Available online at www.sciencedirect.com

ScienceDirect

Physics Procedia 70 (2015) 712 – 715

Physics Procedia

2015 International Congress on Ultrasonics, 2015 ICU Metz

Backward collinear acousto-optic interaction in germanium crystal in terahertz spectral range

P.A. Nikitin*, V.B. Voloshinov

Faculty of Physics, Lomonosov Moscow State University, Leninskie Gory 1, 119991 Moscow, Russia

Abstract

The purpose of this research was to find a way and to observe the so-called "backward" collinear acousto-optic (AO) interaction in case of bulk optic and acoustic waves. We found that single crystal germanium is so far one of the best materials to observe the backward interaction in the terahertz (THz) range. This type of diffraction takes place when wave vectors of electromagnetic and acoustic waves are collinear while the diffracted and incident electromagnetic waves propagate in opposite directions. A theoretical analysis of the bulk AO interaction is presented. The model takes into account attenuation of acoustic and electromagnetic waves in the medium. Results of our calculations were used to develop a prototype of a tunable collinear AO filter based on germanium.

Peer-review under responsibility of the Scientific Committee of ICU 2015

Keywords: acousto-optics, backward collinear diffraction, terahertz radiation, germanium crystal

Introduction

The AO effect is widely used to control parameters of electromagnetic radiation. The devices based on the AO effect have many applications in optical engineering, laser technology, astronomy, medicine, etc. The majority of scientific papers are devoted to development of AO devices operating in the ultraviolet, visible and infrared spectrum ranges. However, the number of studies devoted to AO interactions in the far-IR range (and especially in the THz range) is limited, despite the interest of researchers in these ranges of electromagnetic radiation. In the papers Durr and Schmidt (1985) as well as Vogel and Dodel (1985), the quasi-orthogonal AO diffraction in germanium (Ge) single crystal was experimentally investigated at the wavelength of electromagnetic radiation $\lambda = 119\ \mu m$ and deflection angle about 1° and 7° respectively. The possibility of designing an AO deflector capable of deflecting THz radiation by angles of several tens of degrees was experimentally confirmed for the first time by Voloshinov et al. (2013).

The AO effect is well known since the year 1922, when it was predicted theoretically by Brillouin (1922). There are a number of geometry types utilized in AO devices: quasi-orthogonal, quasi-collinear, parallel-tangent non-collinear, off-axial and others. The backward-collinear interaction is an exception. The latter geometry means that the diffracted

* Corresponding author. P.A. Nikitin Tel.: +7-495-939-4404
E-mail address: nikitin.pavel.a@gmail.com

Peer-review under responsibility of the Scientific Committee of ICU 2015
doi:10.1016/j.phpro.2015.08.101

and incident electromagnetic waves propagate in opposite directions. Therefore GHz acoustic frequencies F are needed. This leads to strong sound attenuation and low diffraction efficiency. However, in THz spectral range F is about hundreds MHz and attenuation becomes admissible.

In general, a theoretical description of the AO interaction does not depend on a particular spectral range, but there are two problematic items to consider if we examine the AO diffraction in the THz spectral domain. The first problem is to find a transparent material suitable for the AO applications, while the second one is related to the inverse ratio between diffraction efficiency and wavelength of the radiation. However, invention of powerful sources of monochromatic coherent THz radiation such as free-electron lasers makes it possible to observe the effect in this spectrum range, in spite of the extremely low diffraction efficiency.

Coupled-wave equations for the backward AO interaction

In order to take into account absorption of electromagnetic waves in the medium, we introduce real n' and imaginary n'' parts of refractive index $n = n' - in''$. The acoustic wave propagating along z-direction induces the small perturbation $\Delta n' \cos(\Omega t - \boldsymbol{K}\boldsymbol{r})$, where Ω – circular frequency and $\boldsymbol{K}$ – wave vector of acoustic wave. If absorption is not strong and $n'' \ll n'$, one can write down the expression for the permittivity as following:

$$\varepsilon = n'^2 - 2in'n'' + 2n'\Delta n' \cos(\Omega t - \boldsymbol{K}\boldsymbol{r}), \tag{1}$$

Under collinear geometry of AO interaction, an electromagnetic and acoustic wave propagate in the same direction. We find the solution of wave equation in the form of plane monochromatic waves with slowly varying amplitudes (SVA) $C_0(z)$ and $C_{\pm1}(z)$ in the case of Bragg regime. So the electric field represents itself in the sum:

$$E(z) = C_0(z) \exp[i(\omega_0 t - \boldsymbol{k_0}\boldsymbol{r})] + C_{\pm1}(z) \exp[i(\omega_{\pm1} t - \boldsymbol{k_{\pm1}}\boldsymbol{r})], \tag{2}$$

where ω_m – circular frequency and $\boldsymbol{k}_m$ – wave vector of electromagnetic wave in m-th diffraction order.

The second derivatives appearing in the wave equation could be neglected in case of the SVA method and frequency of the acoustic wave much lower than that of the electromagnetic wave $\Omega \ll \omega_m$. The stationary solution should be true in any moment and therefore independent on time t. This condition is valid only if the frequency of diffracted radiation has a Doppler shift $\omega_{\pm1} = \omega_0 \pm \Omega$. As the relative value of the shift is mach smaller than unity, one can state that $|\boldsymbol{k}_0| = |\boldsymbol{k}_1| = k$ in absence of birefringence of medium. The next step is introducing wave-vector mismatch η, which is equal to z-projection of $\boldsymbol{k}_0 \pm \boldsymbol{K} - \boldsymbol{k}_{\pm1}$ indicating violation of Bragg condition. In the case of backward collinear geometry, one should take into account that the electromagnetic waves in the first and zero diffraction orders propagate in opposite directions. Finally one should equate terms with equal time dependent parts and then obtain the following coupled-wave equations:

$$\frac{dC_0}{dz} = -\frac{\alpha}{2}C_0 + i\frac{q}{2}C_{\pm1}\exp(i\eta z), \quad \frac{dC_{\pm1}}{dz} = \frac{\alpha}{2}C_{\pm1} - i\frac{q}{2}C_0\exp(-i\eta z), \quad C_0(0) = 1, \quad C_{\pm1}(L) = 0, \tag{3}$$

where $\alpha = 4\pi n''/\lambda$ is the absorption coefficient of electromagnetic radiation, $q = k\Delta n'/n'$ is the coupling parameter and L is the length of AO interaction.

Analytical solution and numerical approximation

In order to take into account attenuation of acoustic wave, the coupling parameter has to be multiplied by a factor depending on orientation of $\boldsymbol{k}_0$ and $\boldsymbol{K}$: 1) if $\boldsymbol{k}_0 \uparrow\uparrow \boldsymbol{K}$ than the factor is equal to $\exp(-\alpha_s z/2)$; 2) in the other case $\boldsymbol{k}_0 \uparrow\downarrow \boldsymbol{K}$ one should use the factor $\exp[-\alpha_s(L-z)/2]$, where α_s is the attenuation coefficient. The schematic as well as vector diagram for anisotropic AO diffraction is shown in Fig. 1. Note that the angle of incidence of the electromagnetic radiation is wider than the refraction angle in accordance with Snell law since the refractive index of a medium is sufficiently larger than unity. For example, in germanium (Ge), the refractive index has no dispersion in the wide spectral range from 5 to 500 μm and is equal to $n = 4$ as shown by Potter (1997). That is why additional optical elements are required. To simplify AO setup, the crystals must have small cut-off angles, as shown in Fig. 2. Unfortunately this geometry requires double transit of sound over the interaction length L.

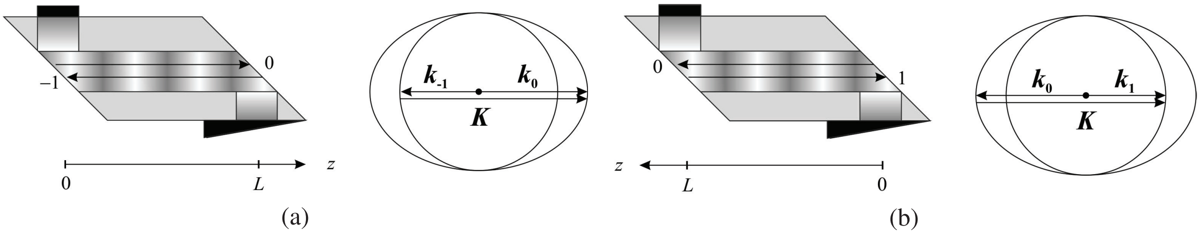

Fig. 1. Scheme of AO interaction and vector diagram corresponding to single transit of sound: (a) $\boldsymbol{k}_0 \uparrow\uparrow \boldsymbol{K}$; (b) $\boldsymbol{k}_0 \uparrow\downarrow \boldsymbol{K}$.

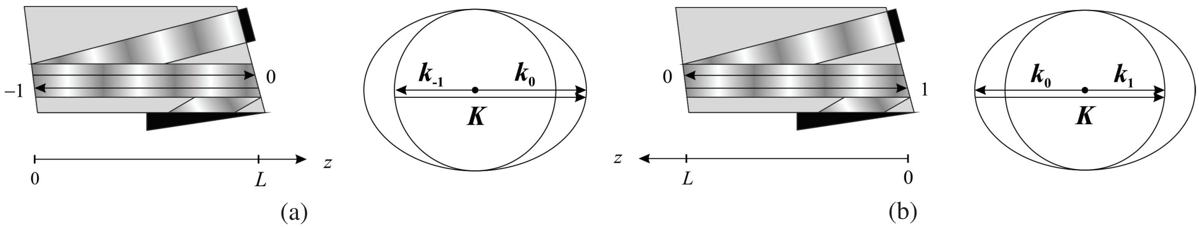

Fig. 2. Scheme of AO interaction and vector diagram corresponding to double transit of sound: (a) $\boldsymbol{k}_0 \uparrow\uparrow \boldsymbol{K}$; (b) $\boldsymbol{k}_0 \uparrow\downarrow \boldsymbol{K}$.

If Bragg condition is satisfied and $\eta = 0$ the analytical solution of the coupled wave-equation is represented respectively by the modified Bessel functions $I_\nu(x)$ and $K_\nu(x)$ of the first and second kind. If Bragg condition is not satisfied and $\eta \neq 0$, the order and argument of the Bessel functions in the solution become complex making analysis of such relations rather difficult. So the only way to get a simple relation is examination of low efficiency AO interaction when the intensity of diffracted radiation is much lower than unity $I_{\pm 1} = |C_{\pm 1}(0)|^2 \ll 1$.

Solution of eq. (3) $C_{\pm 1}(0)$ can be easily obtained analytically. Unfortunately, there is no analytical expression for the bandwidth $\Delta\eta$ at the level 0.5 relative to maximal value of I_1. The numerical calculation shows that $\Delta\eta$ as well as $I_{\pm 1}$ depends on combination $\alpha + \alpha_s/2$ for $\boldsymbol{k}_0 \uparrow\uparrow \boldsymbol{K}$, and on $\alpha - \alpha_s/2$ for $\boldsymbol{k}_0 \uparrow\downarrow \boldsymbol{K}$. The function $\Delta\eta(\alpha, \alpha_s, q, L)$ can be numerically fitted by linear and quadratic relations with respect to these combinations using the least square method.

For single transit of sound, the solution of the coupled-wave equations (3) is as follows:

$$\begin{aligned}
\boldsymbol{k}_0 \uparrow\uparrow \boldsymbol{K}: \quad & I_{-1} = \frac{q^2}{4\left[\left(\dfrac{\alpha_s}{2}+\alpha\right)^2+\eta^2\right]}\left\{1+\exp\left[-2L\left(\frac{\alpha_s}{2}+\alpha\right)\right]-2\exp\left[-L\left(\frac{\alpha_s}{2}+\alpha\right)\right]\cos(\eta L)\right\},\\
& \Delta\eta L = 0.09L^2\left(\frac{\alpha_s}{2}+\alpha\right)^2+0.9\pi \ \ (qL\le 0.1), \quad \Delta\eta L = L\left(\frac{\alpha_s}{2}+\alpha\right)+0.9\pi \ \ (qL>0.1);\\
\boldsymbol{k}_0 \uparrow\downarrow \boldsymbol{K}: \quad & I_{1} = \frac{q^2\exp(-2\alpha L)}{4\left[\left(\dfrac{\alpha_s}{2}-\alpha\right)^2+\eta^2\right]}\left\{1+\exp\left[-2L\left(\frac{\alpha_s}{2}-\alpha\right)\right]-2\exp\left[-L\left(\frac{\alpha_s}{2}-\alpha\right)\right]\cos(\eta L)\right\},\\
& \Delta\eta L = 0.09L^2\left(\frac{\alpha_s}{2}-\alpha\right)^2+0.9\pi \ \ (qL\le 0.1), \quad \Delta\eta L = L\left|\frac{\alpha_s}{2}-\alpha\right|+0.9\pi \ \ (qL>0.1).
\end{aligned} \tag{4}$$

In the case of double transit of sound, the coupling parameter q has the same dependence on z but it has to be reduced by an extra factor $\exp(-\alpha_s L/2)$. For better understanding of the phenomenon, the schematic and wave-vector diagrams are shown in Fig. 2. Since the extra factor is independent on the coordinate z, one can use the solution (4) but replacing q^2 by $q^2\exp(-\alpha_s L)$. As follows from eq. (4), the intensities $I_{\pm 1}$ of the diffracted electromagnetic radiation in the cases of $\boldsymbol{k}_0 \uparrow\uparrow \boldsymbol{K}$ and $\boldsymbol{k}_0 \uparrow\downarrow \boldsymbol{K}$ are different. It may be analytically derived that this non-reciprocal effect is an essential feature of the backward collinear diffraction and can not be observed using forward collinear and orthogonal geometry of AO interaction. Moreover, the ratio I_{+1}/I_{-1} increases exponentially with α and α_s if sound attenuation and light absorption are sufficiently strong. Another peculiarity is a simultaneous dependence of I and $\Delta\eta$ on the same combination of parameters α and α_s for a given direction of $\boldsymbol{k}_0$ and $\boldsymbol{K}$. The latter character is broadening of bandwidth $\Delta\eta$ of the AO interaction in presence of absorption of the electromagnetic radiation and of acoustic attenuation.

The maximal diffraction efficiency can be obtained at the optimal length $L = L^{\text{opt}}$. In order to find the length, one should take a derivative of the expression for $I_{\pm 1}$ on L and equate it to zero. The results are shown in Table 1.

Table 1. Optimal crystal length for different geometries of AO interaction

Type	$\boldsymbol{k}_0 \uparrow\uparrow \boldsymbol{K}$	$\boldsymbol{k}_0 \uparrow\downarrow \boldsymbol{K}$
single transit of sound	$L^{\text{opt}} = \infty$	$L^{\text{opt}} = \dfrac{2}{\alpha_s - 2\alpha} \ln\left(\dfrac{\alpha_s}{2\alpha}\right)$
double transit of sound	$L^{\text{opt}} = \dfrac{2}{\alpha_s + 2\alpha} \ln\left(\dfrac{2\alpha}{\alpha_s} + 2\right)$	$L^{\text{opt}} = \dfrac{2}{\alpha_s - 2\alpha} \ln\left(\dfrac{2\alpha_s}{\alpha_s + 2\alpha}\right)$

These simple relations can be used to estimate the optimal length of AO interaction even if the coupling parameter q is unknown. Note that for the backward collinear AO diffraction, in germanium the acoustic frequency $F = 2nV/\lambda =$ 300 MHz at $n = 4$, $\lambda = 130\ \mu$m and $V = 4920$ m/s, as shown by Bond et al. (1964), where V is the velocity of a longitudinal acoustic wave propagating along [100] axis. The [100] geometry is optimal since the AO figure of merit reaches its maximal value $100 \cdot 10^{-15}$ s^3/kg resulting in the strongest AO interaction. The value of the absorption coefficient of germanium is equal $\alpha = 0.75$ 1/cm, as shown by Voloshinov et al. (2013), while Mason and Bateman (1964) gave the value 2.8 dB/cm of the sound attenuation for the longitudinal acoustic wave propagating in [100] direction at the frequency 300 MHz corresponding to $\alpha_s = 0.64$ 1/cm. Since the refractive index of Ge equals to $n = 4$, only double transit sound geometry can be used. The calculated optimal length is about $L^{\text{opt}} = 1.4$ cm. As the divergence of the electromagnetic radiation in the THz range is 100 times stronger than that in the optical range, wide acoustic beams with cross-section of about 5 mm × 5 mm are needed. If the acoustic power equals to 1 W, the computation gives the following values of the intensities of the diffracted radiation $I_{-1} = 2.5 \cdot 10^{-5}$ and $I_1 = 2.2 \cdot 10^{-5}$.

Preliminary experiments carried out in Novosibirsk by free-electron laser has shown that the diffracted radiation was not registered due to high noise level. To obtain good signal to noise ratio, it is necessary to use acoustic beams with small cross-section 3 mm × 3 mm and power 5 W. The investigation of the backward-collinear interaction is in progress and it will be the first observation of Bragg reflection by means of bulk acoustic waves.

Conclusion

The mathematical model of AO interaction in the medium taking into account absorption of electromagnetic radiation and attenuation of acoustic waves is proposed. We showed that the optimal length of AO interaction depends only on the above mentioned parameters. Finally, we predict that the backward collinear AO diffraction has a few intrinsic features of interest to experts in fundamental and applied science.

Acknowledgements

The research was supported by the grant N 14-12-00380 of the Russian Scientific Fund.

References

Bond, W.L., Mason, W.P., McSkimin, H.J., Olsen, K.M. and Teal, G.K., 1950. The elastic constants of germanium single crystals. Physical Review 78, 176.

Brillouin, L., 1922. Diffusion of light and X-rays by a transparent homogeneous body. Annales de Physique 17, 88.

Durr,W.,Schmidt,W., 1985. Measurement of acousto-optic interaction in germanium in the far infrared. International Journal of Infrared Millimeter Waves 6,1043.

Mason, W.P., Bateman, T.B., 1964. Ultrasonic-wave propagation in pure silicon and germanium. Journal of Acoustical Society of America 36, 644.

Potter, R.F., 1997. Germanium (Ge) in *Handbook of optical constants of solids*. In: Palik, E.D. (Ed.). Academic Press, New York, V. 1, pp. 465-478.

Vogel, T., Dodel, G., 1985. Acousto-optic modulation in the far-infrared. Infrared Physics 25, 315.

Voloshinov, V.B., Nikitin, P.A., Gerasimov, V.V., Knyazev, B.A., Choporova, Yu.Yu., 2013. Deflection of a monochromatic THz beam by acousto-optic methods. Quantum Electronics 43, 1139.