiotemporal intensity profile of the spectrally limited optical signal introduced into the crystal determines the profile of the generated field of a broadband terahertz pulse.

Recently, A.P. Sukhorukov posed a question on the effect of the phase of an optical signal on the generation of terahertz radiation. This effect was analyzed in [5], where it was shown that the effect of phase modulation can be decisive under certain conditions. In this case, the spatiotemporal profile of the generated terahertz signal is determined by the character of variation of the phase of the optical pulse and the spectrum of terahertz radiation can be both broadband and quasi-monochromatic. Such features of generation are due exclusively to the dispersion of quadratic nonlinearity.

The mechanism associated with phase modulation can be dominant for nanosecond optical pulses [6], whereas the simpler generation mechanism based on amplitude modulation is manifested in the case of femtosecond and subpicosecond signals [3, 7, 8]. However, a further shortening of the duration of the optical pulse with the same carrier frequency should lead to the enhancement of the role of its phase. This particularly concerns signals with a duration of a few oscillation periods.

The self-consistent consideration of the generation process includes the inverse effect of the terahertz signal on the optical pulse. If the effect of the phase of the optical pulse is disregarded, such a generation process is described by the integrable Yajima–Oikawa system of equations [9]. The general physical problems of the interaction of long and short waves lead to this system, which is the version of Zakharov equations [10] reduced to the case of unidirectional propagation. In this case, the optical pulse and terahertz signal serve as the short- and long-wave components, respectively.

The aforementioned integrability of the Yajima–Oikawa system of equations means that the inverse scattering method [11, 12] is applicable to this system. The discrete part of scattering data completely describes soliton solutions of such equations.

Generation of terahertz radiation owing to phase modulation was studied in [5] in the approximation of a given field of the optical pulse. At the same time, by analogy with the case of spectrally limited signals, the reverse effect of terahertz radiation on the optical pulse should be expected. For this reason, the correct consideration of this problem below is based on its self-consistent analysis based on equations generalizing the Yajima–Oikawa system of equations.

In this work, an integrable generalization of the system describing the interaction between optical and terahertz pulses is obtained in the case of the inclusion of the dispersion of the quadratic nonlinearity of a medium. The generation of terahertz radiation is analyzed with the use of its soliton solutions.

† Deceased.

Let an optical pulse and a generated terahertz signal propagate along the z axis perpendicular to the optical axis of a uniaxial crystal. In this case, pulses do not have the longitudinal component of the electromagnetic field. If the electric field of the input pulse lies in the plane formed by the optical axis and z axis, the plane of polarization of a pulse does not change in the process of its propagation [13]. Then, the total electric field E of the pulse in the crystal can be represented in the scalar form

$$E = E_{\mathrm{T}} + \Psi e^{i(\omega t - kz)} + \Psi^* e^{-i(\omega t - kz)}, \quad (1)$$

where E_{T} and Ψ are the field of the terahertz component and the complex envelope of the electric field of the optical component, respectively, and ω and k are the carrier frequency and wavenumber of the optical component, respectively.

We perform the standard procedure of the separation of the electromagnetic field into the optical and terahertz components in the initial wave equation. In the slowly varying envelope approximation for the former component [14, 15] and in the unidirectional propagation approximation for the latter component [16, 17], we arrive at the system of equations

$$\frac{\partial E_{\mathrm{T}}}{\partial z} = \alpha \frac{\partial^3 E_{\mathrm{T}}}{\partial \tau^3} - \beta E_{\mathrm{T}} \frac{\partial E_{\mathrm{T}}}{\partial \tau} - \sigma \frac{\partial}{\partial \tau}(|\Psi|^2) + ig\frac{\partial}{\partial \tau}\left(\Psi^* \frac{\partial \Psi}{\partial \tau} - \Psi \frac{\partial \Psi^*}{\partial \tau}\right), \quad (2)$$

$$i\frac{\partial \Psi}{\partial z} = -\frac{k_2}{2}\frac{\partial^2 \Psi}{\partial \tau^2} + i\frac{k_3}{6}\frac{\partial^3 \Psi}{\partial \tau^3} + aE_{\mathrm{T}}\Psi - ib\Psi\frac{\partial E_{\mathrm{T}}}{\partial \tau} - i\mu E_{\mathrm{T}}\frac{\partial \Psi}{\partial \tau} + \varepsilon|\Psi|^2\Psi. \quad (3)$$

Here, $\tau = t - z/v_{\mathrm{T}} = t - z/v_{\mathrm{g}}$ and the phase velocity v_{T} of the terahertz component and the group velocity v_{g} of the optical component are given by the expressions

$$\frac{1}{v_{\mathrm{T}}} = \frac{1 + 2\pi\chi_{\mathrm{T}}}{c},$$

$$\frac{1}{v_{\mathrm{g}}} = \frac{\partial k}{\partial \omega} = \frac{1}{c}\left[1 + 2\pi\left(\chi_{\omega} + \omega\frac{\partial \chi_{\omega}}{\partial \omega}\right)\right],$$

where χ_{T} and χ_{ω} are the linear susceptibilities of the medium in the terahertz band and at the frequency ω, respectively. Furthermore, we assume that the Zakharov–Benney resonance condition is satisfied [10, 18]:

$$v_{\mathrm{g}} = v_{\mathrm{T}}.$$

The coefficients of Eqs. (2) and (3) are specified by the expressions

$$\alpha = \frac{\pi}{c}\left(\frac{\partial^2 \chi_{\mathrm{T}}}{\partial \omega_{\mathrm{T}}^2}\right)_{\omega_{\mathrm{T}} = 0}, \quad \beta = \frac{4\pi\chi^{(2)}(0, 0)}{c},$$

$$\sigma = \frac{4\pi\chi^{(2)}(\omega, -\omega)}{c}, \quad g = \frac{4\pi}{c}\left(\frac{\partial \chi^{(2)}}{\partial \omega_1}\right)_{\substack{\omega_1 = -\omega \\ \omega_2 = \omega}},$$

$$k_2 = \frac{\partial^2 k}{\partial \omega^2} = \frac{2\pi}{c}\left(2\frac{\partial \chi_{\omega}}{\partial \omega} + \omega\frac{\partial^2 \chi_{\omega}}{\partial \omega^2}\right),$$

$$k_3 = \frac{\partial^3 k}{\partial \omega^3} = \frac{2\pi}{c}\left(3\frac{\partial^2 \chi_{\omega}}{\partial \omega^2} + \omega\frac{\partial^3 \chi_{\omega}}{\partial \omega^3}\right),$$

$$a = \frac{4\pi\omega}{c}\chi^{(2)}(\omega, 0), \quad b = \frac{4\pi}{c}\chi^{(2)}(\omega, 0),$$

$$\mu = \frac{4\pi}{c}\left[\chi^{(2)}(\omega, 0) + \omega\frac{\partial \chi^{(2)}(\omega, 0)}{\partial \omega}\right],$$

$$\varepsilon = \frac{6\pi\omega}{c}\chi^{(3)}(\omega, \omega, -\omega),$$

where $\chi^{(2)} = \chi^{(2)}(\omega_1, \omega_2)$ and $\chi^{(3)} = \chi^{(3)}(\omega_1, \omega_2, \omega_3)$ are the nonlinear susceptibilities of the second and third orders, respectively, and k_2 and k_3 are the second- and third-order dispersion parameters of the group velocity, respectively.

Equations (2) and (3) were derived under the assumption that the spectra of optical and terahertz components lie in the transparency region of the crystal, where nonlinearity and dispersion are relatively weak. For this reason, taking into account linear and nonlinear dispersions, we used the standard procedure of expansion in a small delay time [5, 15].

Equation (2) for the terahertz component generalizes the equation obtained in [5] to the case with dispersion and terahertz quadratic nonlinearity (first and second terms on the right-hand side, respectively). The phase of the optical pulse is taken into account in Eq. (2) in the last term with the coefficient g. Equation (3) generalizes the equation of the Yajima–Oikawa system for the short-wave component to the case with the third-order group dispersion (the second term on the right-hand side), dispersion of quadratic nonlinearity (the fourth and fifth terms), and dispersionless cubic nonlinearity (the last term). Setting $\alpha = \beta = g = k_3 = b = \mu = \varepsilon = 0$ in Eqs. (2) and (3), we arrive at the well-known, previously studied Yajima–Oikawa system of equations.

We note that the short- and long-wave components in Eqs. (2) and (3), as well as in the Yajima–Oikawa system of equations, play different roles. An optical pulse propagating in a nonlinear medium generates a terahertz pulse even if this pulse was absent at the initial time. The reverse process is impossible. In the absence of the optical pulse (i.e., at $\Psi = 0$), the dynamics of the terahertz pulse of the field is described by the well-known Korteweg–de Vries (KdV) equation [11, 12]. This equation is obtained in the problems of propagation of nonlinear waves with a small amplitude in weakly dispersive media.

The Yajima–Oikawa system of equations is one of the member of the hierarchy of integrable equations which correspond to a common third-order matrix spectral problem of a special form in the inverse scattering method [11, 19]. The right-hand sides of Eqs. (2) and (3) contain terms whose orders of derivatives and nonlinearities correspond to both the Yajima–Oikawa system of equations and the next member in the hierarchy of integrable equations under consideration. Consequently, the integrability of the system of Eqs. (2) and (3) can be expected within the same hierarchy that includes the Yajima–Oikawa system of equations. Indeed, integrability occurs under the conditions

$$\beta = \frac{12\alpha a}{k_2}, \quad g = \frac{6\alpha\sigma}{k_2}, \quad k_3 = 24\alpha,$$
$$b = \frac{6\alpha a}{k_2}, \quad \mu = \frac{12\alpha a}{k_2}, \quad \varepsilon = \frac{12\alpha a\sigma}{k_2^2}. \tag{4}$$

In this case, the system of Eqs. (2) and (3) can be represented in the form of the zero-curvature condition

$$\frac{\partial\hat{L}(\lambda)}{\partial z} - \frac{\partial\hat{A}(\lambda)}{\partial\tau} + [\hat{L}(\lambda), \hat{A}(\lambda)] = 0$$

with the matrices $\hat{L}(\lambda)$ and $\hat{A}(\lambda)$ of the form

$$\hat{L}(\lambda) = \begin{pmatrix} -\lambda & \Psi & -\frac{2ia}{k_2}E_T \\ 0 & 0 & \frac{2a\sigma}{k_2^2}\Psi^* \\ i & 0 & \lambda \end{pmatrix}, \tag{5}$$

$$\hat{A}(\lambda) = \begin{pmatrix} A_- & A_1 & -\frac{2ia}{k_2}A_2 \\ \frac{2a\sigma}{k_2^2}\delta^* & 8i\frac{\alpha a\sigma}{k_2^2}|\Psi|^2 & \frac{2a\sigma}{k_2^2}A_1^* \\ i\Delta & \delta & A_+ \end{pmatrix}, \tag{6}$$

where λ is the spectral parameter,

$$\delta = 4i\alpha\frac{\partial\Psi}{\partial\tau} - \frac{k_2}{2}\Psi, \quad \Delta = 4\alpha\left(\lambda^2 - \frac{a}{k_2}E_T\right),$$

$$A_\pm = \pm\lambda\Delta + \frac{ik_2}{2}\lambda^2 \pm \frac{2\alpha a}{k_2}\left(\frac{\partial E_T}{\partial\tau} \mp \frac{2i\sigma}{k_2}|\Psi|^2\right),$$

$$A_1 = 4\alpha\lambda^2\Psi + i\lambda\delta - i\frac{\partial\delta}{\partial\tau} - \frac{4\alpha a}{k_2}E_T\Psi,$$

$$A_2 = \Delta E_T - 2\alpha\lambda\frac{\partial E_T}{\partial\tau} + \alpha\frac{\partial^2 E_T}{\partial\tau^2} + \frac{\sigma}{k_2}\Re(\Psi\delta^*).$$

The matrix $\hat{L}(\lambda)$ given by Eq. (5) specifies the spectral problem for the hierarchy of integrable equations containing the Yajima–Oikawa system of equations. The matrix $\hat{A}(\lambda)$ given by Eq. (6) corresponds to the Yajima–Oikawa system of equations and the next member of this hierarchy. The case $\Psi = 0$ leads to the matrix representation of the Lax pair for the KdV equation.

The one-soliton solution of the integrable variant of the system of Eqs. (2) and (3) in the laboratory coordinate system has the form

$$\Psi = \frac{k_2}{\tau_p}\sqrt{\frac{\Omega}{\alpha\sigma}}e^{-i(\Omega t - qz)}\operatorname{sech}\left(\frac{t - z/v}{\tau_p}\right), \tag{7}$$

$$E_T = -\frac{k_2}{\alpha\tau_p^2}\operatorname{sech}^2\left(\frac{t - z/v}{\tau_p}\right), \tag{8}$$

where τ_p is the characteristic duration of the soliton; Ω and q are the nonlinear additions to the carrier frequency and wavenumber of the optical pulse, respectively, which are related to each other as

$$q = \frac{\Omega}{v_g} - \left(k_2 + \frac{k_3\Omega}{3}\right)\frac{\Omega^2}{2} + \frac{k_2 - k_3\Omega}{2\tau_p^2}, \tag{9}$$

and v is the velocity of the soliton given by the expression

$$\frac{1}{v} = \frac{1}{v_g} - k_2\Omega + \frac{k_3}{6}\left(3\Omega^2 - \frac{1}{\tau_p^2}\right). \tag{10}$$

The coefficient k_3 in Eqs. (9) and (10) is related to α by the third relation in Eqs. (4).

We note that the terahertz component of the soliton under consideration undergoes larger compression than the optical component, because $|E_T| \sim |\Psi|^2$.

The soliton solution given by Eqs. (7)–(10) is generally two-parametric. The duration τ_p and quantity Ω are taken as free parameters. The physical meaning of the latter parameter is as follows. The nonlinear susceptibility does not change sign with variation of the frequency in the spectral transparency region of the crystal. Therefore, as can be seen from the definitions of the coefficients a and σ, their product is positive. Since the radicand in Eq. (7) is nonnegative, $\Omega > 0$; i.e., the carrier frequency of the optical pulse is shifted toward the red region (see Eq. (1)). This behavior can be easily explained by the fact that each photon of the propagating optical pulse generates a terahertz photon, thus losing a fraction of its energy. It can also be seen in Eq. (7) that the discussed shift of the frequency is proportional to the intensity $\sim|\Psi|^2$ of the optical signal. This result is also obtained disregarding the phase of the optical pulse. It was previously discussed in theoretical [20–24] and experimental [7] works.

According to Eq. (10), the velocity of the optical–terahertz soliton of the system of Eqs. (2) and (3) generally depends on Ω and τ_p. The qualitative character of the dependence on τ_p is determined by the sign of the third-order group dispersion: with a decrease in the duration of the soliton, its velocity increases at $k_3 > 0$ and decreases at $k_3 < 0$.

If $k_3 = 0$ and the coefficients k_2, a, and σ are nonzero, α and all other coefficients of Eqs. (2) and (3) vanish as follows from Eqs. (4). In this case, the solution given by Eqs. (7)–(10) exactly coincides with the optical–terahertz soliton of the Yajima–Oikawa system of equations. The velocity of such soliton is independent of the duration (see Eq. (10)).

Another limiting case is obtained from the above soliton solution with $\Omega = 0$, which corresponds to $\Psi = 0$. In this case, taking into account the first and third relations in Eqs. (4), we obtain from Eqs. (8) and (10) the soliton solution of the KdV equation to which Eq. (2) is transformed in this case, as was mentioned above. This solution corresponds to the effective spatial separation of the terahertz component from the optical component when the process of generation can be considered as completed. Indeed, the above physical analysis shows that terahertz radiation is not generated at $\Omega = 0$. In the simplest case, it can be assumed that only the terahertz component is present at the entrance of the medium. It can also be assumed that coupled optical–terahertz soliton states and purely terahertz solitons, which propagate at different velocities, can be formed after the introduction of the optical pulse into the crystal. The input conditions under which this behavior occurs can be determined from the further analysis of the system of Eqs. (2) and (3).

It follows from Eqs. (7) and (8) that the ratio r of the peak intensity of the terahertz signal to the peak intensity of the optical pulse is

$$r = \frac{\sigma}{a\Omega\tau_p^2}.$$

It can be seen that this ratio increases with a decrease in both the duration τ_p of the pulse and the frequency shift Ω. The amplitude of the optical pulse can remain unchanged.

Using the first two relations in Eqs. (4), we obtain the following expression for the ratio of peak intensities:

$$r = \frac{2g}{\beta\Omega\tau_p^2}. \quad (11)$$

It can be seen from Eqs. (11) and (4) that the role of the last term on the right-hand side of Eq. (2), which is responsible for the effect of the phase of the optical pulse on the generation of the terahertz signal, is enhanced with a decrease in k_2. The reduction of the duration τ_p of the optical pulse also enhances the role of its phase.

For the typical carrier frequencies of the optical pulse corresponding to the visible range, the above behavior occurs at durations of about 10 fs. At smaller durations, the applicability of the slowly varying envelope approximation and, as a result, the validity of the system of Eqs. (2) and (3) are doubtful.

It can be seen from Eqs. (11) and (4) that the terms on the right-hand sides of Eqs. (2) and (3) corresponding to the dispersion of quadratic nonlinearity except for the term with the coefficient g are responsible for the transfer of the energy of the generated terahertz signal back into the optical pulse. Consequently, these terms reduce the efficiency of generation.

The system of Eqs. (2) and (3) can also be integrable when the relations between its coefficients are different from Eqs. (4). The theory of solitons provides tests of the integrability of nonlinear equations within the inverse scattering method. One of them is the Painlevé test [25–27]. It is undoubtedly of interest to apply such tests to the system of Eqs. (2) and (3) in order to find other integrable cases.

Thus, to describe the self-consistent regime of generation of terahertz radiation with a decrease in the duration of the optical pulse to 10 fs, it is necessary to modify the Yajima–Oikawa system of equations, where the effect of the phase of the optical pulse is disregarded. This is particularly important for small values of the second-order dispersion parameters of the group velocity. The proposed system of Eqs. (2) and (3) solves this problem and becomes integrable under conditions (4). Although Eqs. (4) impose quite stringent constraints on the parameters of the medium, they significantly simplify consideration, allowing an analytical analysis. These constraints can be eliminated in the numerical investigation of the generation of terahertz radiation by femtosecond optical pulses with the system of Eqs. (2) and (3).

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