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Dedicated to E.L. Feinberg
at his 80-th birthday.

FRACTAL 'HOT SPOTS' AND STRUCTURE FUNCTIONS AT LOW X_{Bj}

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Abstract

Saturation of hadron structure functions at small x may proceed faster if 'hot spots' have a fractal structure implied by evolution equations.

INTRODUCTION

Perturbative calculation of the evolution of quark and gluon distributions within a hadron show rapid increase of these functions at low values of the Bjorken x–variable, for a fixed Q^2. However, the unitarity condition should prevent their growth at high parton densities [1]. The partons start to interact and their recombination might balance their number so that structure functions (almost) saturate at small x. It is very important how the partons are spread over volume of a hadron when these processes begin to play a role. It is unnecessary to get high densities over the whole volume. There could appear highly packed 'hot spots' by which one could imply the constituent quarks, for example (for a brief reviews see [2]). Even more, there could exist inhomogeneous regions within such 'hot spots'. All these effects favour faster saturation of the structure functions.

As one of such possibilities we would like to propose the space-time picture of 'hot spots' with a fractal distribution of partons within them. The evolution equations favour the fractal structure of parton cascades[3–5] in momentum space with QCD anomalous dimension being closely related to Renyi dimensions. Therefore, one could await for the fractal structure of the cascading 'tree' in space-time, as well. It means that the parton ladder evolution gives rise to a stable (in the infinite momentum frame) fractal configuration of partons inside of a 'hot spot',so that the same number of partons can provide higher density in some subregions compared to the homogeneous distribution (e.g. the density of branches of an usual tree is much higher than the average density over its whole volume). Herefrom it follows that there could be faster saturation of structure functions for smaller fractal dimensions.

Let us note that the various power laws have been experimentally observed for the strucrure factors of numerous macroscopic fractal aggregates by diffractive scattering of neutrons, electrons and photons [6-9]. The similar regularities may show up for structure functions if the distribution of scatterers within hadrons is a fractal one.

THE MODEL

To get some quantitative estimates , one should use a definite model. We assume that, in soft collisions, a hadron (or a constituent quark) is viewed as a collection of smaller colorless sources [1*] filling in its interior according to a fractal law. It reminds of the model proposed in [11] where each hadron (nucleus) is treated as a dilute system of heavy quark-antiquark pairs . That is why one can boldly use the results of [11] just replacing the assumption about the homogeneous distribution of these pairs by taking into account the fractal distribution of sources. Summing up all successive interactions with sources, one gets [11] the integral relation[2*] between the structure function $xG(x,Q^2)$ and the overall source S in a form

$$D(x,Q^2) = \frac{N_c^2-1}{2\pi^2}\ln\left(\frac{1}{x}\right)\int_0^{R^2} d\mathbf{b}^2 \int_{Q^2{}_0}^{Q^2} dy\,\Bigg[1- \exp\left\{-\frac{C_A\alpha_s}{y}\int_{y_0}^{y}\frac{d\mathbf{k}^2}{\mathbf{k}^2}\int_0^{\sqrt{R^2-b^2}} dz\,\mathbf{S}(\mathbf{k},\mathbf{b},z)\right\}\Bigg] \qquad (1)$$

where $D(x,Q^2) = xG(x,Q^2)$,N_c is the number of colors, $C_A = 3$, α_s is QCD coupling constant, the integration over $\mathbf{b}^2$ and z takes into account the space (transverse and longitudinal) structure of the source S which is normalised to the number of partons so that

$$\frac{N_c^2-1}{(2\pi)^2}\int d^2\mathbf{b}\int dz\int\frac{d\mathbf{k}^2}{\mathbf{k}^2}\mathbf{S}(\mathbf{k},\mathbf{b},z) = D(x,Q^2). \qquad (2)$$

The overall source may be represented as a product of a single (almost pointlike) source $\mathbf{s}(\mathbf{k})$ and of the space density of such sources $\rho(\mathbf{r})$ i.e.

$$\mathbf{S}(\mathbf{k},\mathbf{b},z) = \rho(\mathbf{r})\mathbf{s}(\mathbf{k}) \qquad (3)$$

with a normalization to the total number of the sources A

$$\int d^3\mathbf{r}\,\rho(\mathbf{r}) = A. \qquad (4)$$

[1*] Such a picture could arise in the effective lagrangian approach advocated in [10] where constituent quarks are treated as current quarks surrounded by a pion cloud formed due to cascade-type branching.

[2*] We are considering the gluon densities here. The similar equations are valid for quark-antiquark densities. Due to shortage of space, we refer the reader for details to the paper [11].

Herefrom, it is easy to see that the structure function satisfies the equation

$$\frac{\partial D}{\partial Q^2} = \frac{N_c^2 - 1}{2\pi^2} \ln(\frac{1}{x}) \Big[R^2 - \int_0^{R^2} db^2 \exp\left\{-\frac{2\pi^2 C_A}{N_c^2 - 1}\frac{\alpha_s}{Q^2}\frac{D}{A}\int_0^{\sqrt{R^2-b^2}} dz\,\rho(b,z)\right\}\Big] \tag{5}$$

It is the sources density $\rho(\mathbf{r})$ what should account for the fractal structure of the whole source. The particular form of it is very complicated for different aggregates since $\rho(\mathbf{r})$ should be very irregular function. However, before going into details let us note that the solutions of (5) do not depend on fractal dimension if the exponent in the integral is either small or large. In the first case,using Taylor expansion one gets rid of ρ due to the normalization condition (4) while, in the second one, the integral term may be neglected at all. Thus any dependence on the fractal dimension may appear only for intermediate values of Q^2. This is understandable because at small sizes (large Q^2) one deals with a single source while at large sizes one does not reveal at all the internal structure of the overall source. Thus in both cases the fractal structure is hidden somehow and it can be only resolved by using an appropriate scale as is common for any fractal in nature.

To proceed further, one should use a definite model of $\rho(\mathbf{r})$. The more irregular it is, the larger effects may be observed. Our aim is to demonstrate qualitative effects not to dive into particular details. That is why, instead of using genuine fractal laws, we apply the strongly smoothened inhomogeneous isotropic distribution of $\rho(\mathbf{r})$ depending only on the absolute value of the radius which looks like

$$\rho(\mathbf{r}) = A\frac{d_f}{4\pi}\frac{r^{d_f-3}}{R^{d_f}}\theta(R-r), \tag{6}$$

which for $d_f = 3$ reduced to the homogeneous 3-dimensional one treated in [11]. It corresponds to a fractal with a central symmetry spread over the whole volume filling in empty places. Thus the system becomes less irregular and should show smaller effects compared to a genuine fractal. For an arbitrary d_f, the equation becomes very complicated. Therefore, we demonstrate in the analytical form here the difference between the cases $d_f = 3$ and $d_f = 2$ only even though $d_f = 1$ can be easily treated as well.

For $d_f = 3$ and $\alpha_s = const$, it becomes

$$\frac{\partial D}{\partial Q^2} = \frac{N_c^2 - 1}{2\pi^2} R^2 \ln(\frac{1}{x}) \left[1 - \frac{2}{\chi^2}(1 - e^{-\chi} - \chi e^{-\chi})\right] \tag{7}$$

with

$$\chi = \frac{3\pi C_A \alpha_s}{2(N_c^2 - 1)} \frac{D}{Q^2 R^2},$$

while for $d_f = 2$ it is

$$\frac{\partial D}{\partial Q^2} = \frac{N_c^2 - 1}{2\pi^2} R^2 \ln(\frac{1}{x}) [1- \\ - \frac{3}{\chi + 3} F(3, \frac{\chi}{3} + 1; \frac{\chi}{3} + 2; -1) + \\ + \frac{3}{\chi + 6} F(3, \frac{\chi}{3} + 2; \frac{\chi}{3} + 3; -1) \Big], \tag{8}$$

where F is a hypergeometric function.

For $\alpha_s = const$, the general solution of eqs. (7), (8) may be written as

$$\ln \frac{Q^2}{Q_o^2} = \int d\chi \left[\frac{3\alpha_s C_A}{4\pi} \ln(\frac{1}{x}) \Phi_{d_f}(\chi) - \chi \right]^{-1}, \tag{9}$$

where $\Phi_{d_f}(\chi)$ are the functions appearing in the square brackets in (7) and (8), correspondingly. These equations remind of 'qain-loss' equations with higher order correlations taken into account.

Comparing equations (7) and (8) one easily sees that the limits for both equations at large and small χ coincide while the higher order corrections differ. Namely, for $d_f = 2$ the derivative $\partial D / \partial Q^2$ is slightly lower at small χ than for $d_f = 3$ indicating stronger saturation of the structure function at low dimensions. The corresponding correction factors to the leading linear in χ term are equal to $(1 - 3\chi/8)$ for d_f =3 and $(1 - (2/3) \ln 2\chi)$ for $d_f = 2$. However,the net effect appears to be too small in such a model to be noticed in present day experiments. Computer calculations have shown that the structure functions change by at most $\sim 5\%$ for such a smoothed matter distribution. One should develop more elaborate model to get stronger effects.

Here we aimed at the qualitative results only. Our particular model (6) of the inhomogeneous distribution inspired by fractal laws provides rather small effects but one can hope for larger ones for non-smoothed fractal behaviour. As usually, such effects may be masked by the structure functions at lower Q^2 used as an input 'initial' condition.

In conclusion, we have shown that the fractal distribution of sources within 'hot spots' may give rise to faster saturation of hadronic structure functions in small-x region.

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